Universal laws and architectures:

Theory and lessons from

brains, hearts, cells, grids, nets, bugs,
fluids, bodies, planes, docs, fire, fashion,
earthquakes, music, buildings, cities, art, running,
cycling, throwing, Synesthesia, spacecraft, statistical mechanics

and zombies

John Doyle 道耀
Jean-Lou Chameau Professor
Control and Dynamical Systems, EE, & BioE
Ca#1tech

https://www.cds.caltech.edu/~doyle

Universal laws and architectures:

The videos

The following preview is approved for all audiences. ~~and zombies~~

https://rigorandrelevance.wordpress.com/author/doyleatcaltech/

https://www.cds.caltech.edu/~doyle

- **SDN, IOT, IOE, CPS, etc..**
- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- Weapons
- Bipedalism
- Maternal care
- Warm blood
- Flight
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis

Major transitions

- **SDN, IOT, IOE, CPS, etc..**
- Compute/control technology
- Industrialization
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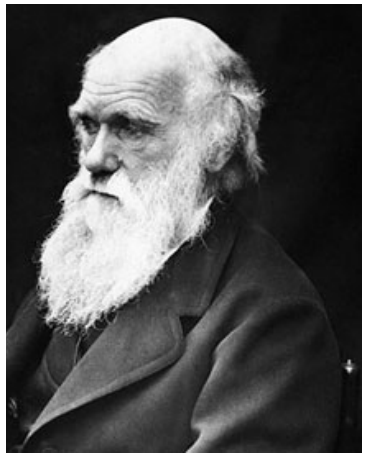
- Oxygen
- Translation (ribosomes)
- Glycolysis

Major transitions

Theory of

- Evolution
- Architecture
- Complexity
- Networks



Evolution

Our heroes

Complexity
Architecture

Compute

Comms for Comp/Cntrl/Bio

Theory of

- Evolution
- Architecture
- Complexity
- Networks

Optimization

Statistics

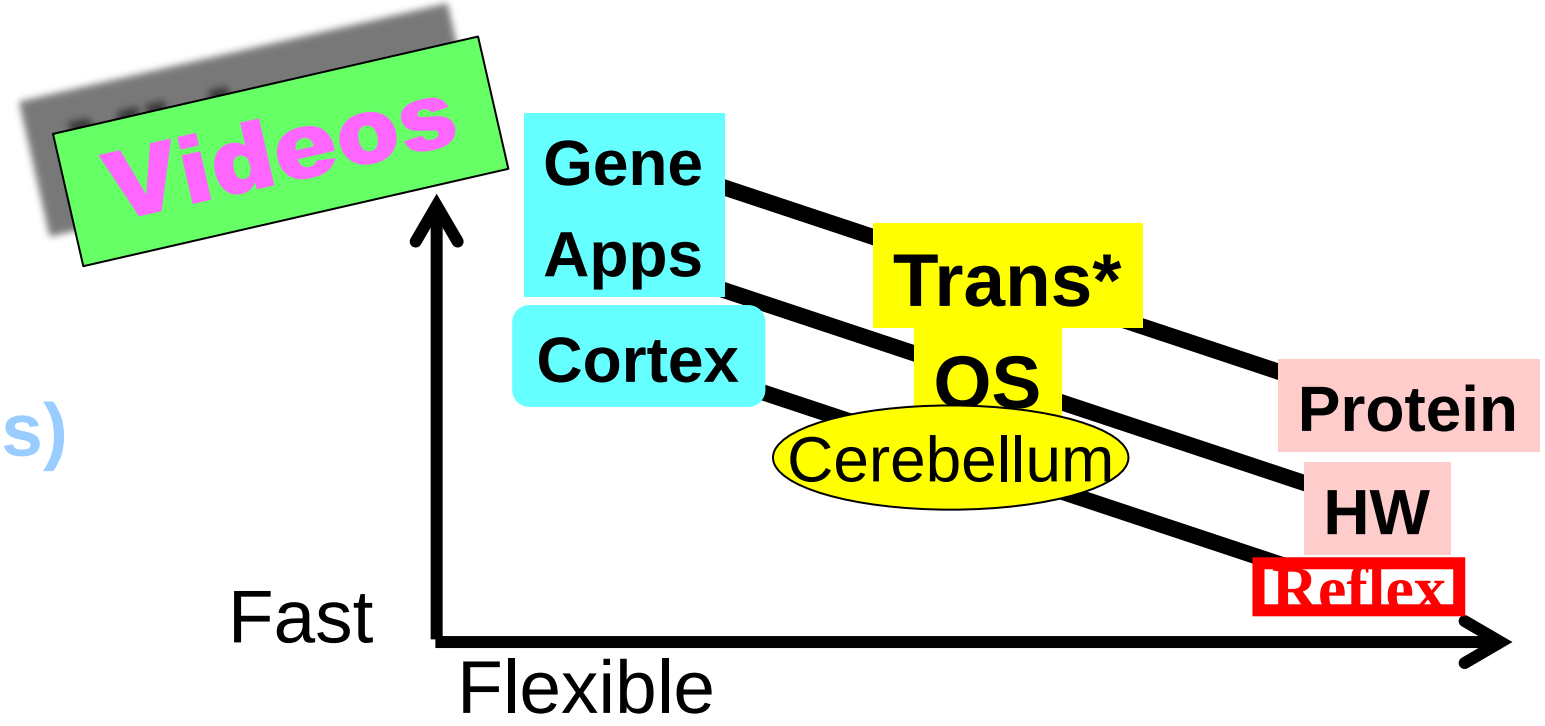
Control, OR

Orthophysics (Eng/Bio/Math)

- SDN, NFV, IOT, IOE, CPS
- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- Weapons
- Bipedalism (balance, cardiovascular physiology)
- Maternal care
- Warm blood
- Flight (turbulence)
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis
- Cells

Theory of

- Evolution
- Architecture
- Complexity
- Networks



- SDN, IOT, IOE, CPS, etc.
- Compute/control technology
- Industrialization
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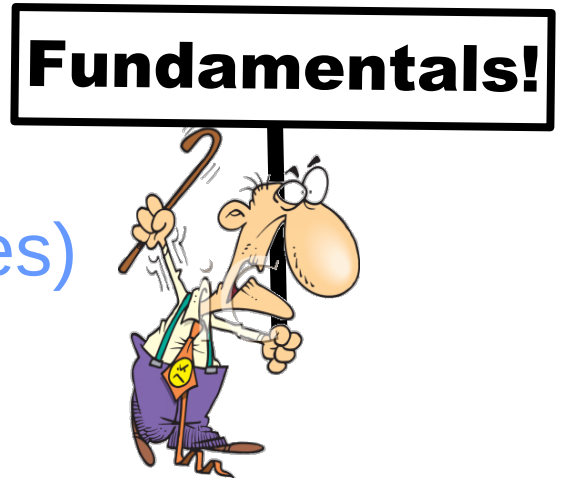
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis

Theorems+

∩{case study}



- Brains
- **Bugs (microbes, ants)**
- **Nets/Grids (cyberphys)**
- **Medical physiology**

- Lots of aerospace
- Wildfire ecology
- Earthquakes
- Physics:
  - **turbulence**,
  - stat mech (QM?)
- "Toy":
  - Lego
  - clothing, fashion
- Buildings, cities
- Synesthesia

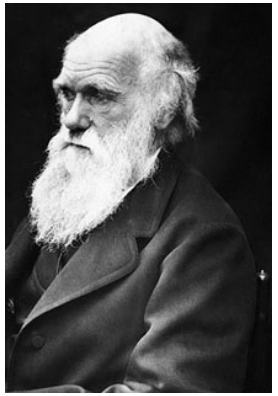
- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- Weapons
- Bipedalism/balance

- Brains
  - **Familiar, accessible**
  - **Live demos!**
  - **Cheap, reproducible**
  - **Open**
  - **Unavoidable**

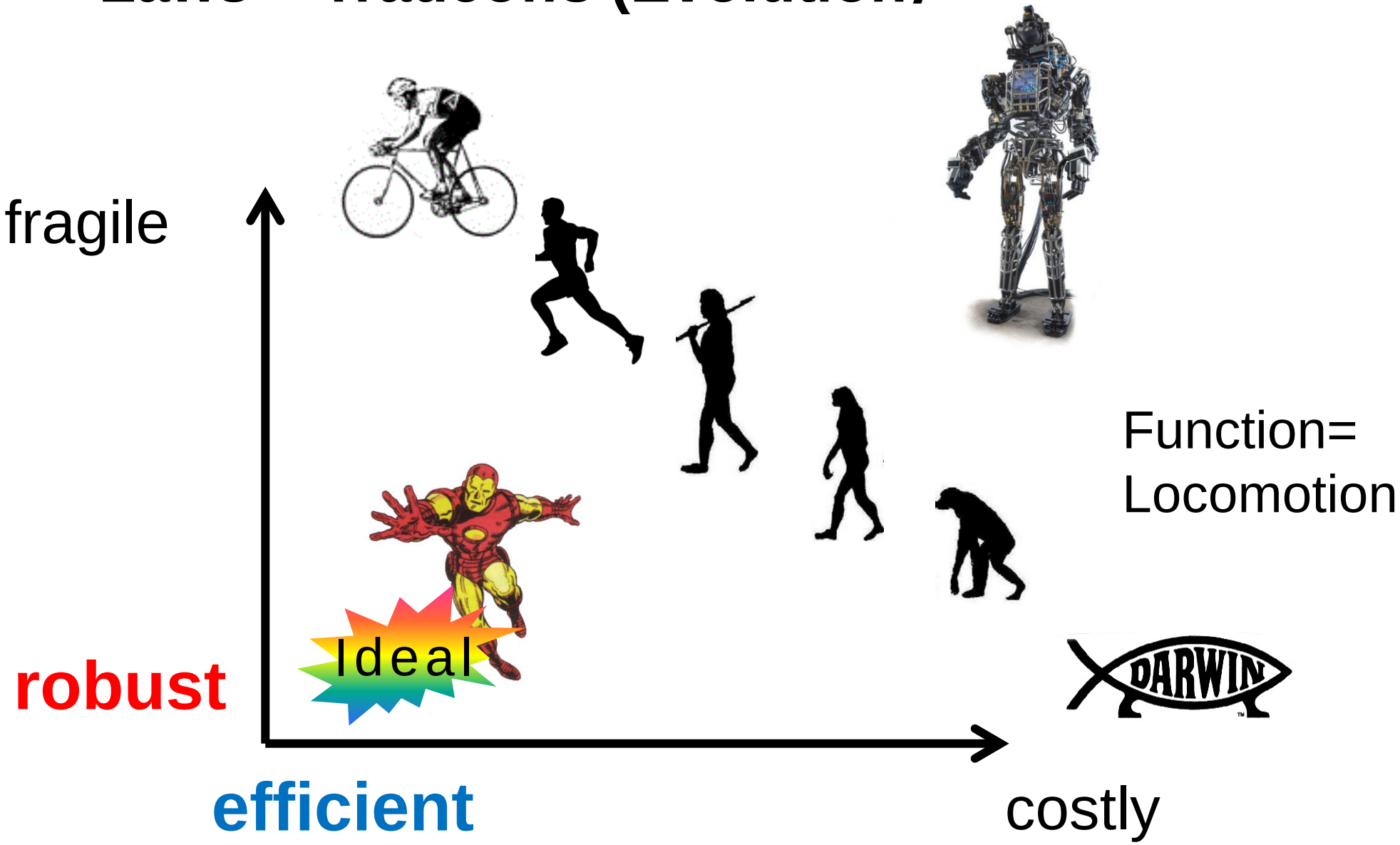
Theorems+

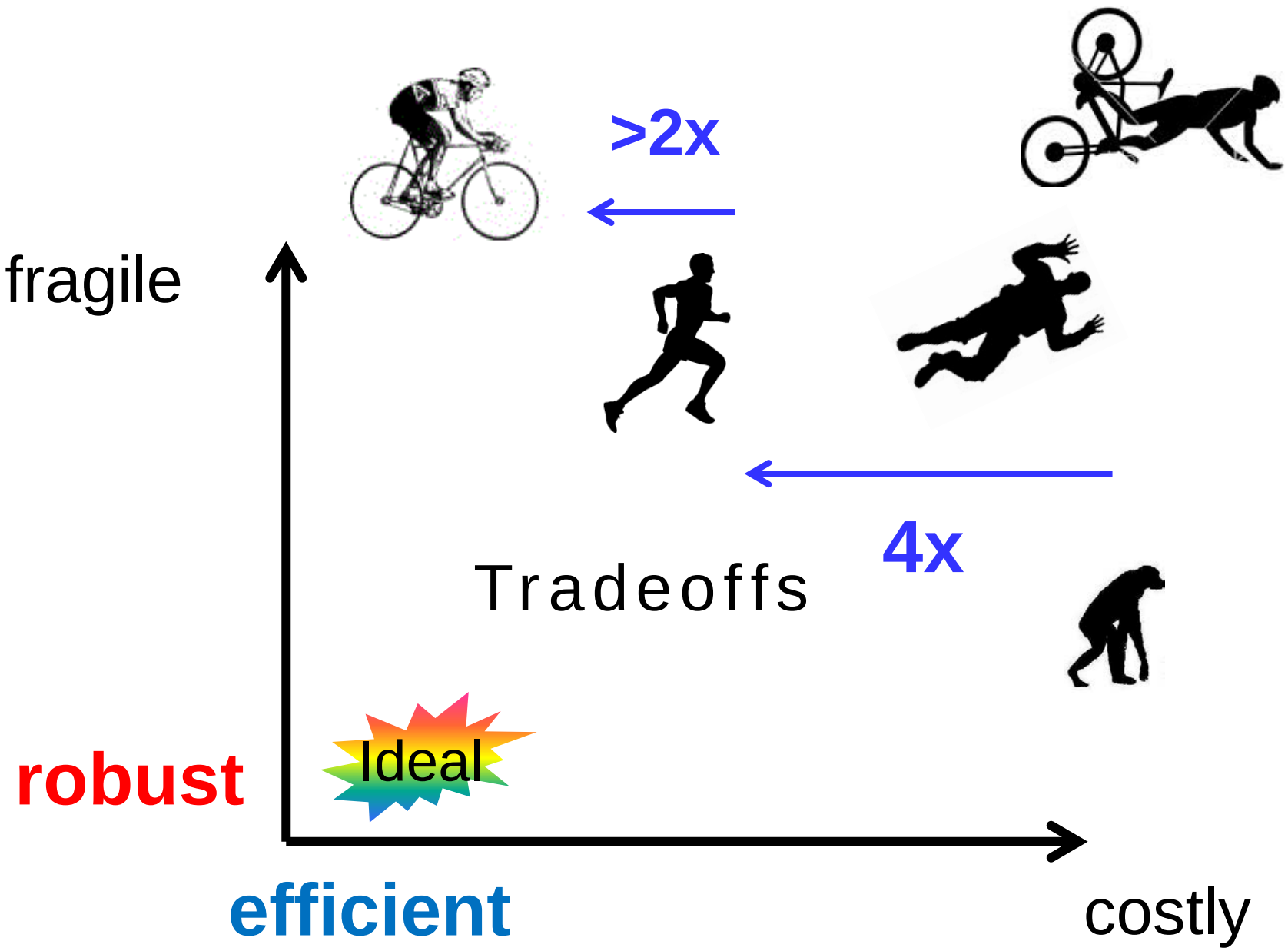
∩{case study}

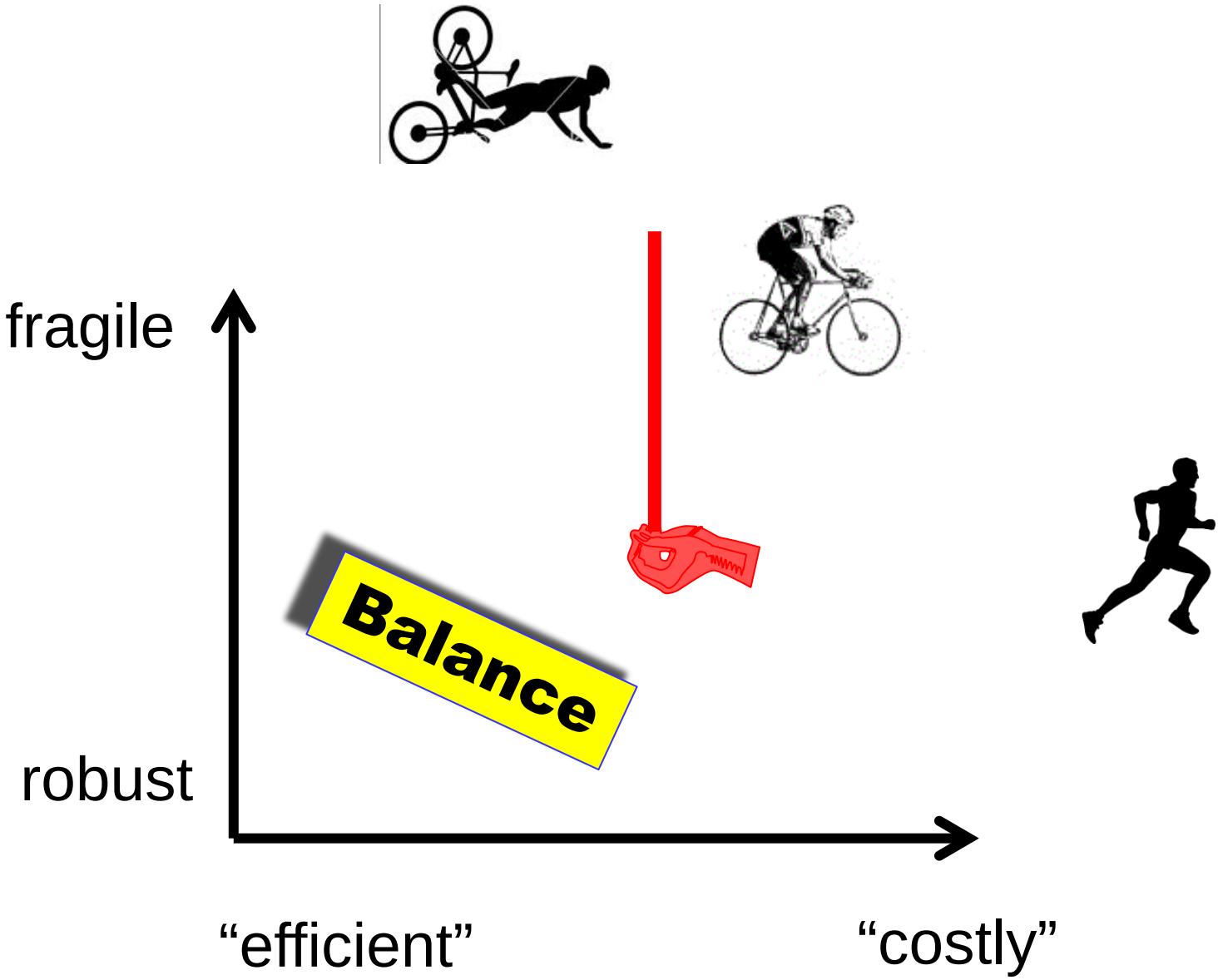
Brains/balance



Laws = Tradeoffs (Evolution)




>2x
fragile
4x
Tradeoffs
Ideal
robust
efficient
costly

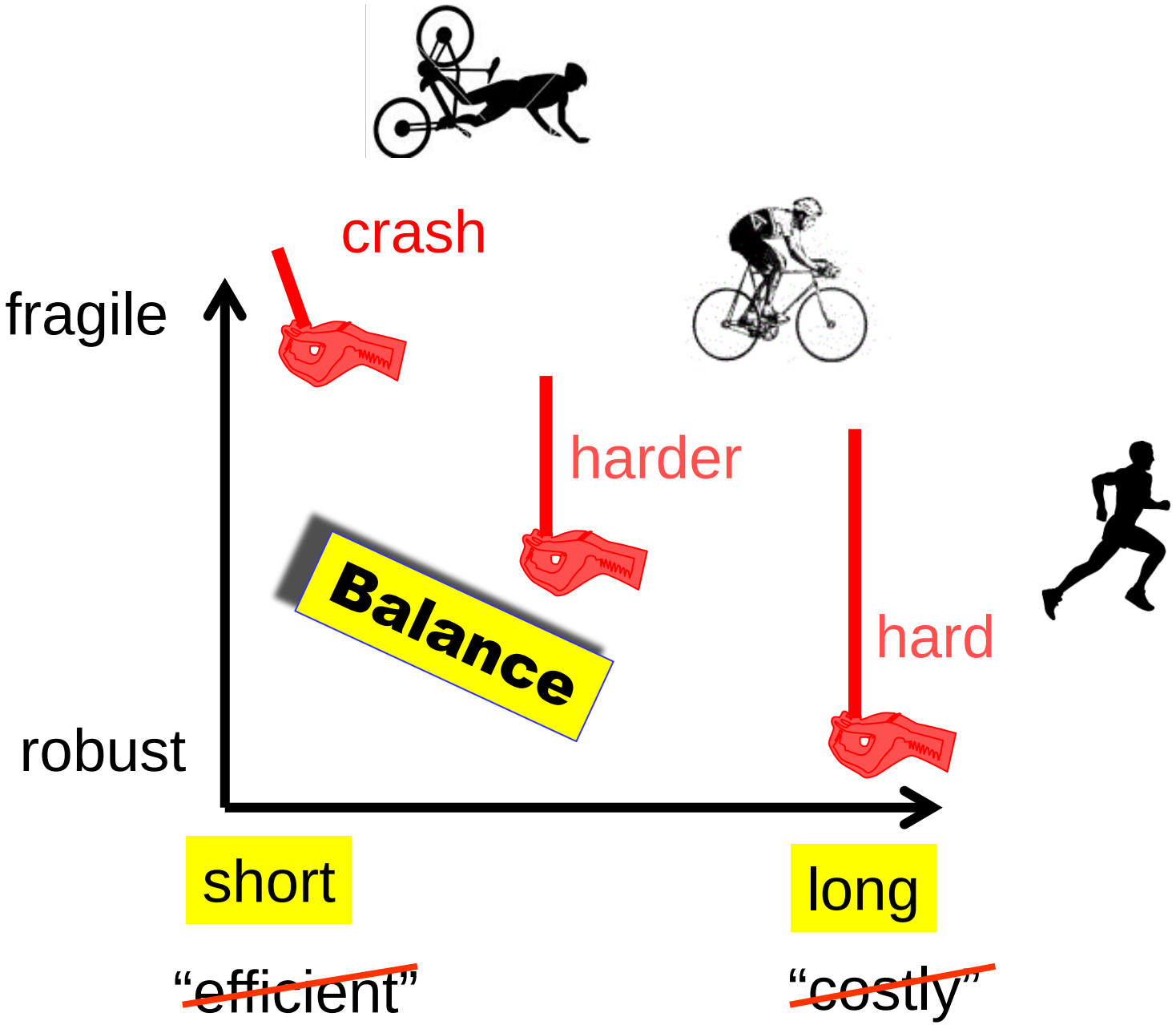


Similar

- Tradeoffs
- Laws
- Mechanisms

But ***simpler***

- Models?
- Experiments?



Similar

- Tradeoffs
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But ***simpler***

- Models
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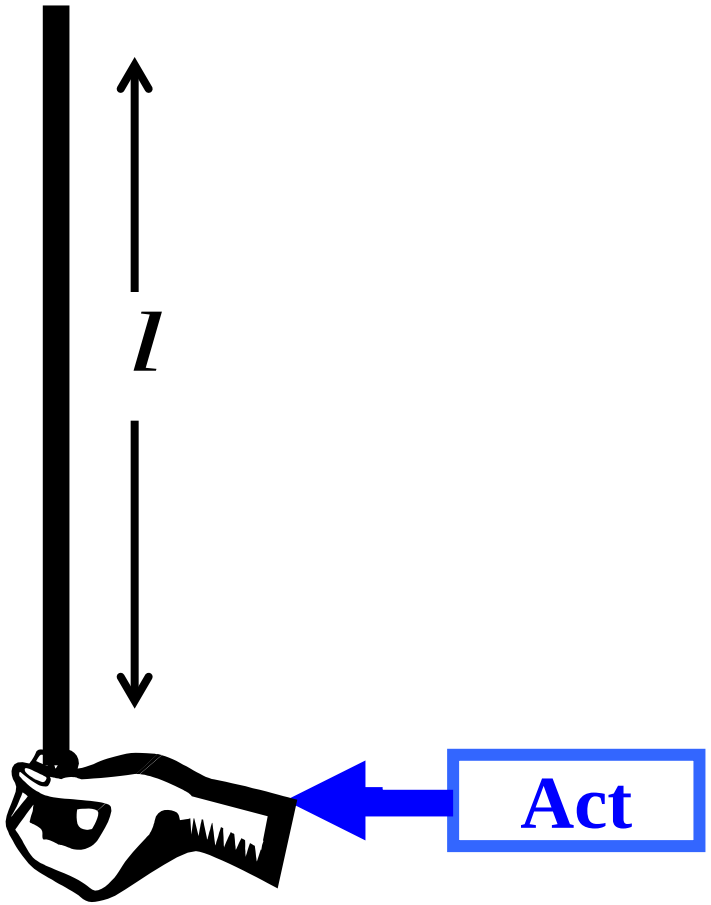
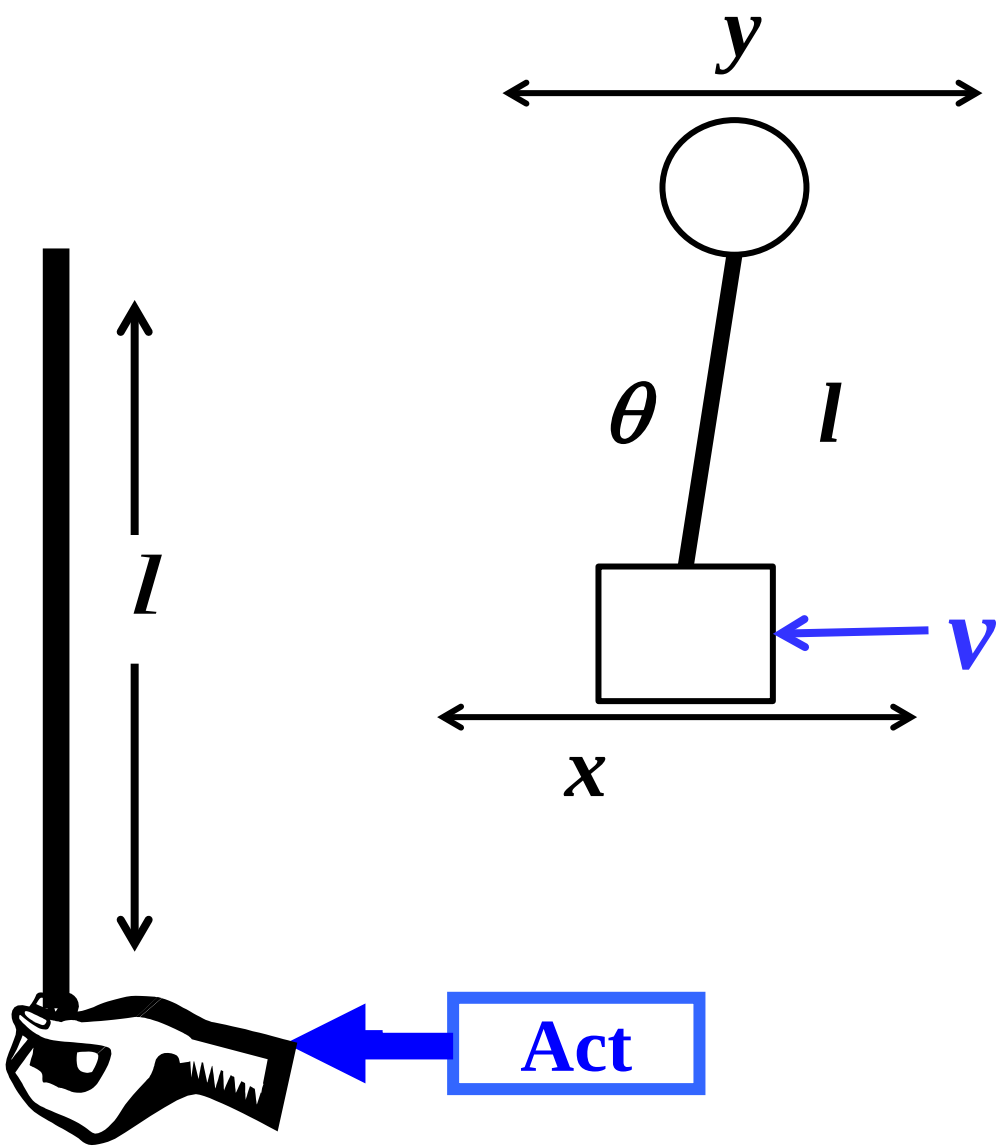
Simplified inverted pendulum



Yoke Peng Leong

Simplified inverted pendulum

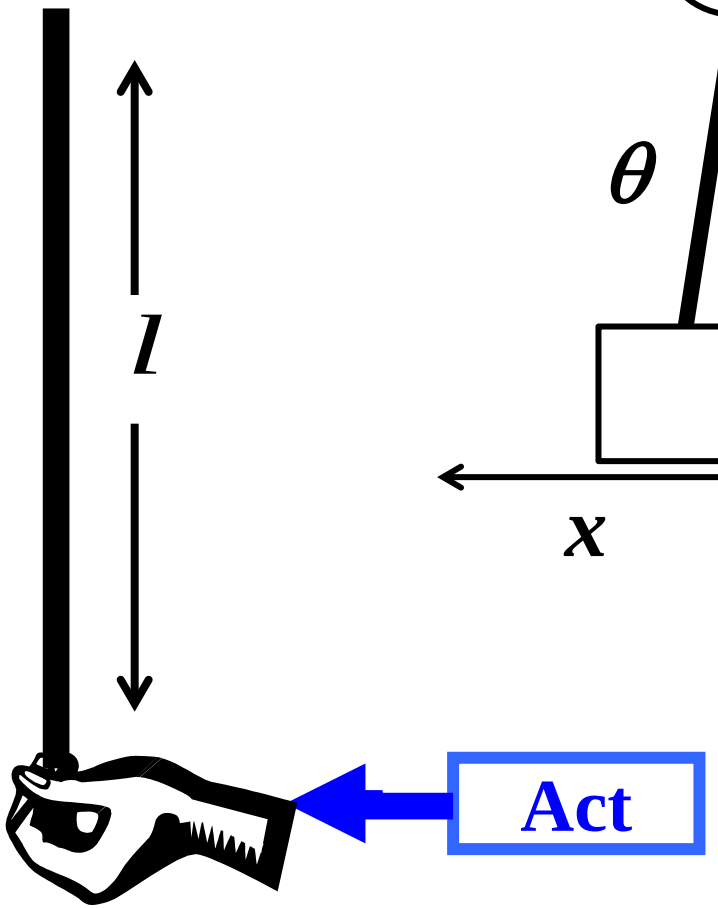


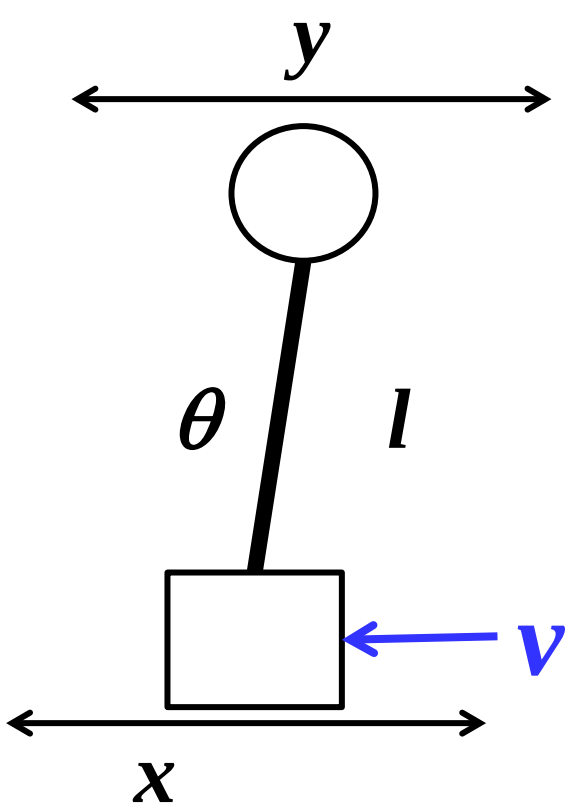
l	*length (to COM)*
g	*gravity*
v	*control (acceleration)*

Simplified inverted pendulum

Instability

$$p = \sqrt{\frac{g}{l}}$$





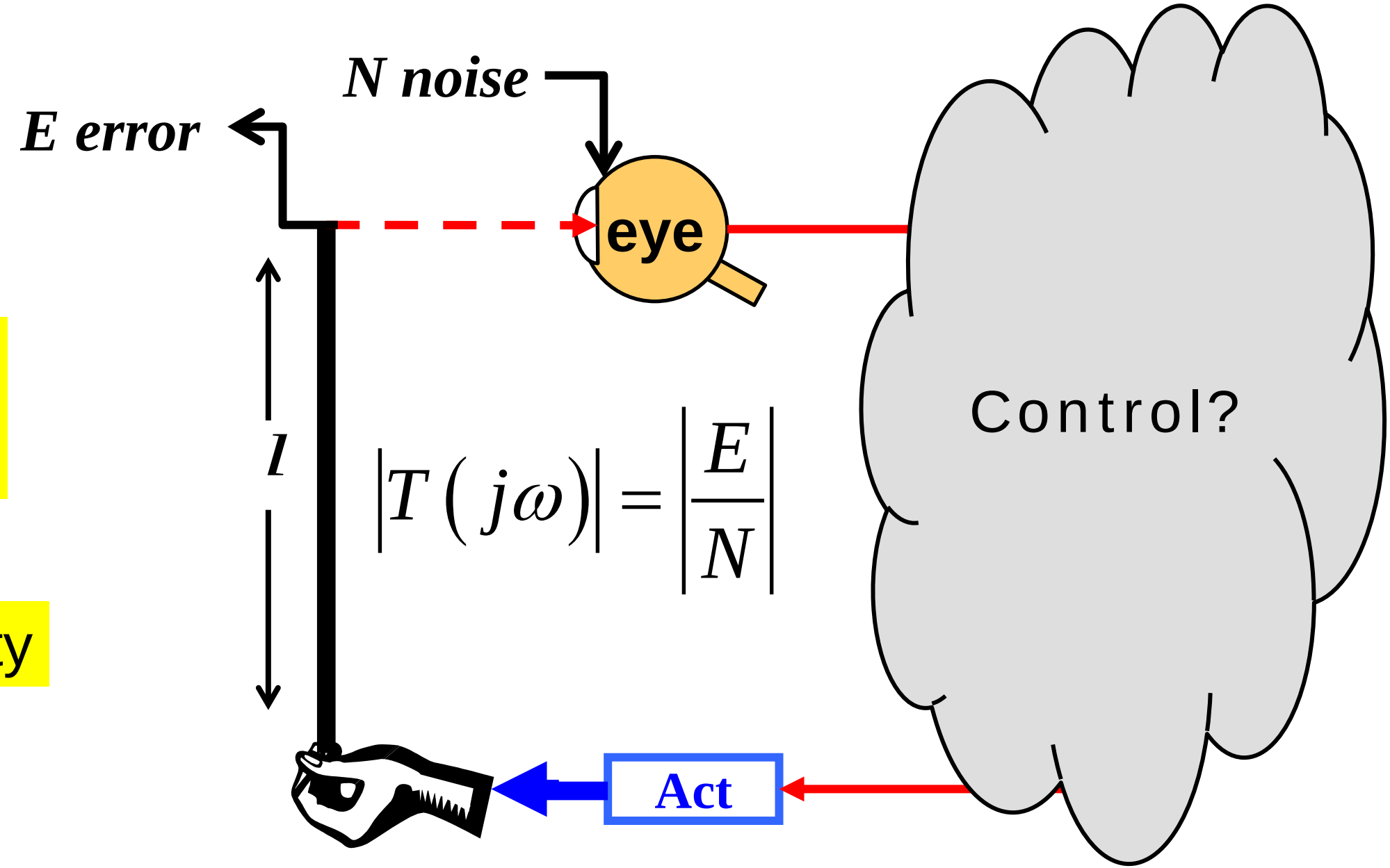
l	***length (to COM)***
g	***gravity***
v	***control (acceleration)***

$$\ddot{x} = v$$

$$l\ddot{\theta} + g\sin\theta = -v\cos\theta$$

$$y = x + l_O \sin\theta$$

Simplified sensorimotor control

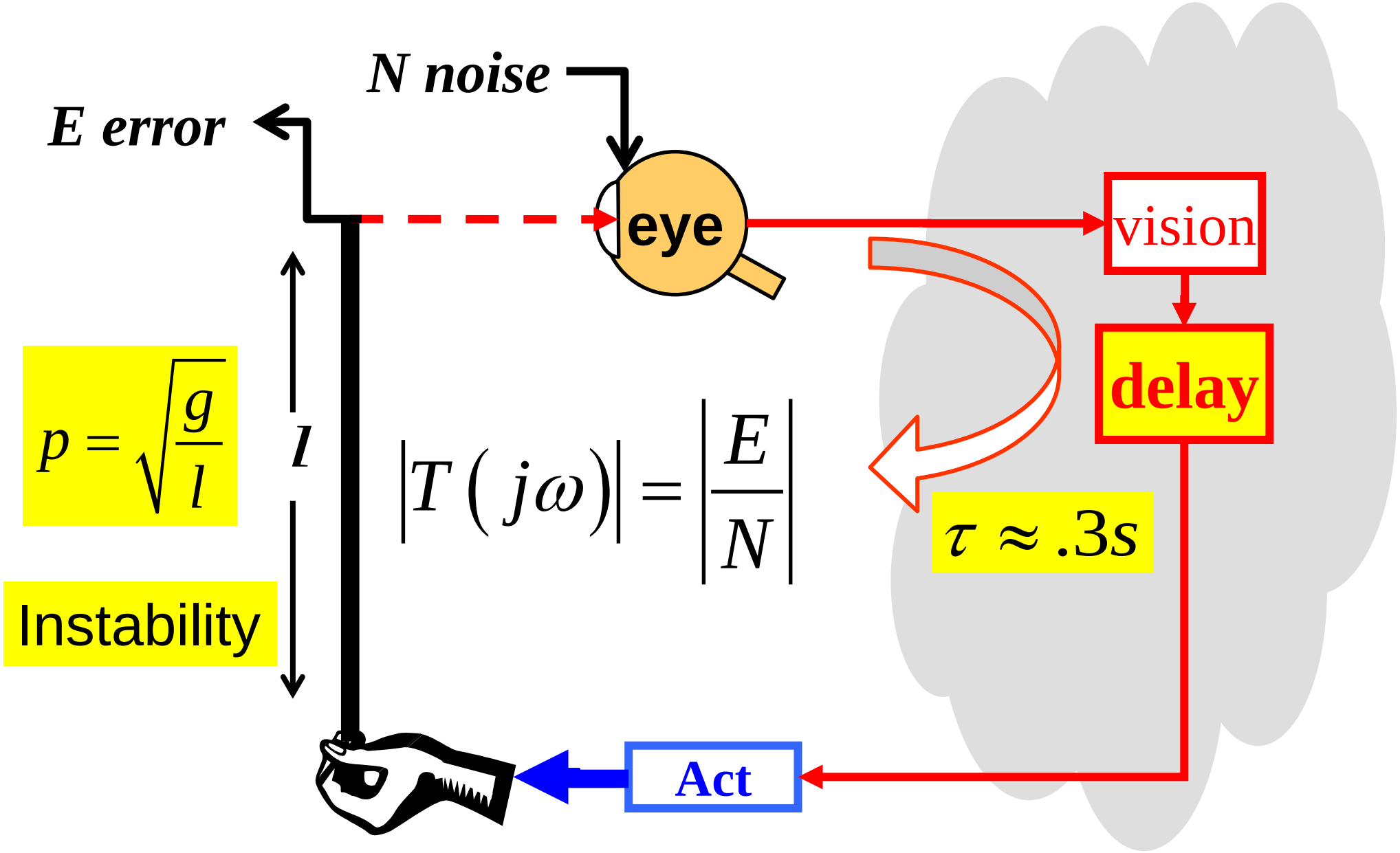


$$p = \sqrt{\frac{g}{l}}$$

$$\left|T(j\omega)\right| = \left|\frac{E}{N}\right|$$

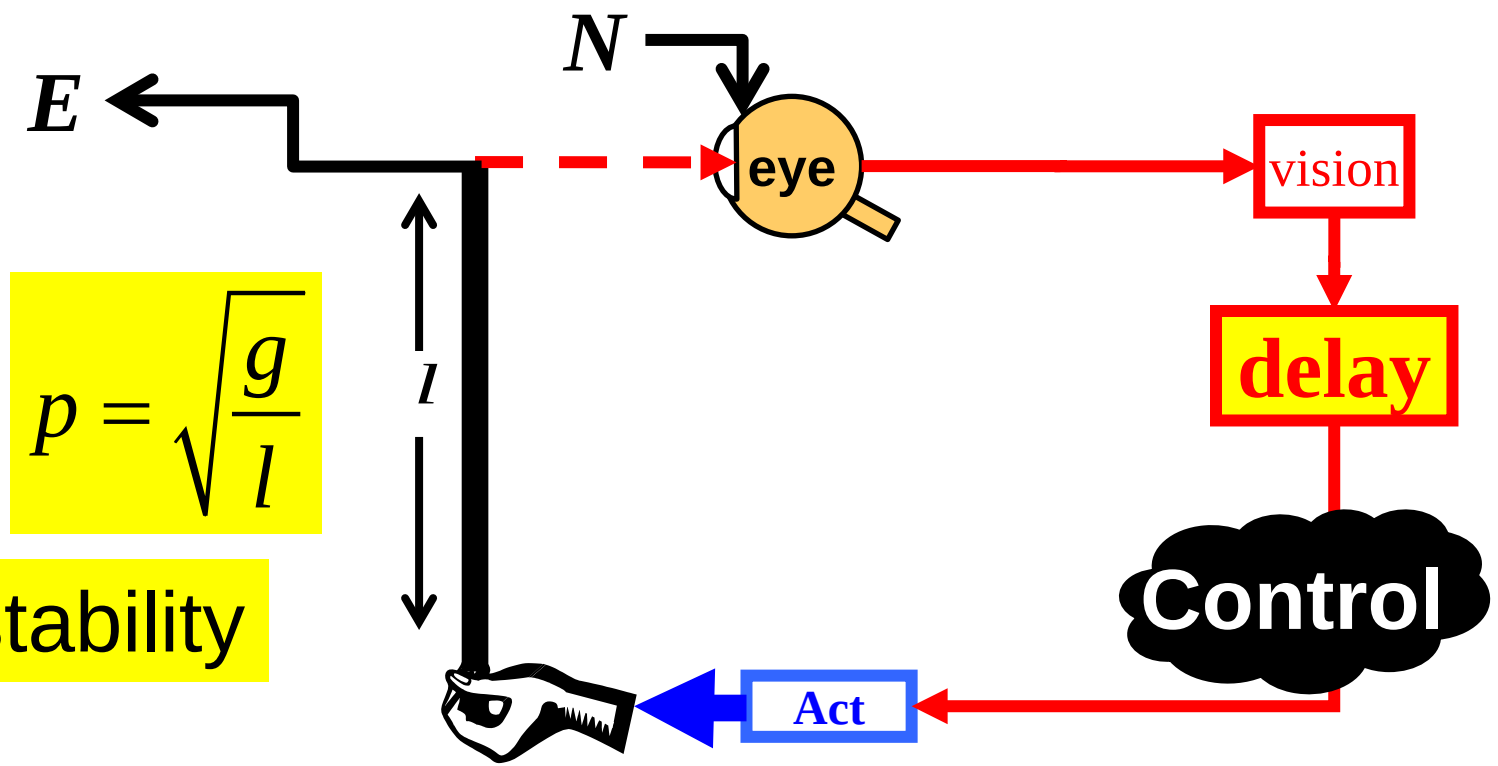
Instability

Simplified sensorimotor control



Amplification (noise to error)?

$$\left|T(j\omega)\right| = \left|\frac{E}{N}\right|$$



$$p = \sqrt{\frac{g}{l}}$$

Instability

$$\tau \approx .3s$$

Amplification (noise to error) *theorem*:

Entropy rate $\exp\left(\int \ln|T|\right)$

Energy (L2) $\|T\|_\infty$

$$\left.\begin{matrix}\exp\left(\int \ln|T|\right)\\ \|T\|_\infty\end{matrix}\right\} \geq \boxed{\exp(p\tau)}$$

$$|T(j\omega)| = \left|\frac{E}{N}\right|$$

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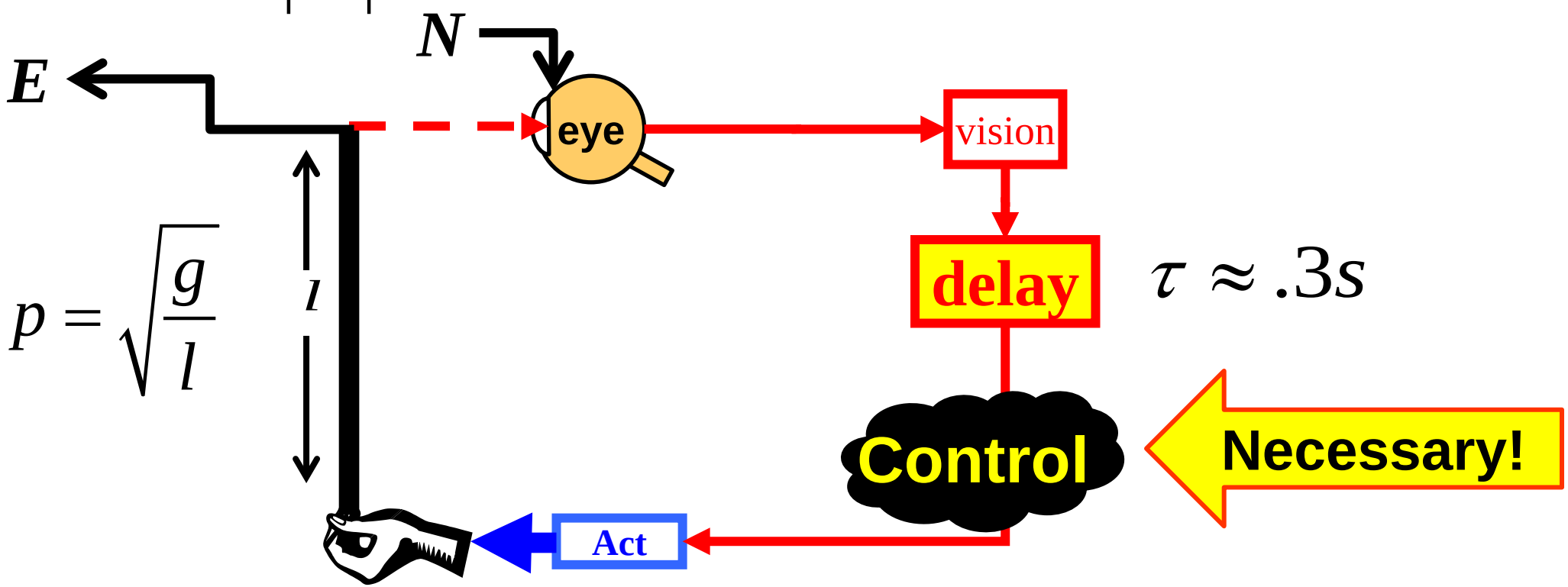

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Entropy rate $\exp\left(\int \ln|T|\right)$

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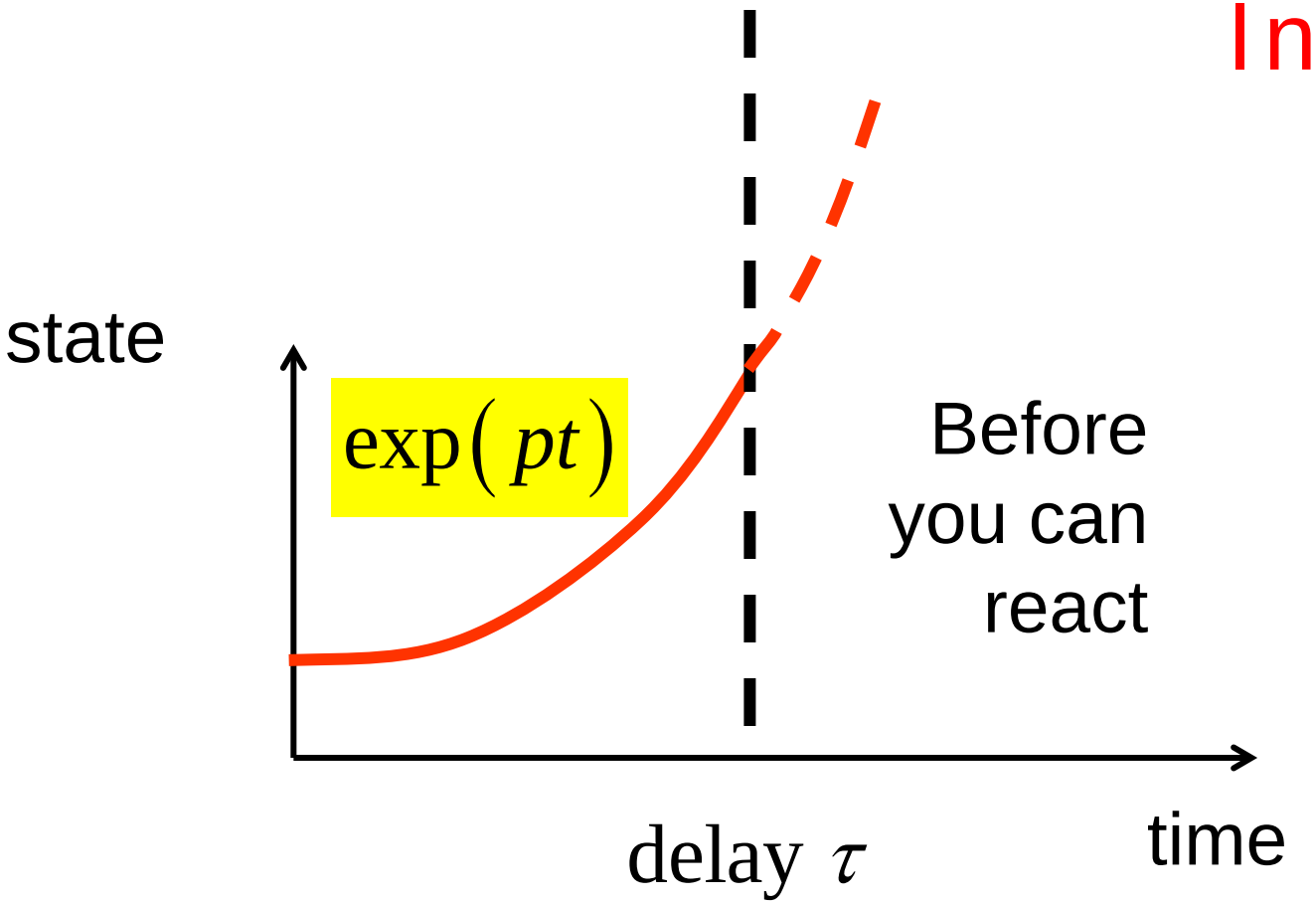
Intuition



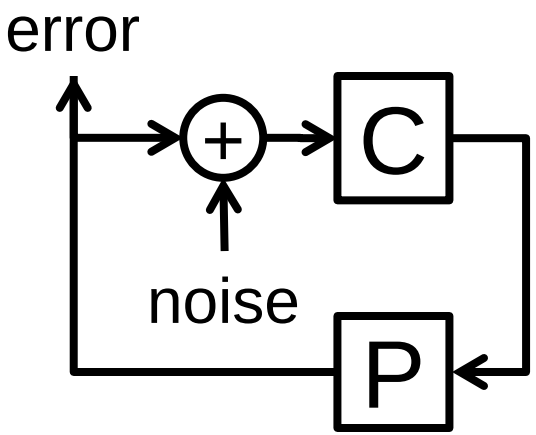
$$p \propto \sqrt{\frac{1}{l}}$$

$$\tau \approx .3s$$

Proof?

$$\|T\|_\infty = \sup_\omega |T(j\omega)| = \sup\left\{ |T(s)| \;\middle|\; \mathrm{Re}(s) \geq 0 \right\}$$

Max modulus



$$|T(j\omega)| = \left|\frac{E}{N}\right|$$

$$T(s) = M(s)\Theta(s) \quad |\Theta(j\omega)| = 1$$

$$\Theta(s) = \exp(-\tau s)$$

$$P(p) = \infty \Rightarrow T(p) = 1$$

$$\Rightarrow M(p) = \Theta(p)^{-1}$$

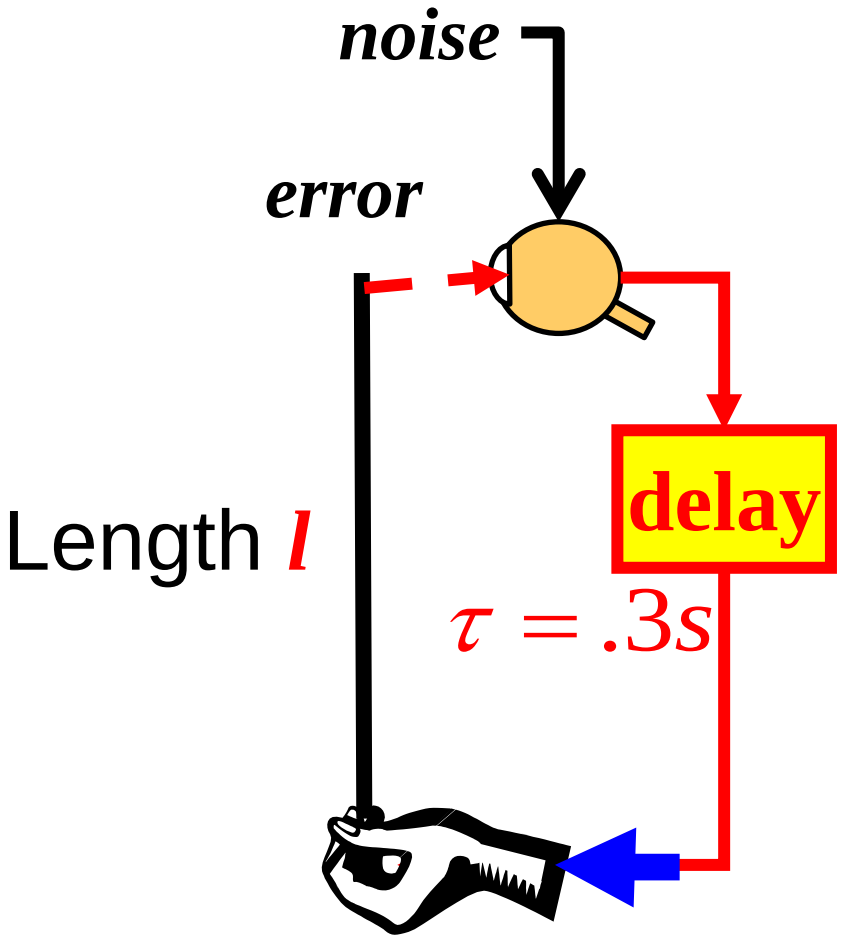
$$P(s) = P_M(s)\exp(-\tau s) \Rightarrow$$

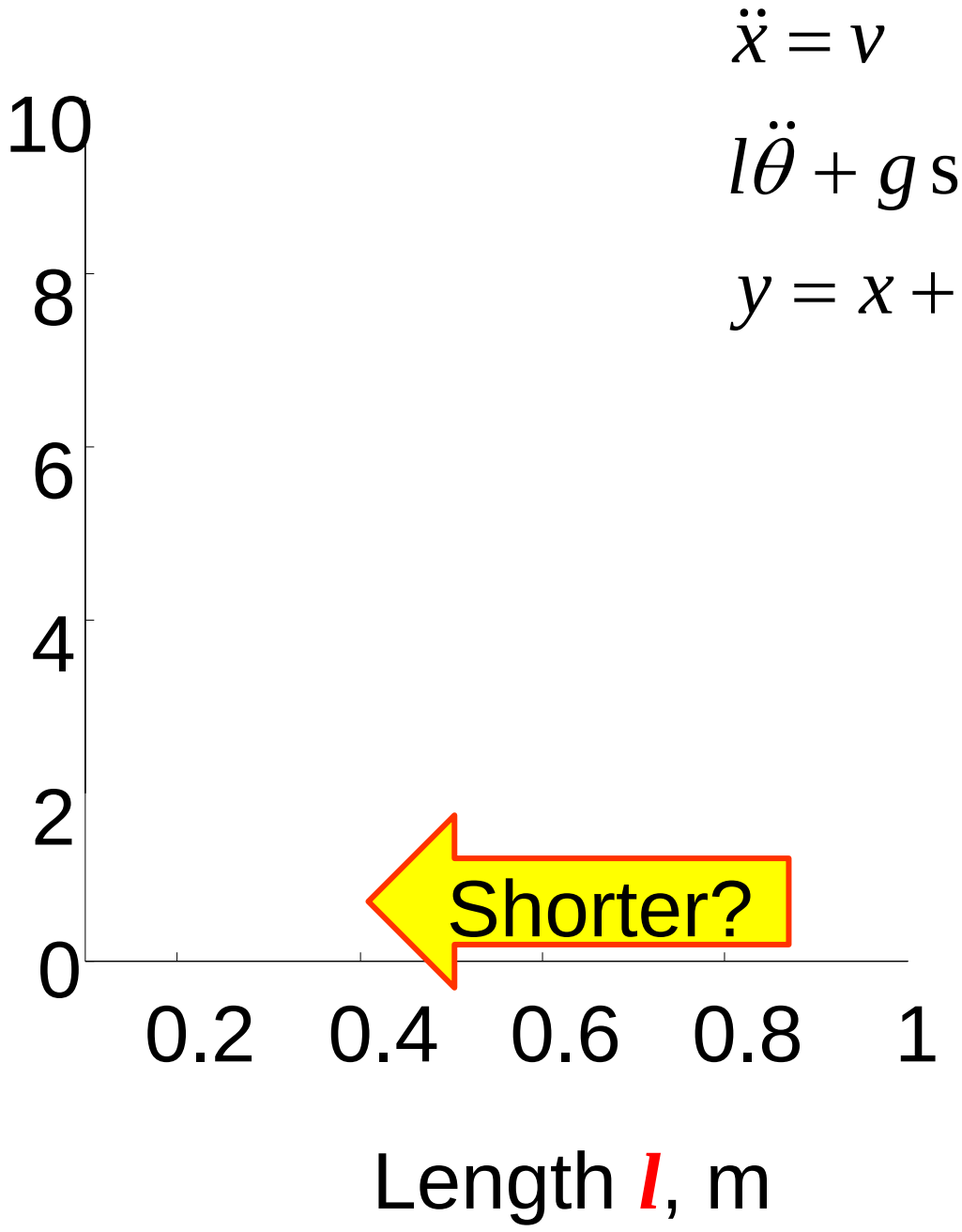
$$\|T\|_\infty = \|M\|_\infty \geq |M(p)| \geq \left|\Theta(p)^{-1}\right| \geq \exp(\tau p)$$

$$\Rightarrow \|T\|_\infty \geq \exp(\tau p)$$

Undergrad math

$$\|T\| \geq \boxed{\exp(p\tau)}$$





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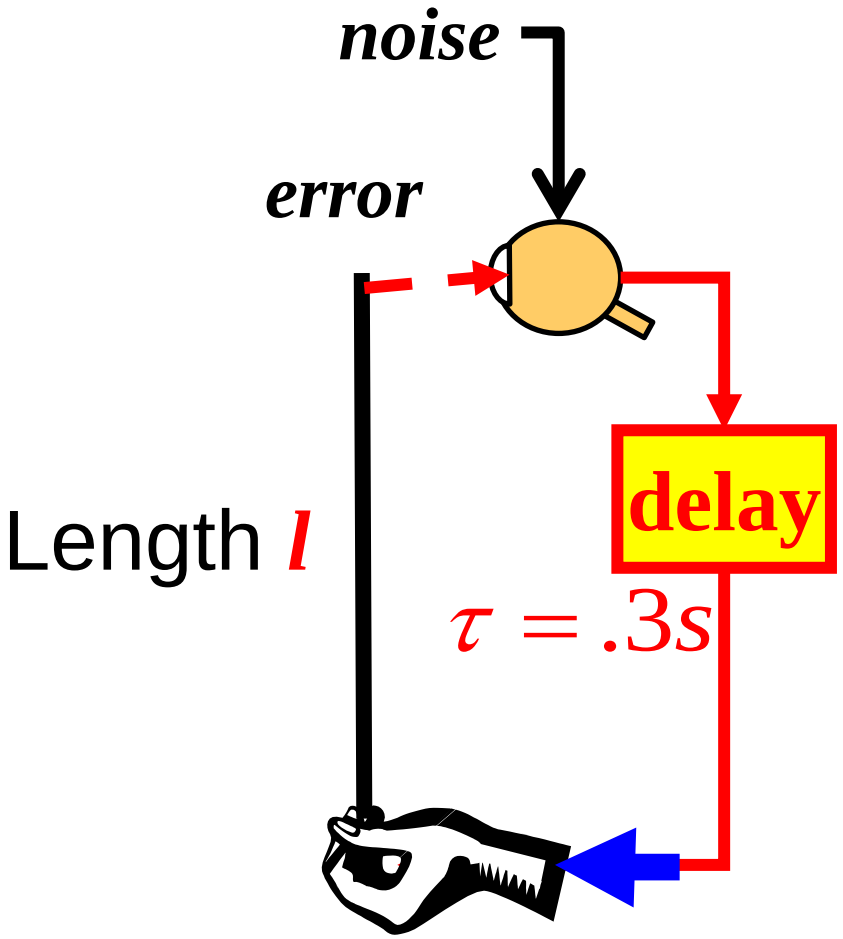

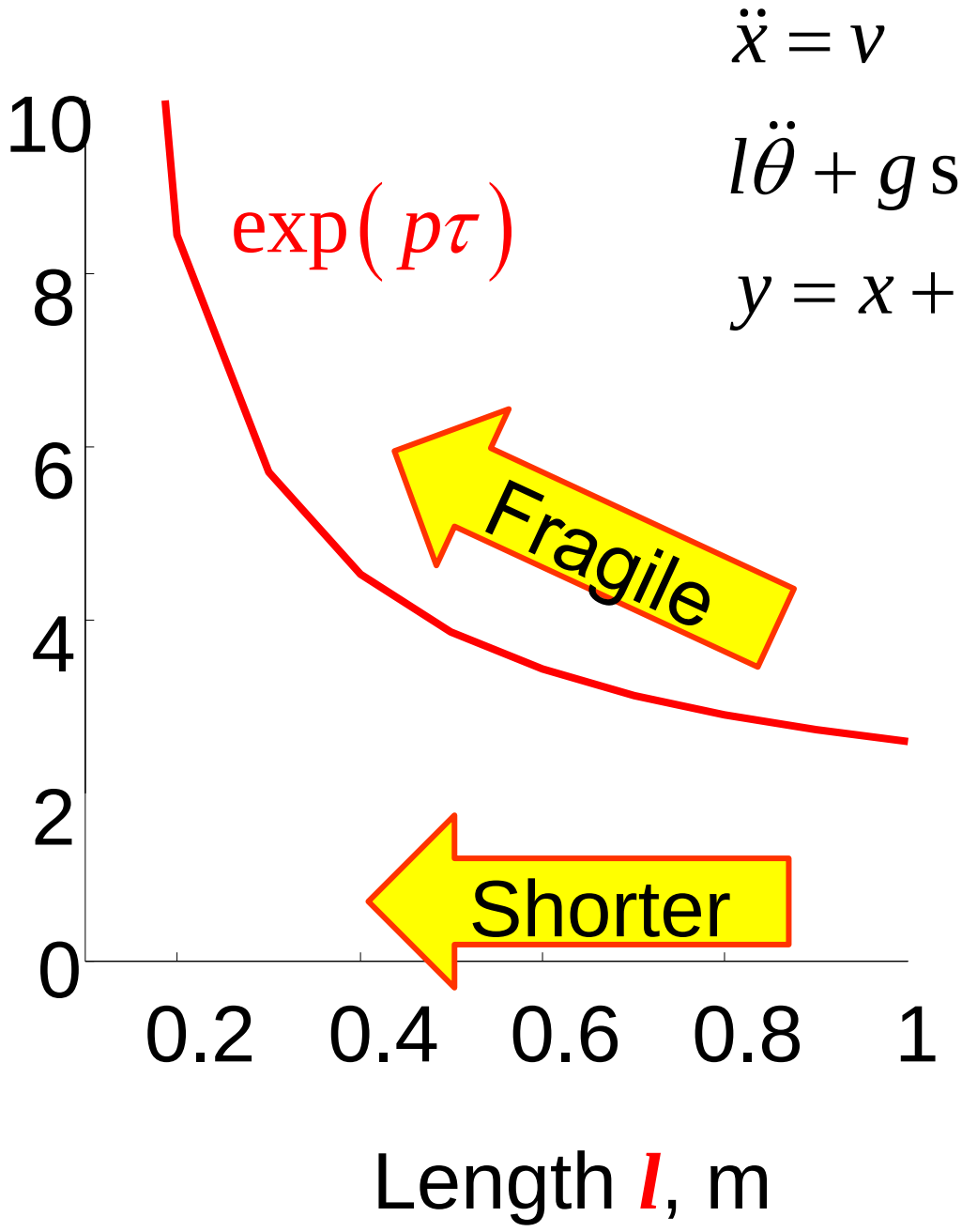

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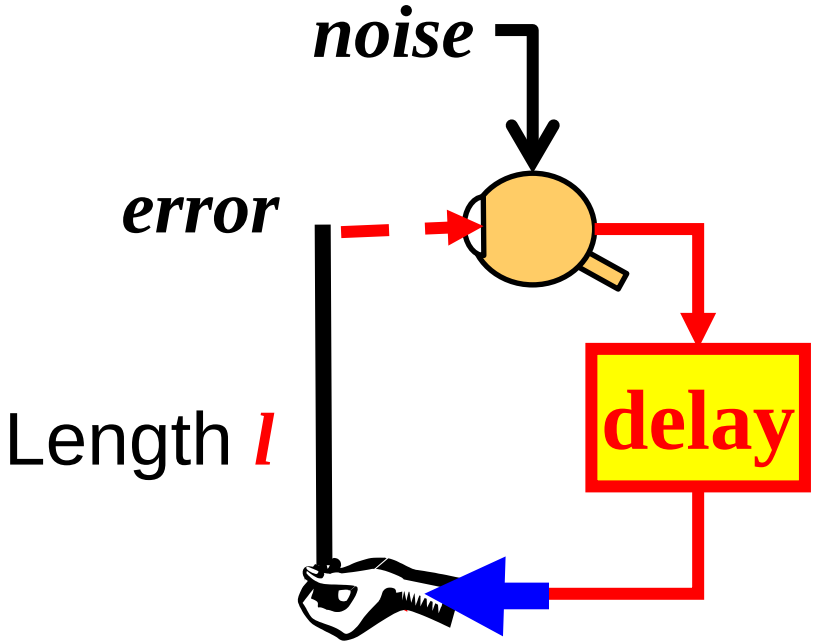
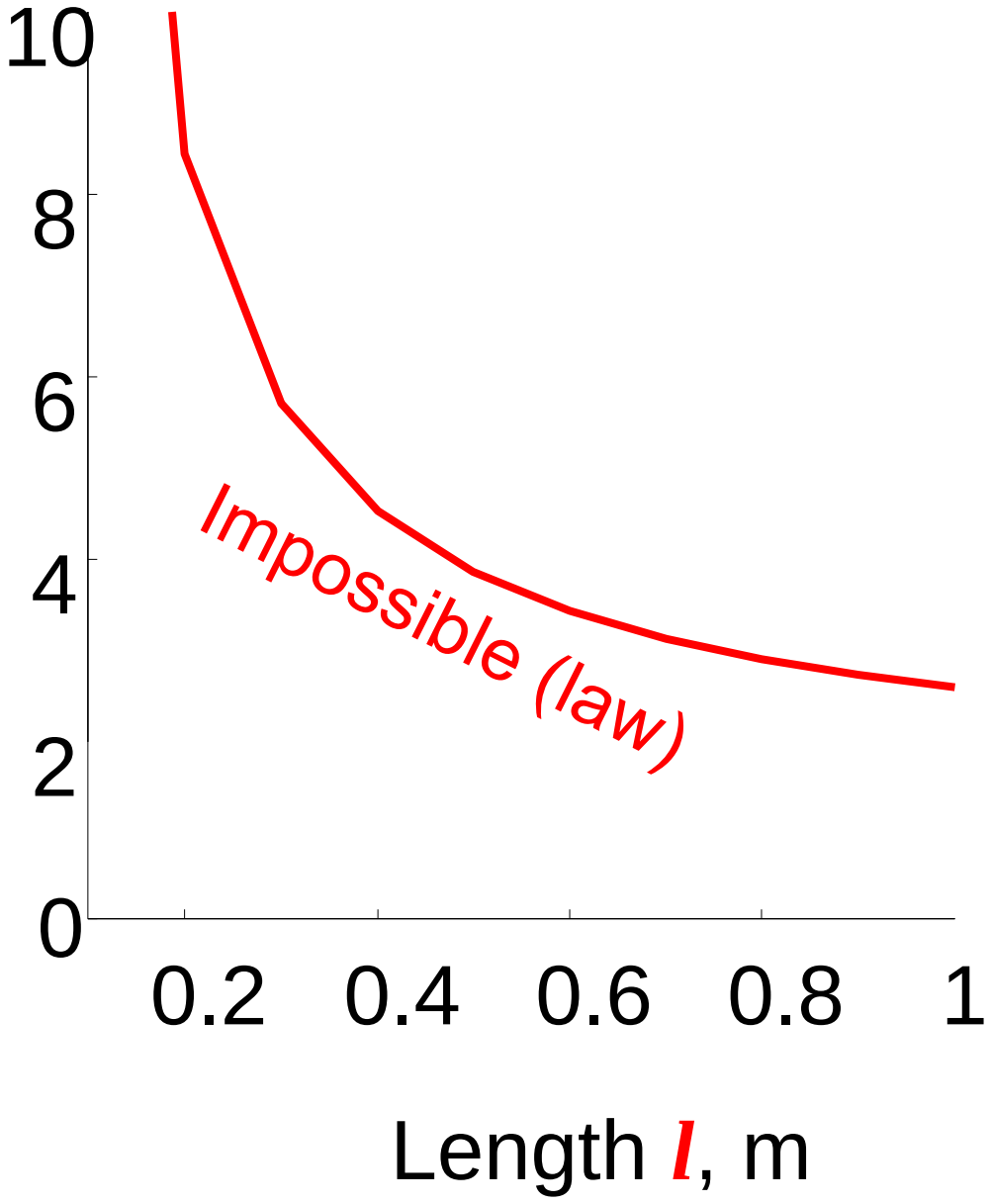
$$\left.\begin{array}{c}\exp\left(\int \ln|T|\right)\\ \|T\|_\infty\end{array}\right\} \geq \exp(p\tau)$$

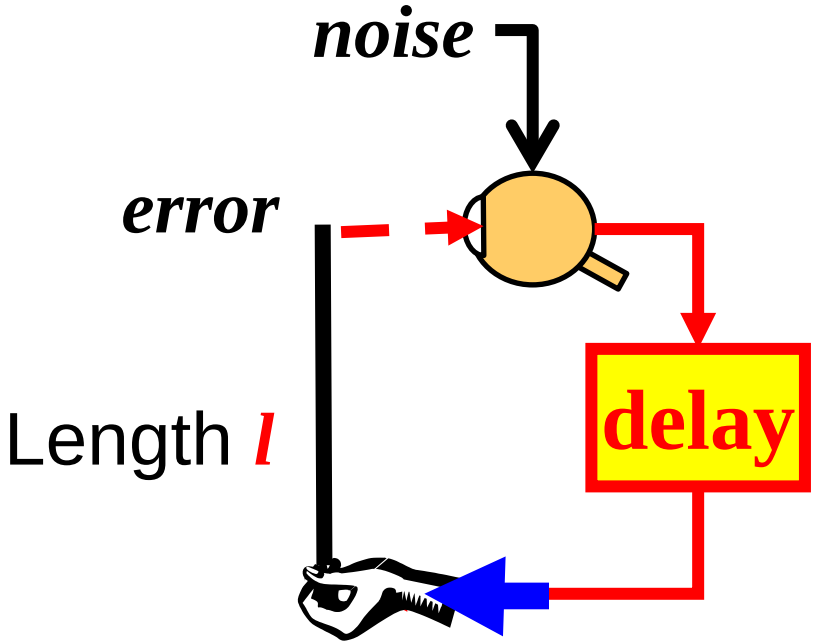
Theory

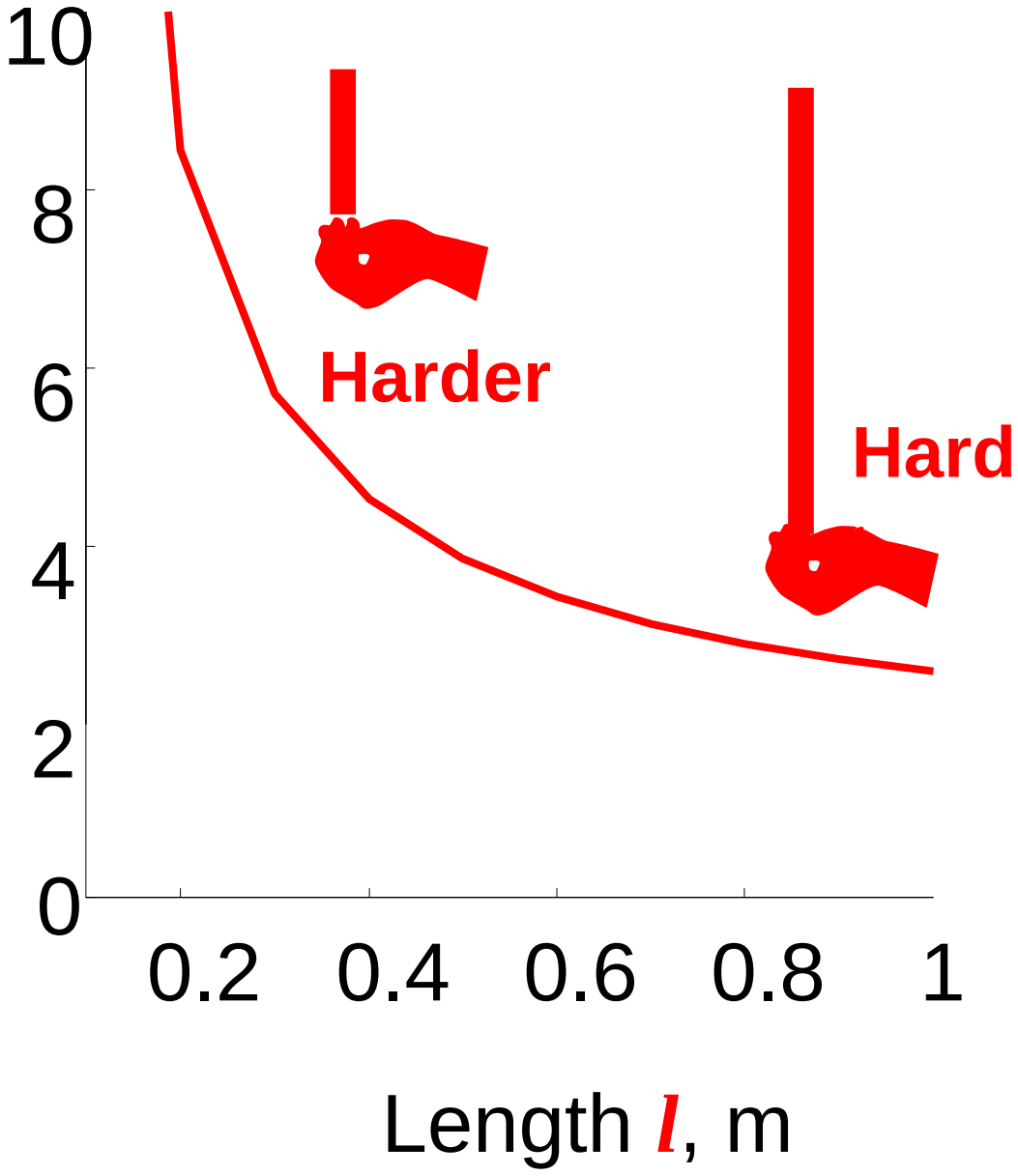




$$\left.\begin{array}{c}\exp\left(\int \ln|T|\right)\\ \|T\|_\infty\end{array}\right\} \geq \exp(p\tau)$$

Theory & Data





Robust to
"noise" model

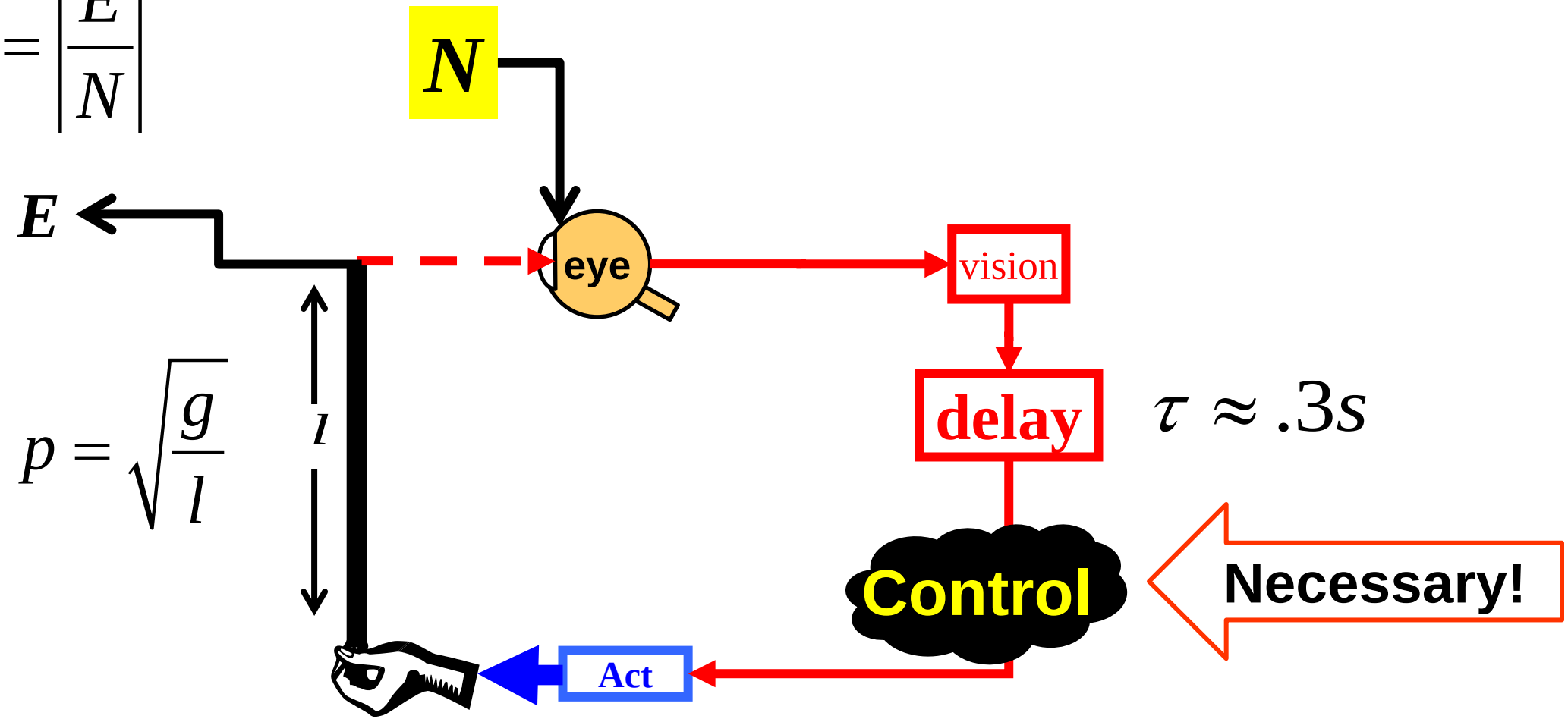
Entropy rate $\exp\left(\int \ln|T|\right)$

Energy (L2) $\|T\|_\infty$

$$\left.\begin{matrix}\exp\left(\int \ln|T|\right)\\ \|T\|_\infty\end{matrix}\right\} \geq \boxed{\exp(p\tau)}$$

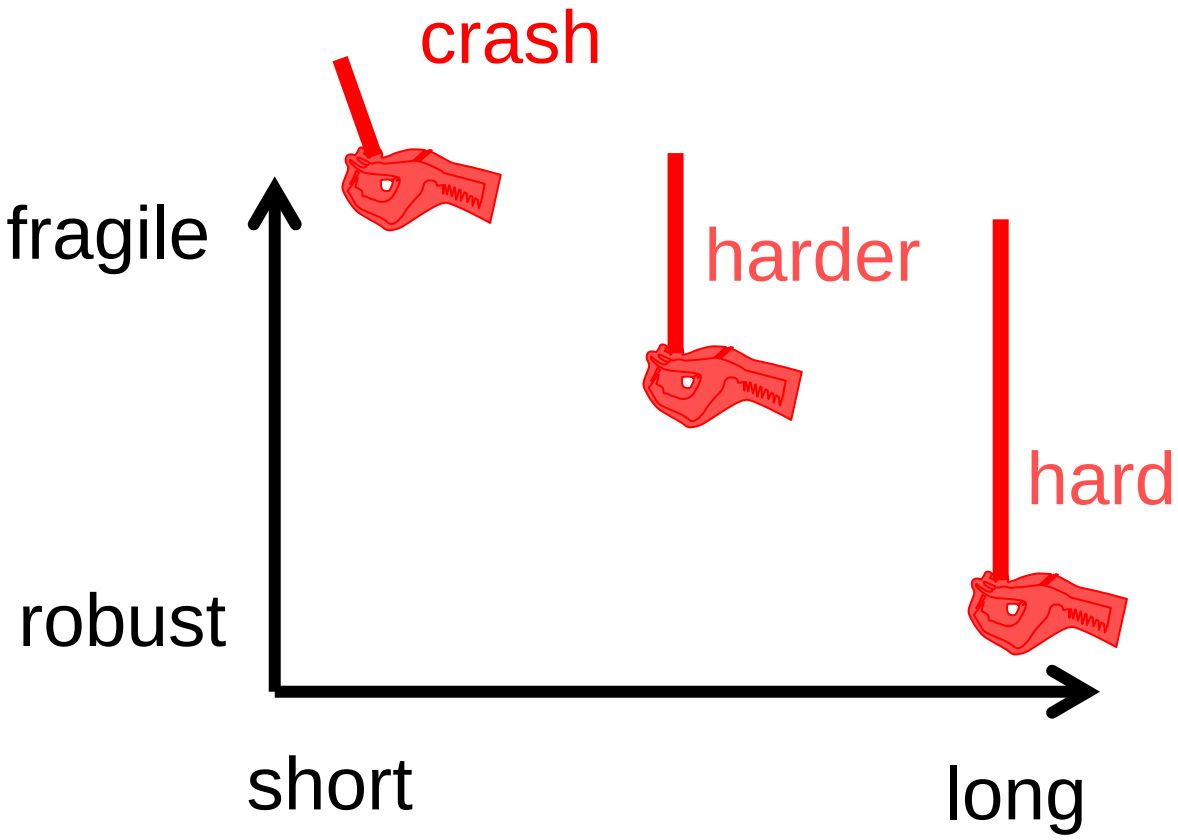
$$|T(j\omega)| = \left|\frac{E}{N}\right|$$

$$p = \sqrt{\frac{g}{l}}$$



$$\tau \approx .3s$$

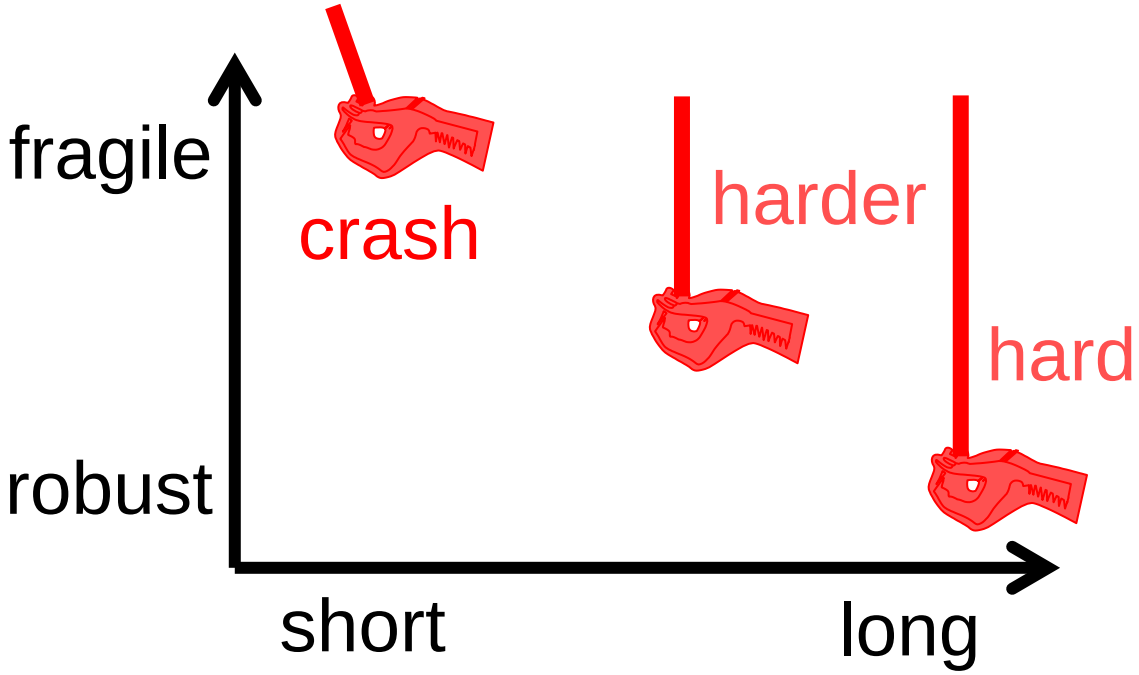
Law #1 : Mechanics
Law #2 : Gravity
Law #3 : Light/vision
Law #4 : $\boxed{\|T\| \geq \exp(p\tau)}$

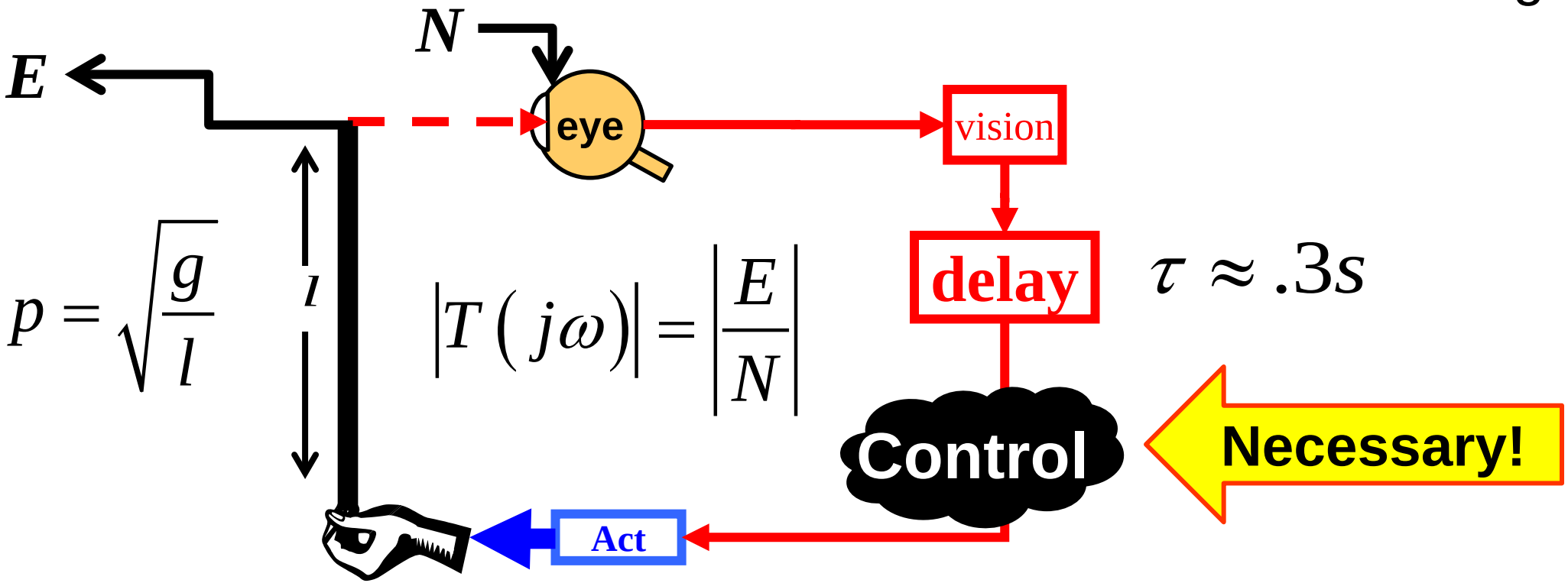


Universal laws

Mechanics+
Gravity +
Light +

$$\left.\begin{array}{c}\exp\left(\int \ln|T|\right)\\ \|T\|_\infty\end{array}\right\} \geq \exp(p\tau)$$





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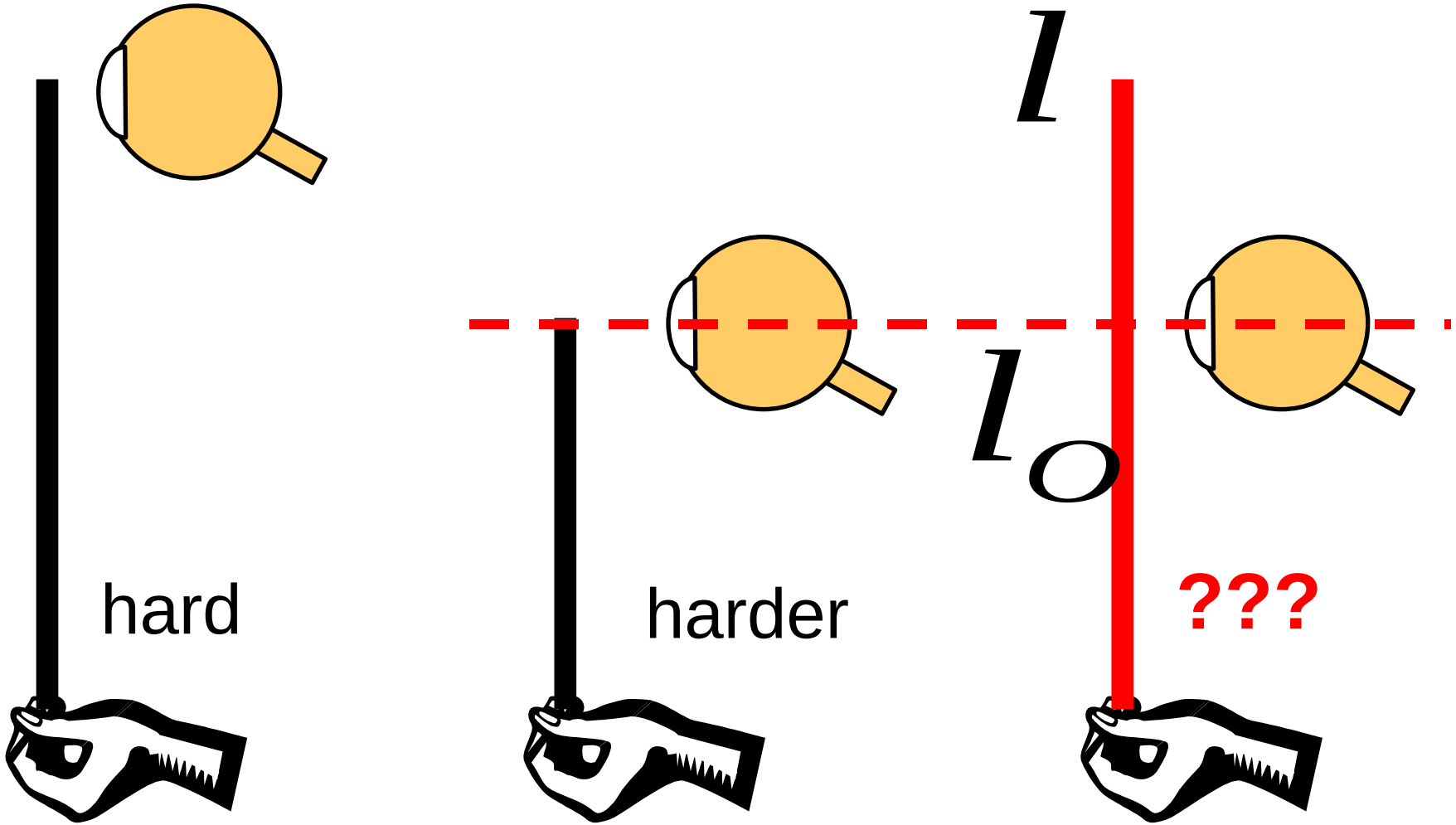
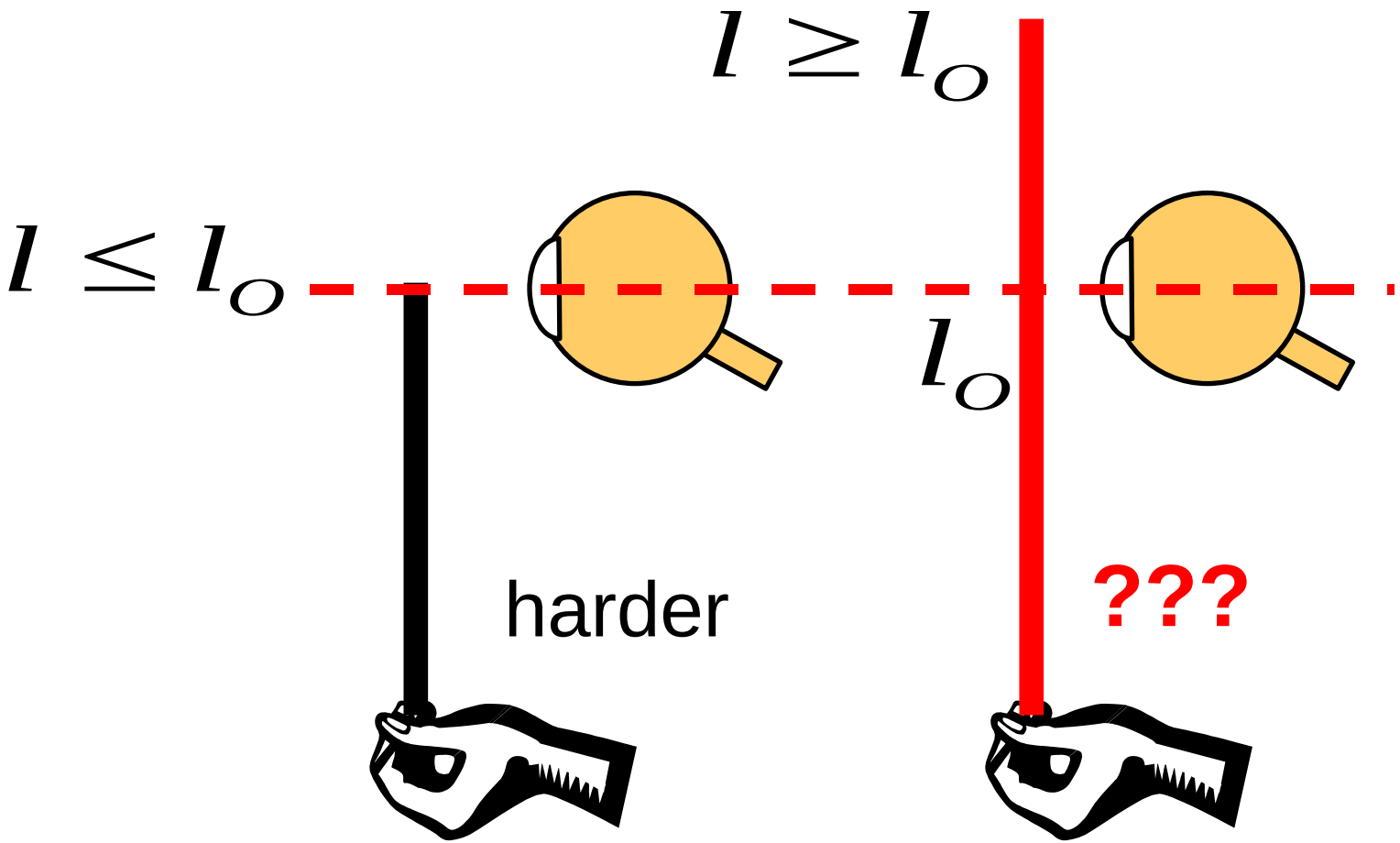

l
l_o
hard
harder
???


$l \geq l_o$
$l \leq l_o$
l_o
harder
???

$$\left.\begin{array}{c}\exp\left(\int \ln|T|\right)\\ \|T\|_\infty\end{array}\right\} \geq \exp(p\tau)\left|\frac{z+p}{z-p}\right|$$

Unstable zeros

Simple analytic formulas

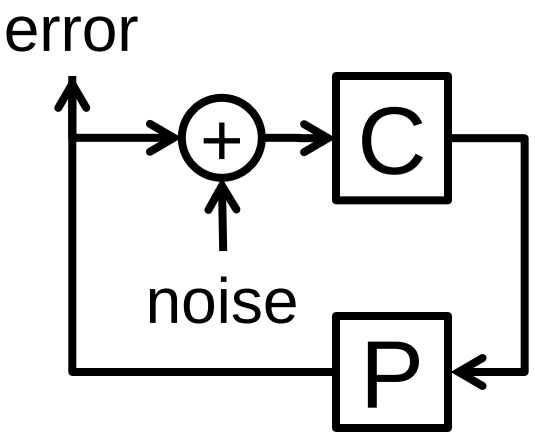
$$z = \sqrt{\frac{g}{l - l_O}} \qquad p = \sqrt{\frac{g}{l}}$$

$$\frac{z+p}{z-p} = \frac{\sqrt{l} + \sqrt{l - l_O}}{\sqrt{l} - \sqrt{l - l_O}}$$

$$\alpha = \frac{l_O}{l} \rightarrow \frac{z+p}{z-p} = \frac{1+\sqrt{1-\alpha}}{1-\sqrt{1-\alpha}} = \frac{\left(1+\sqrt{1-\alpha}\right)^2}{\alpha}$$

Proof?

$$\|T\|_\infty = \sup_\omega |T(j\omega)| = \sup\left\{ |T(s)| \;\middle|\; \mathrm{Re}(s) \geq 0 \right\}$$



$$T(s) = M(s)\Theta(s) \quad |\Theta(j\omega)| = 1$$

$$\Theta(s) = \exp(-\tau s)\frac{s-z}{s+z}$$

$$P(s) = P_M(s)\left[\exp(-\tau s)\frac{s-z}{s+z}\right] \Rightarrow$$

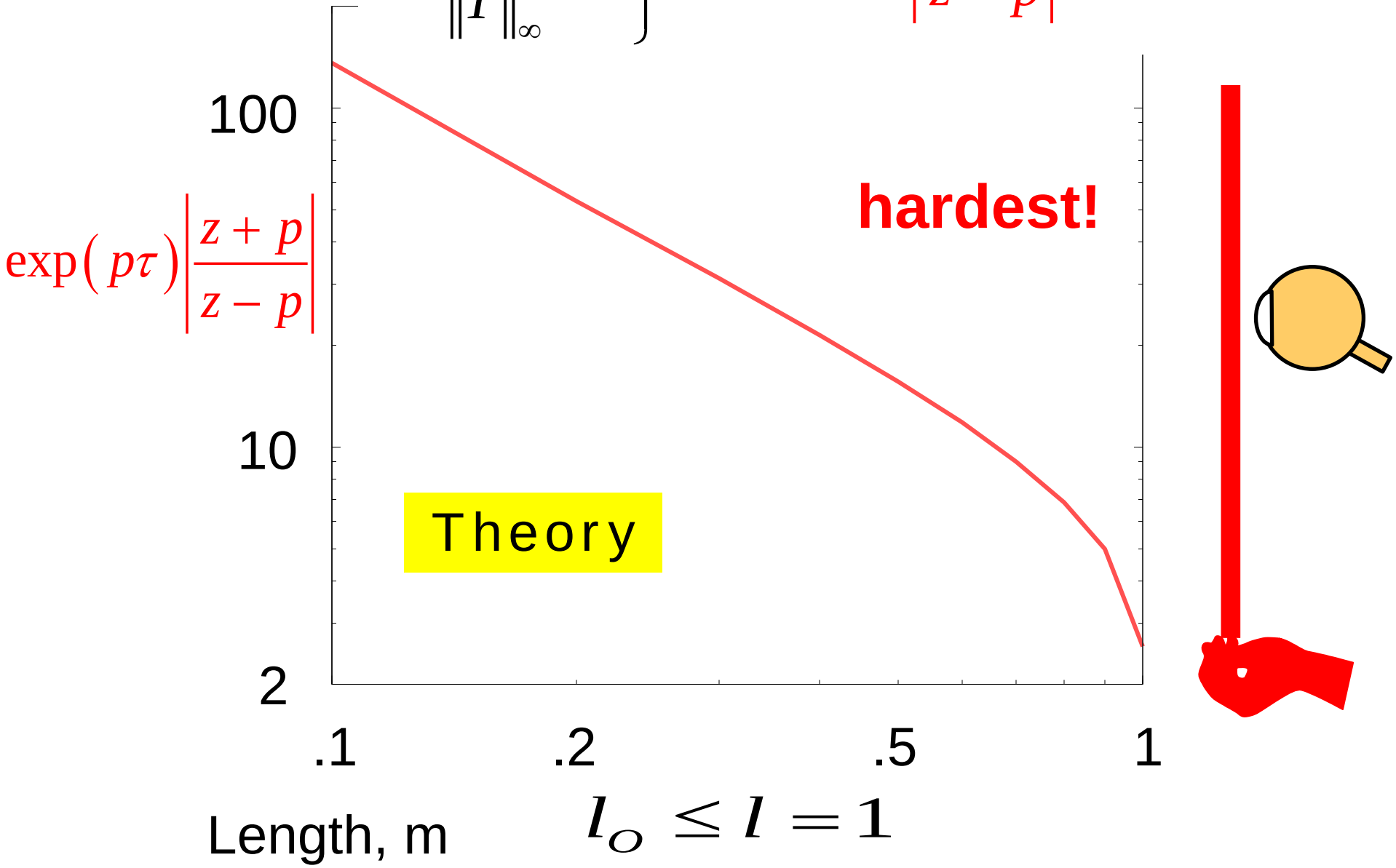
Undergrad math

$$\|T\|_\infty = \|M\|_\infty \geq |M(p)| \geq \left|\Theta(p)^{-1}\right| \geq \exp(\tau p)\left|\frac{z+p}{z-p}\right|$$

$$\Rightarrow \|T\|_\infty \geq \exp(\tau p)\left|\frac{z+p}{z-p}\right|$$

$$\tau = .3s$$

$$\left.\begin{matrix}\exp\left(\int \ln|T|\right) \\ \|T\|_\infty\end{matrix}\right\} \geq \exp(p\tau)\left|\frac{z+p}{z-p}\right| \geq \exp(p\tau)$$



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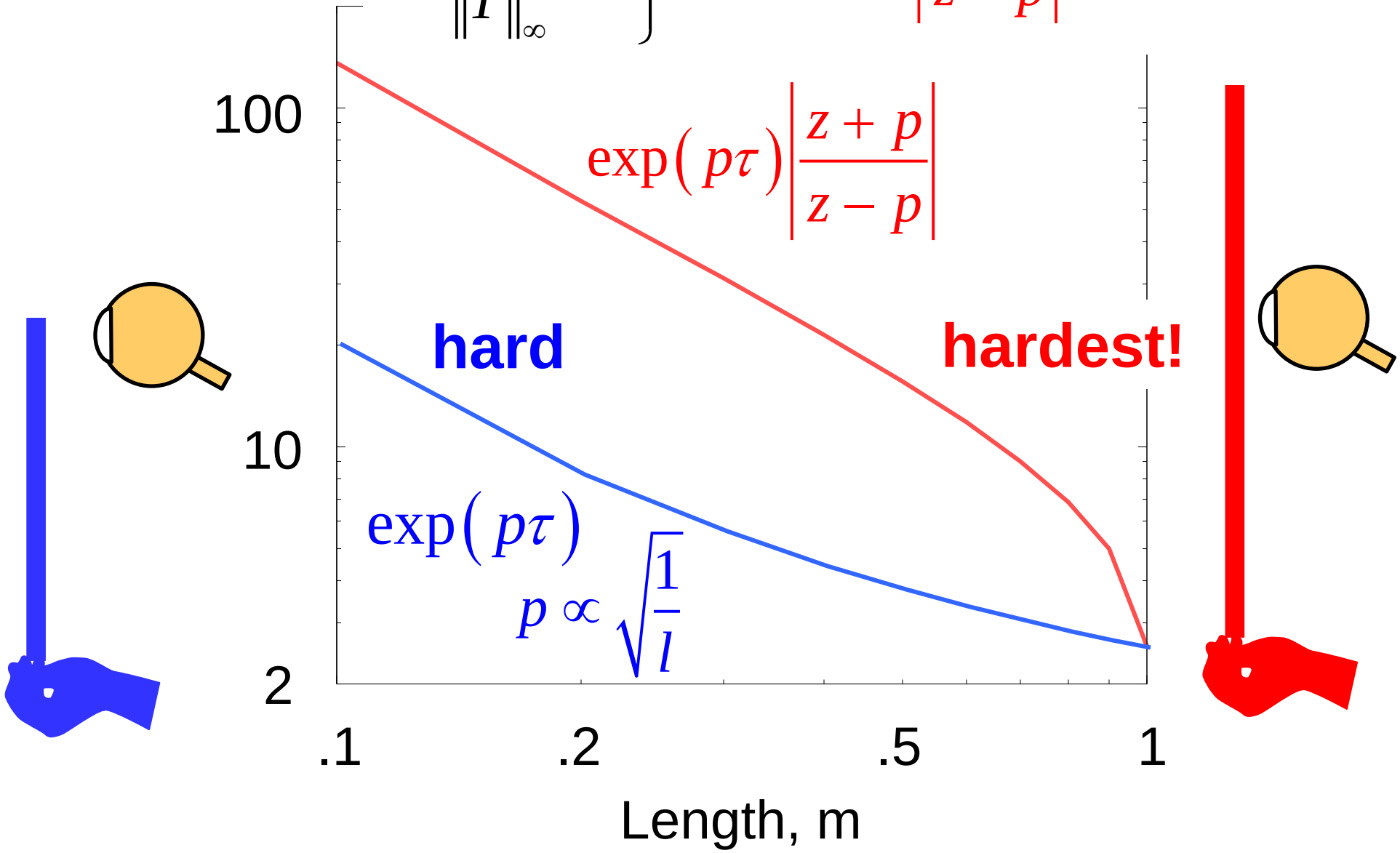
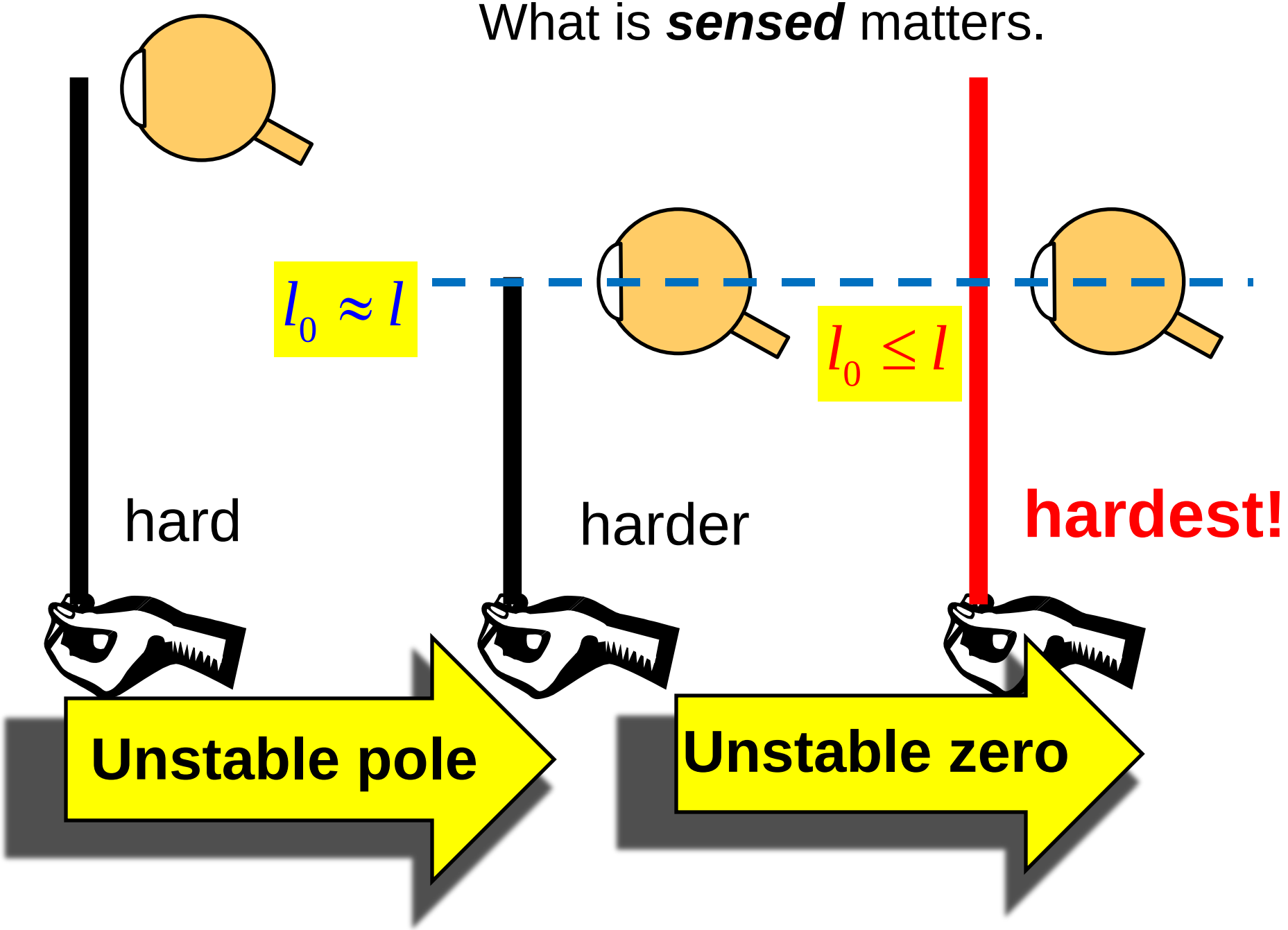


What is ***sensed*** matters.



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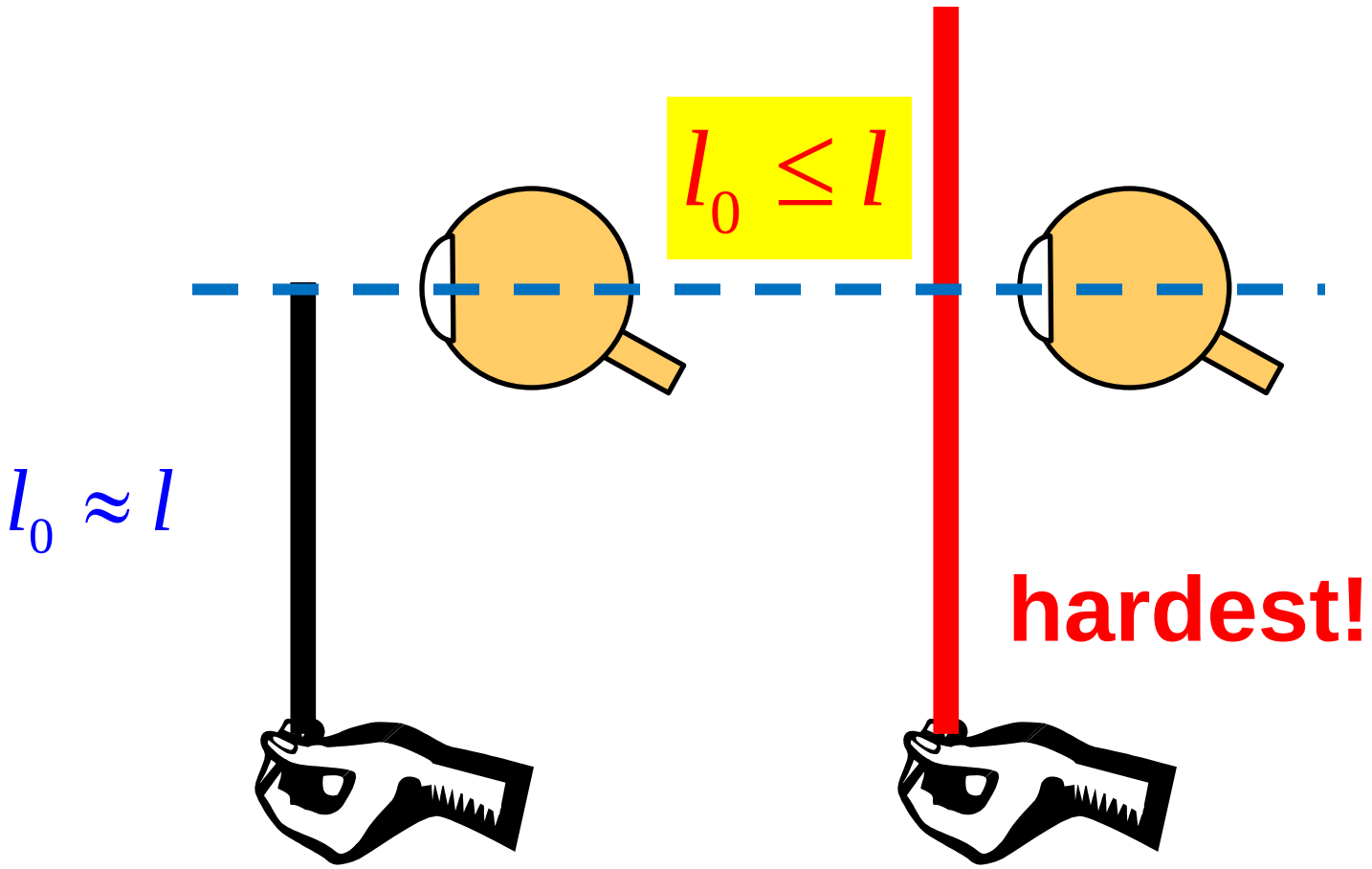

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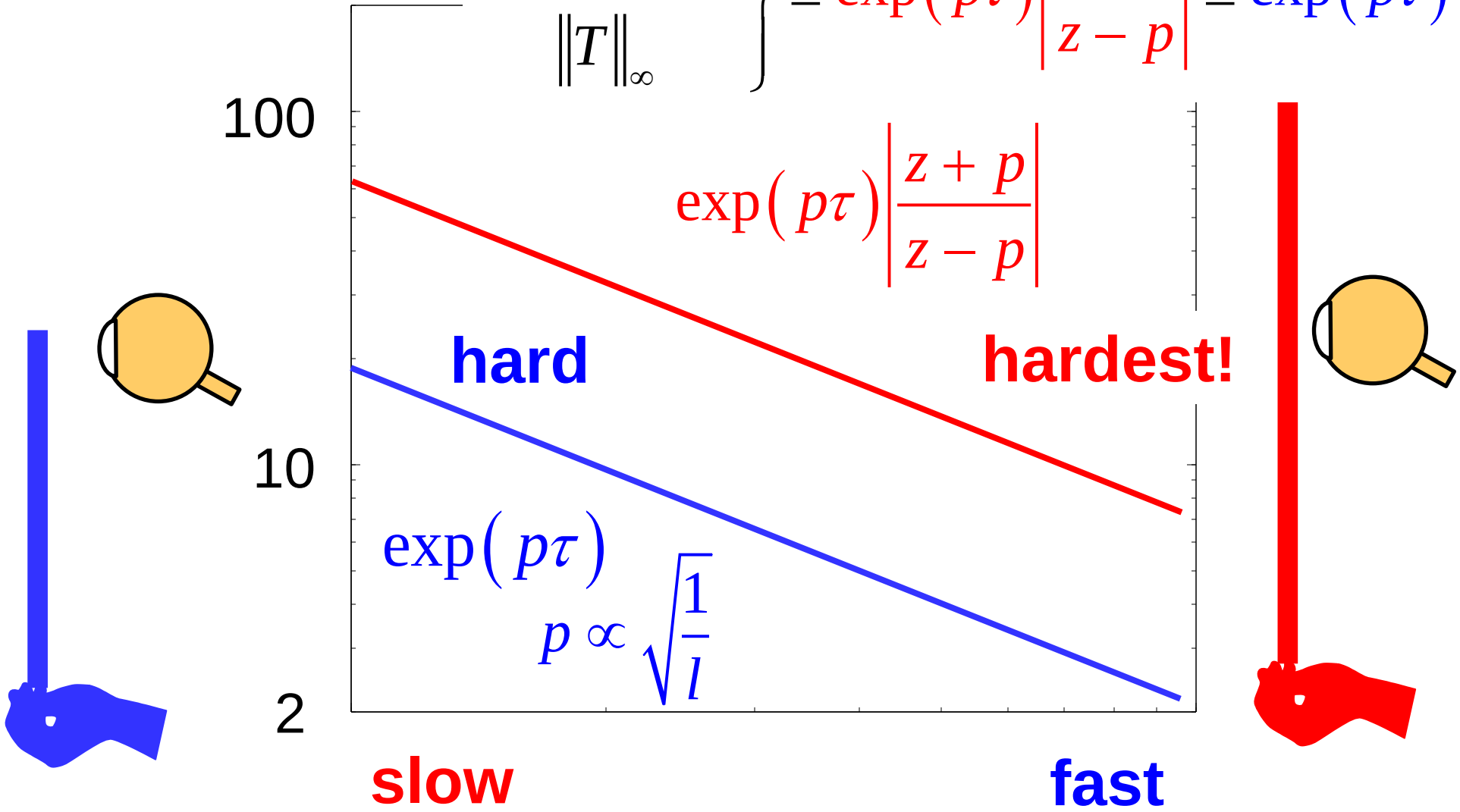
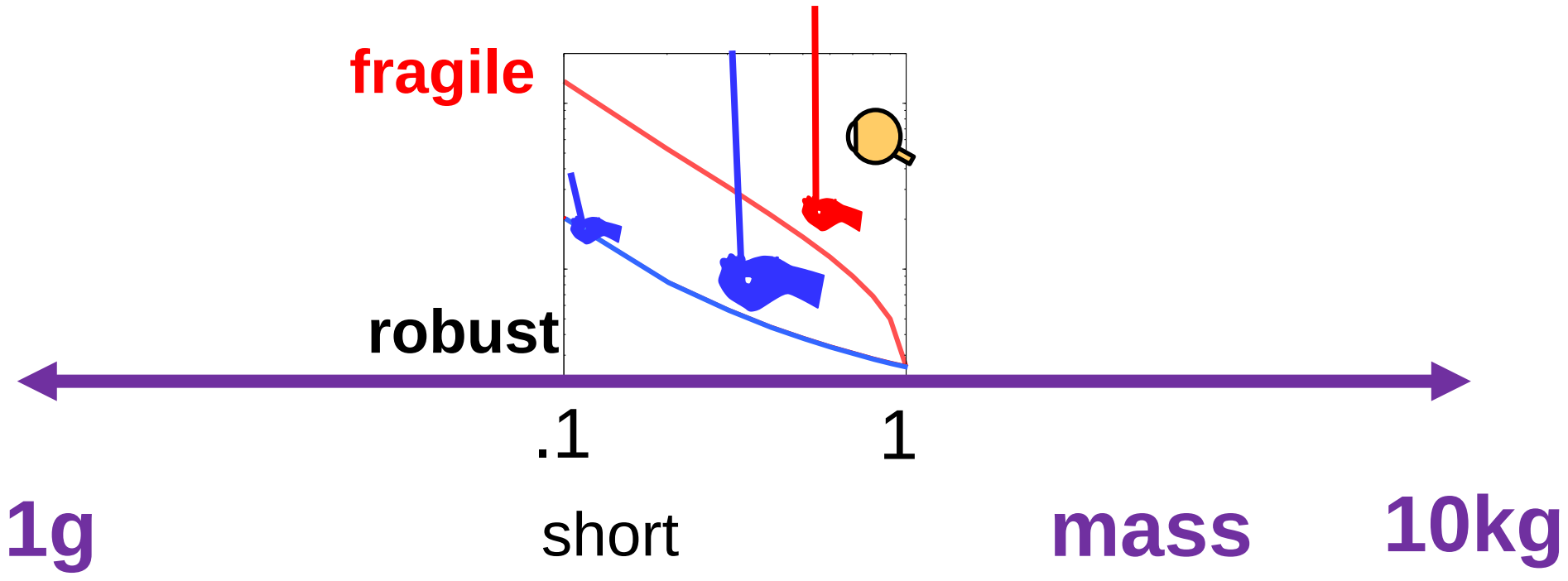
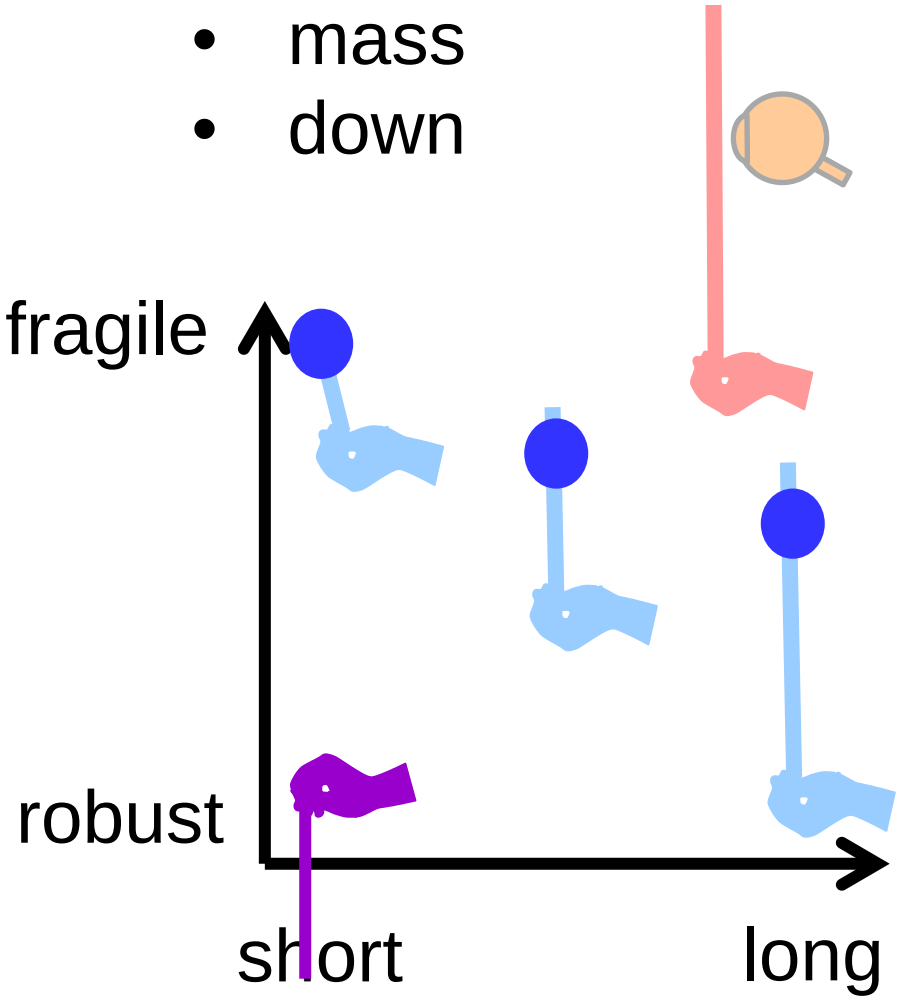


- delay (-τ))


fragile
robust
.1
1
1g
short
mass
10kg

Robust to

- mass
- down



Fragile to

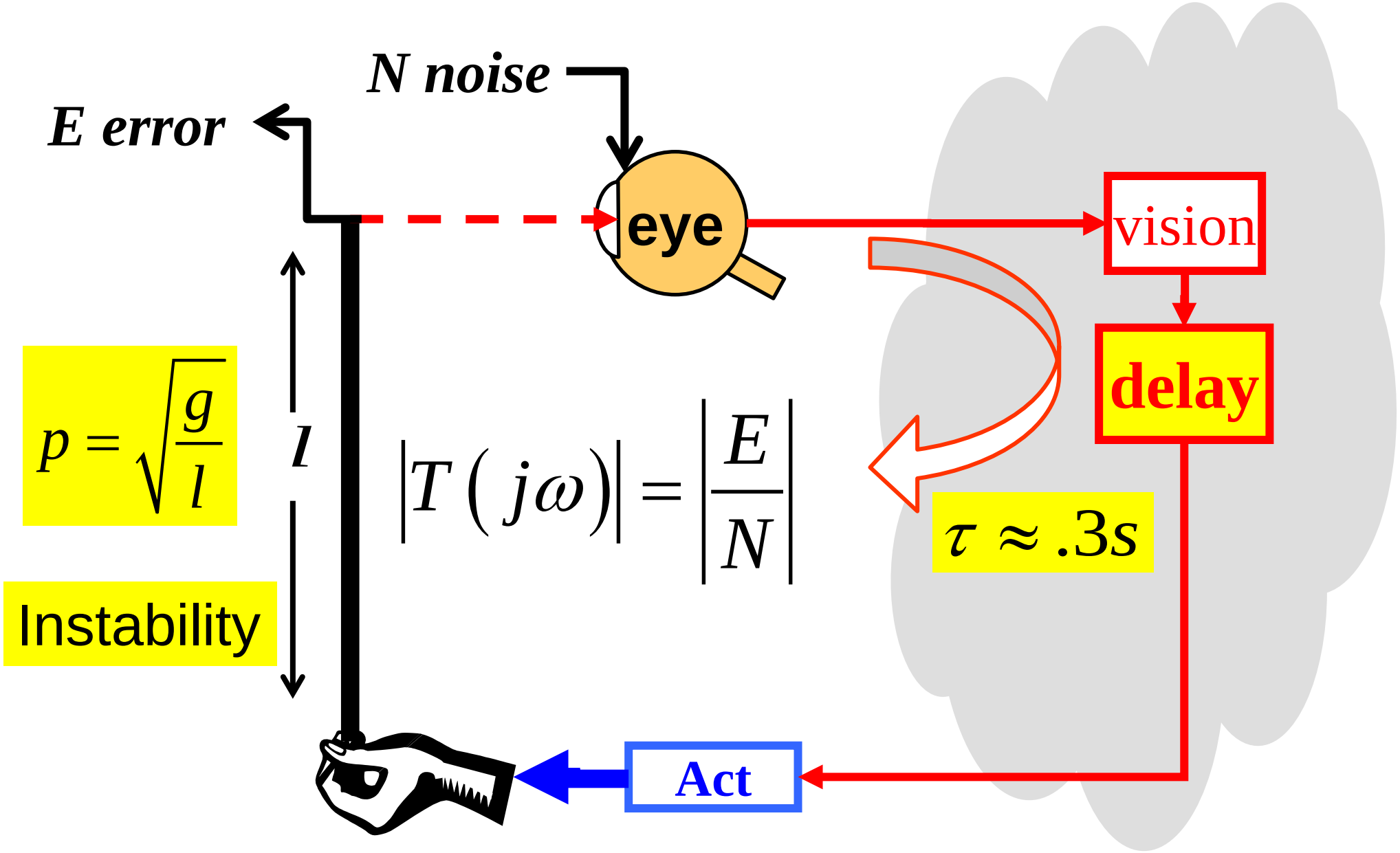
- Up
- Length l
- Length l_o (sense)
- Noise
- Medial direction
- Close one eye
- Stand one leg
- Alcohol
- Age
- Sensorimotor delay
- Overeducation?

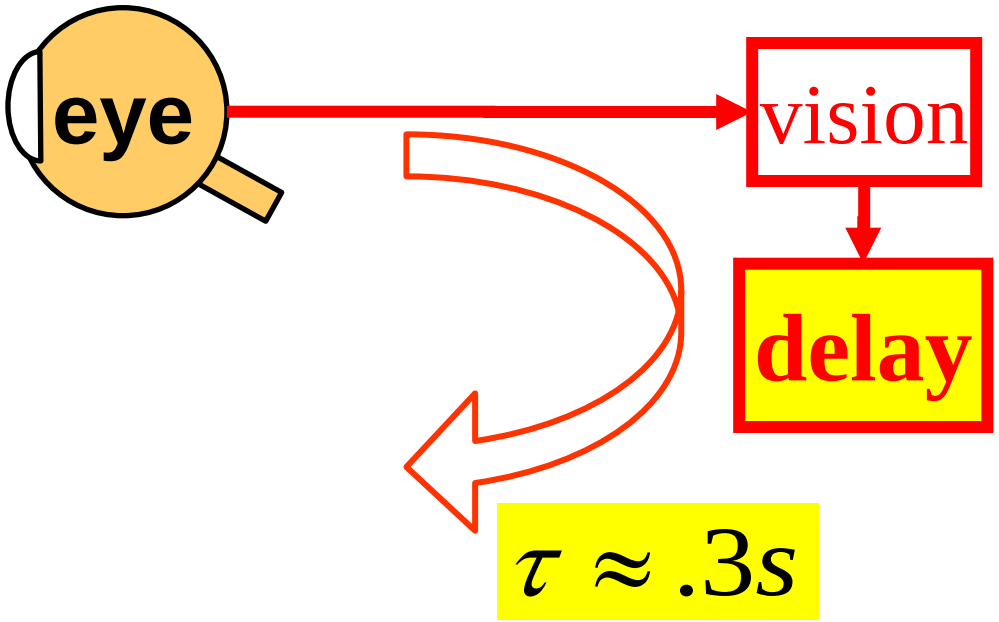
Instability

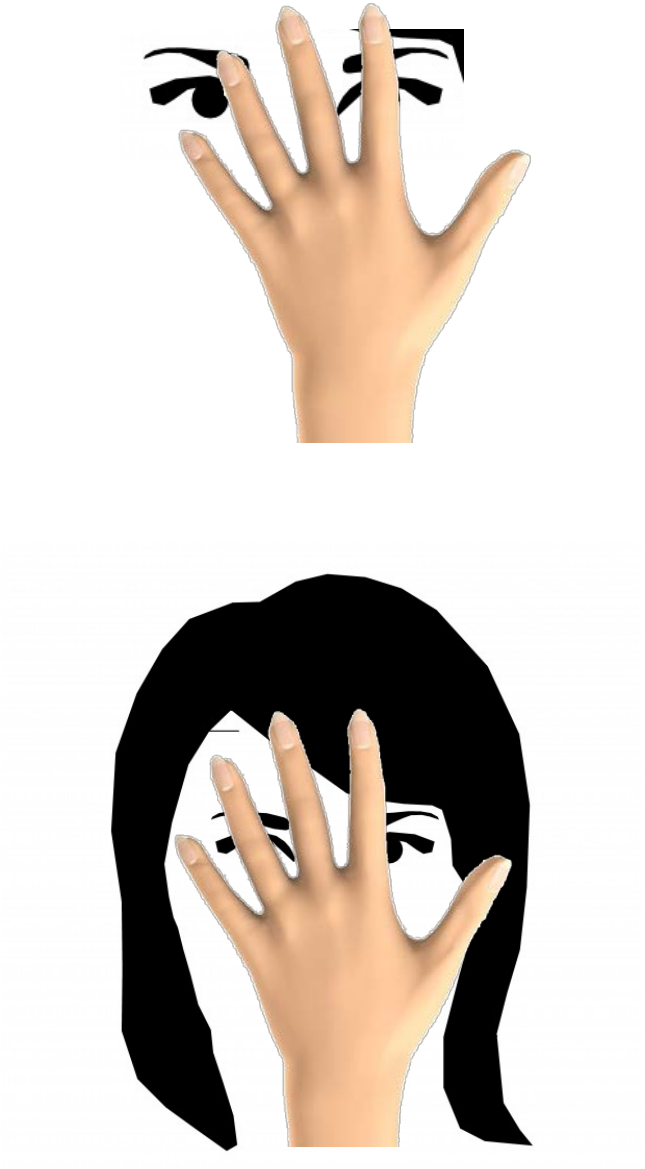
Sensors/ actuators

Delay

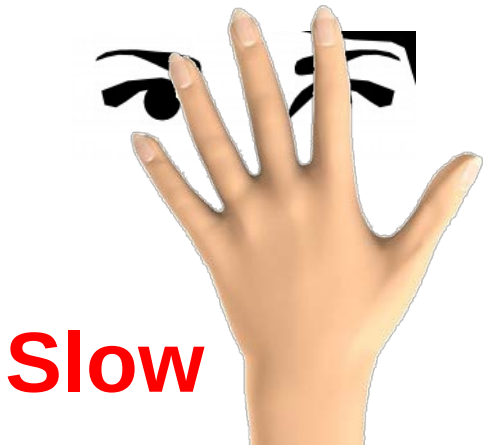
Simplified sensorimotor control

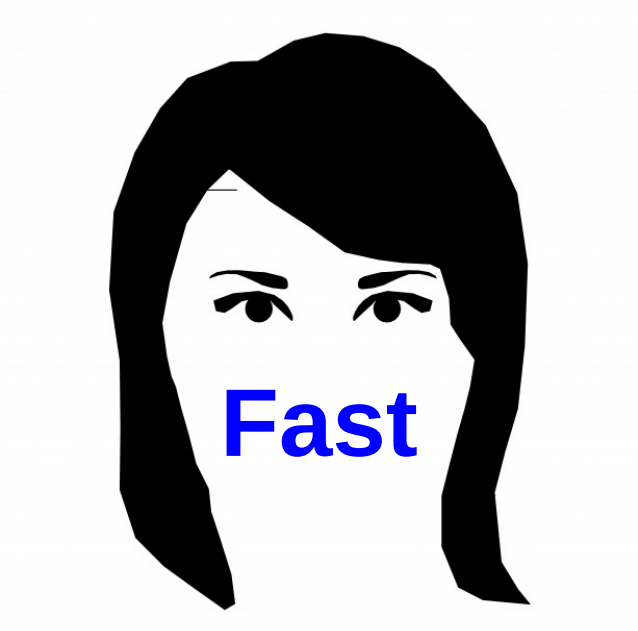



eye
vision
delay
$\tau \approx .3s$



- Brains
  - Familiar, accessible
  - Live demos!
  - Cheap, reproducible
  - Open
  - Unavoidable

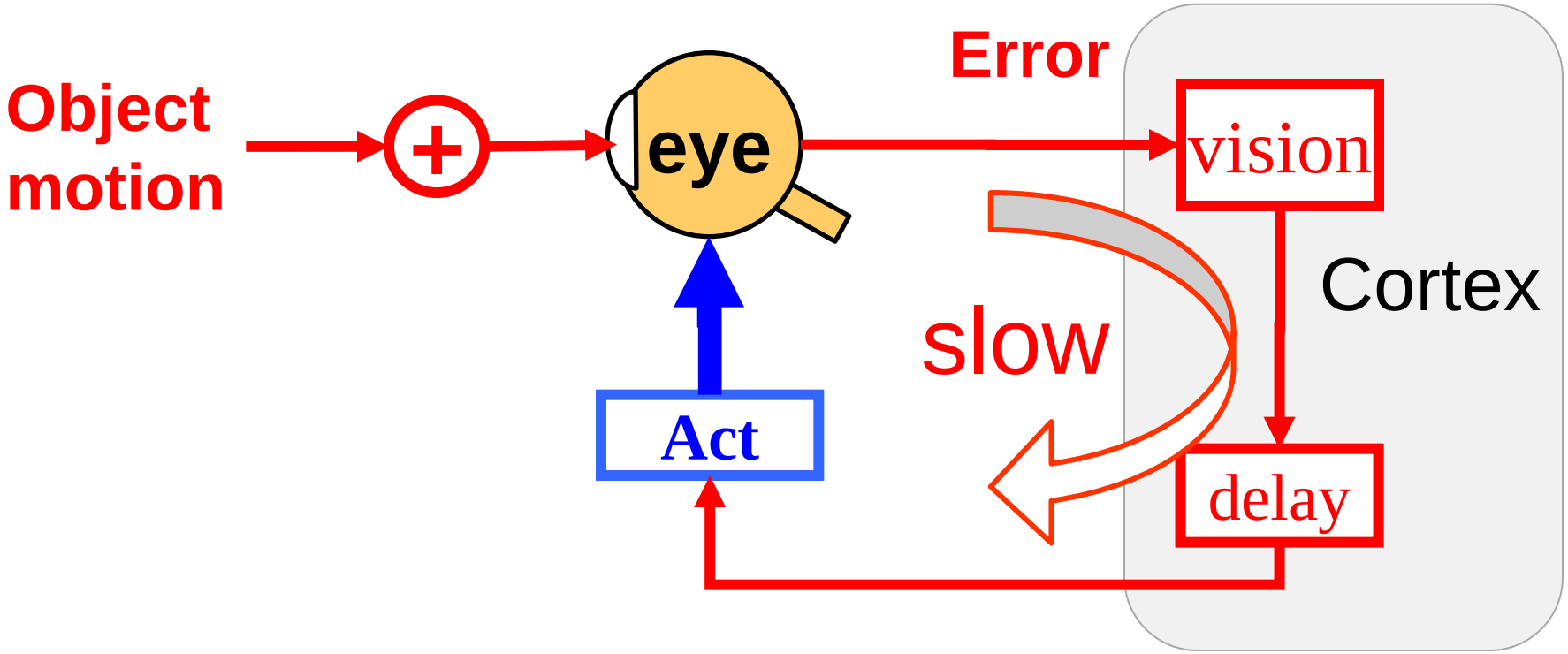


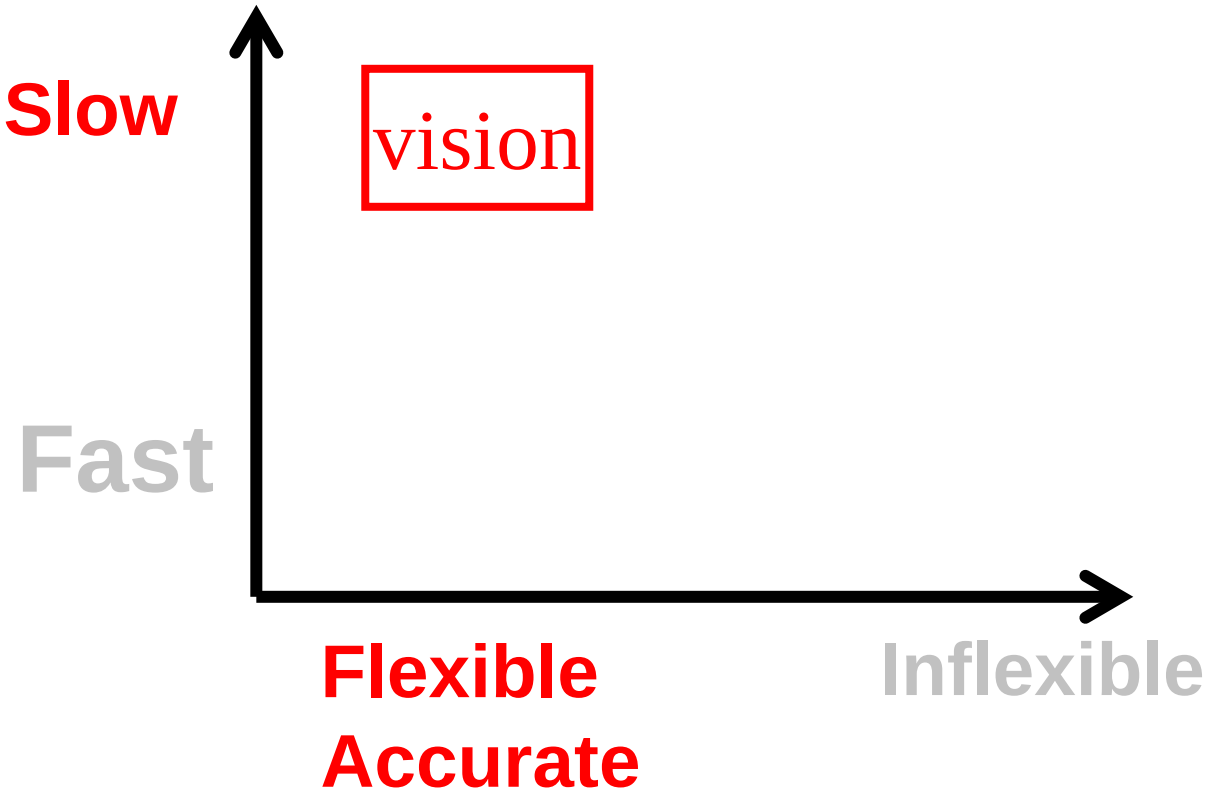


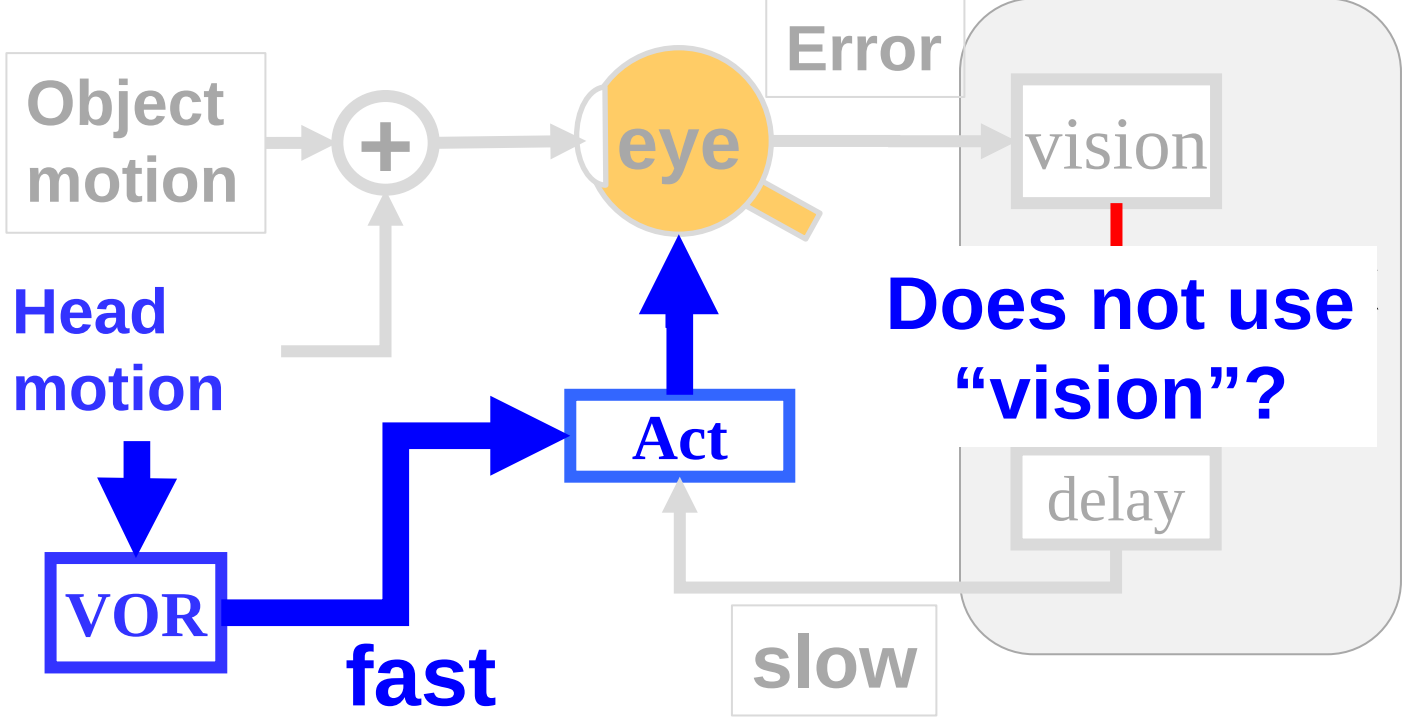
How? Why?

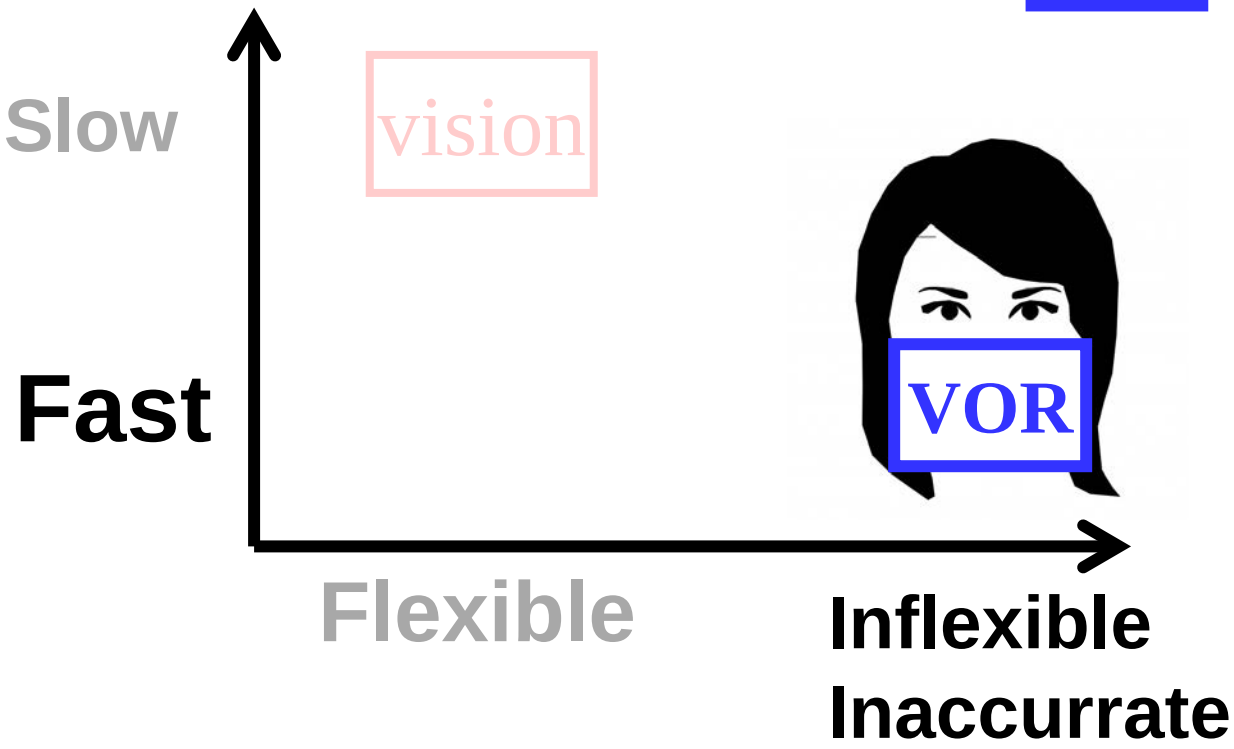
- Mechanism
- Tradeoff

- Architecture
- Speed
- Flexibility
- Robustness
- Virtualization
- Diversity


Object
motion
eye
Error
vision
Cortex
slow
Act
delay

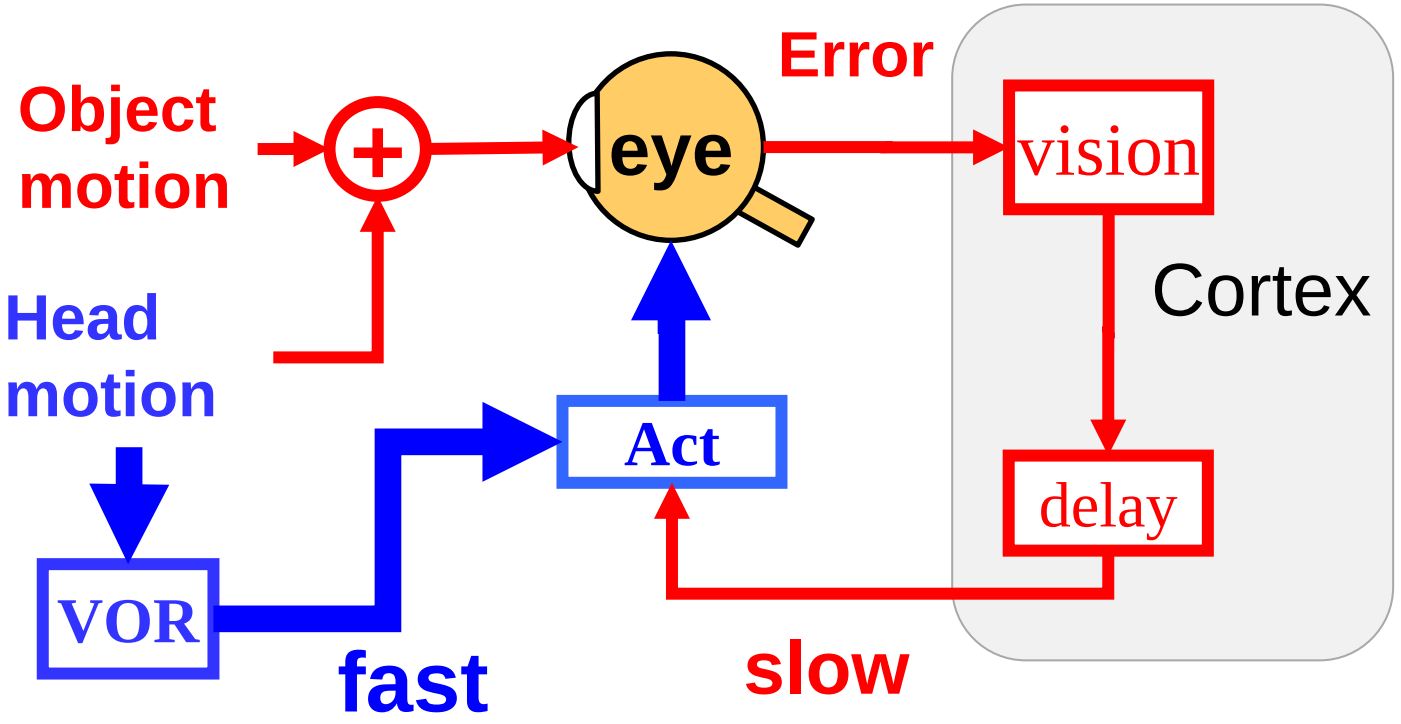

Slow
vision
Fast
Flexible
Accurate
Inflexible



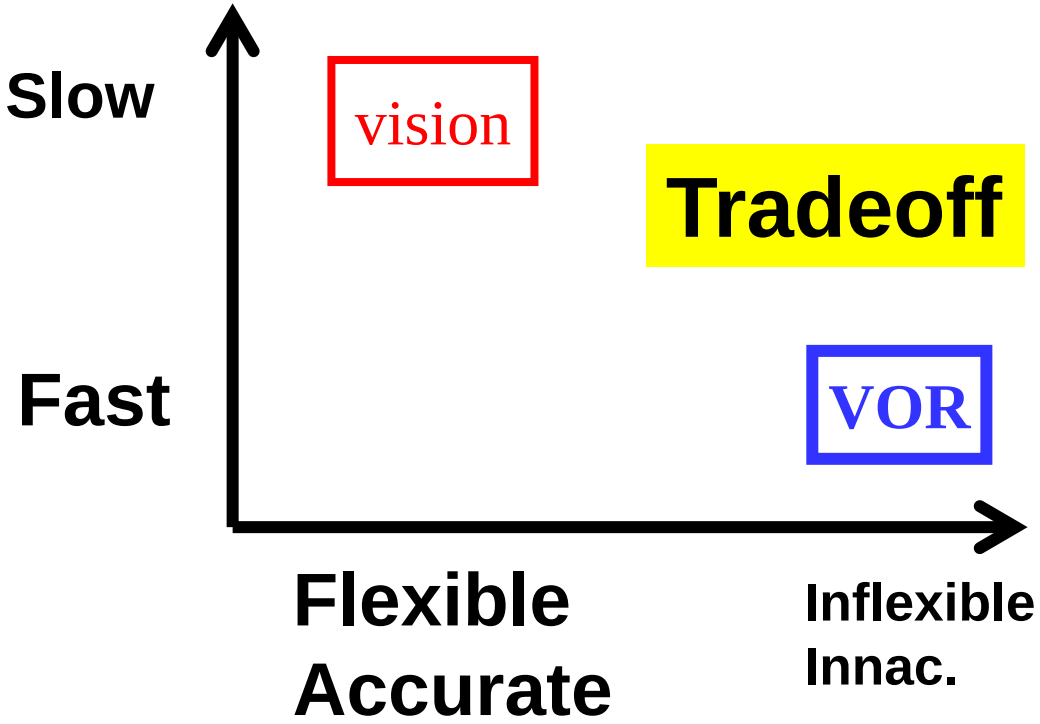


Vestibular Ocular Reflex (VOR)

Mechanism



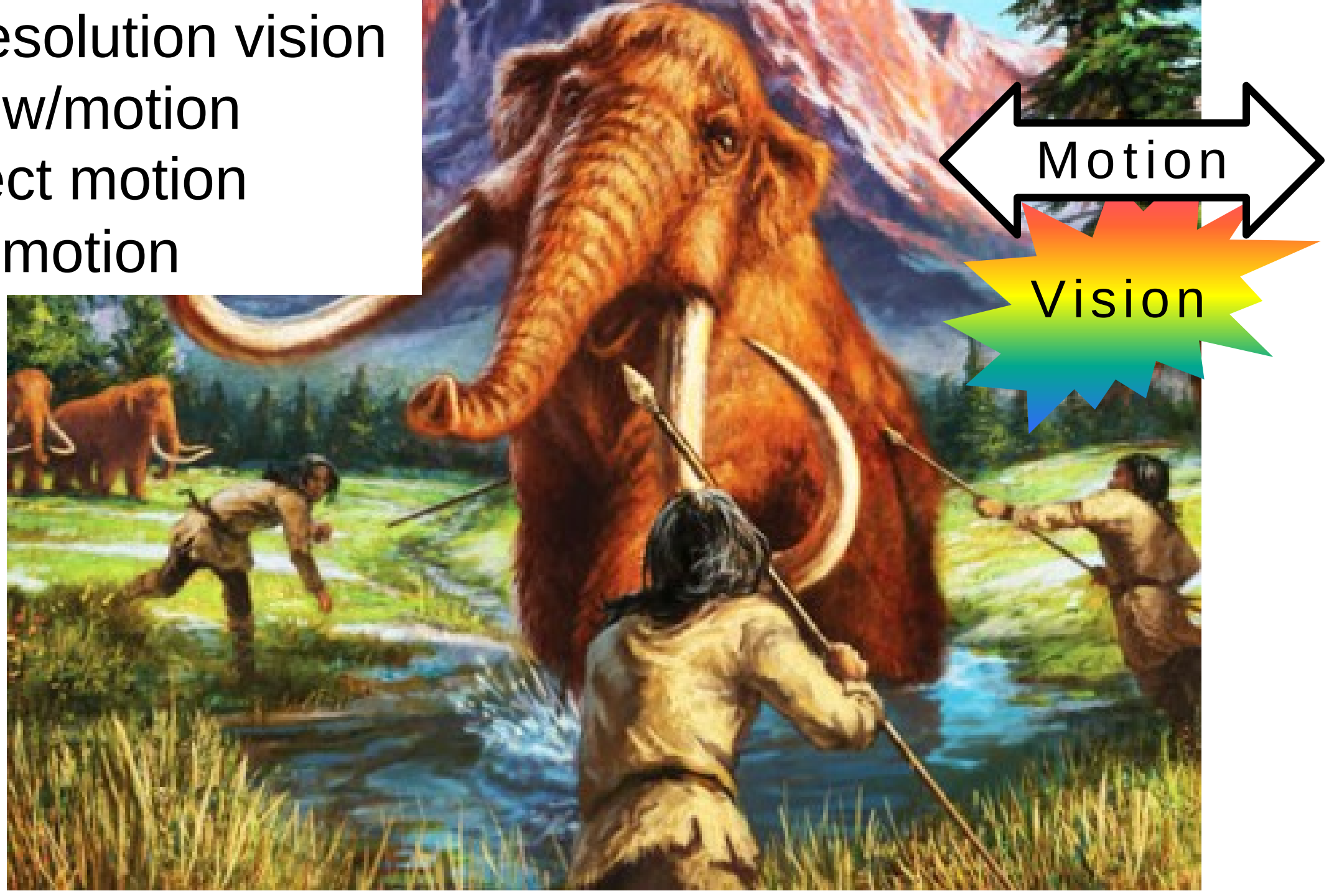
Tradeoff



Minimal cartoon

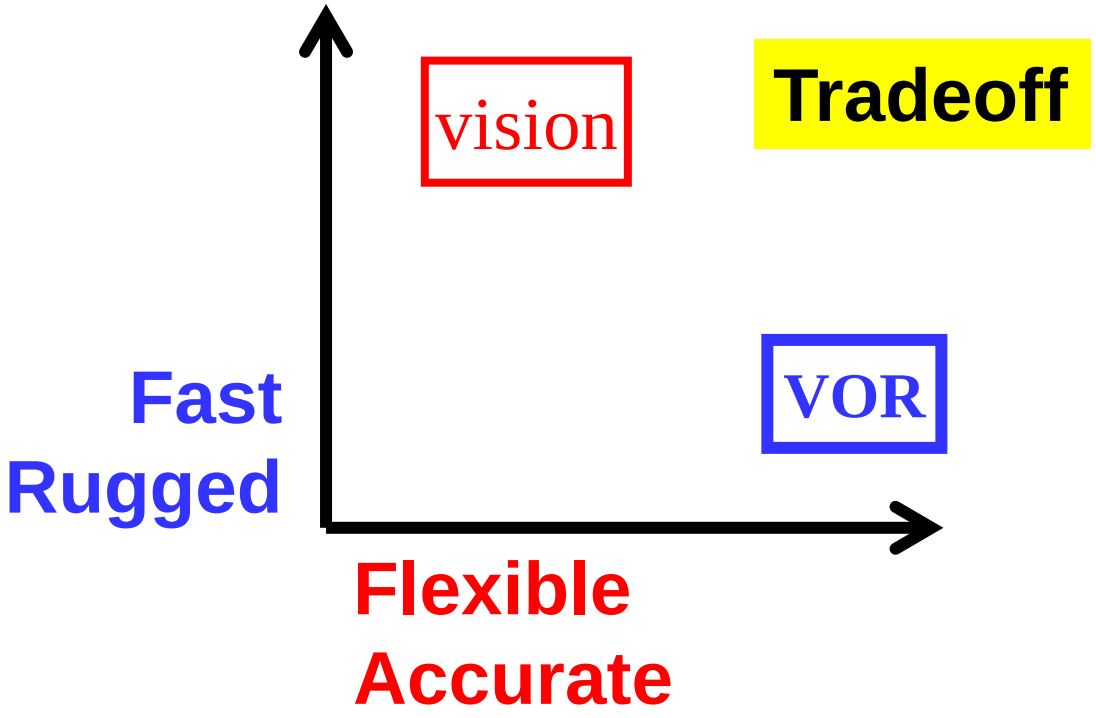
High resolution vision robust w/motion

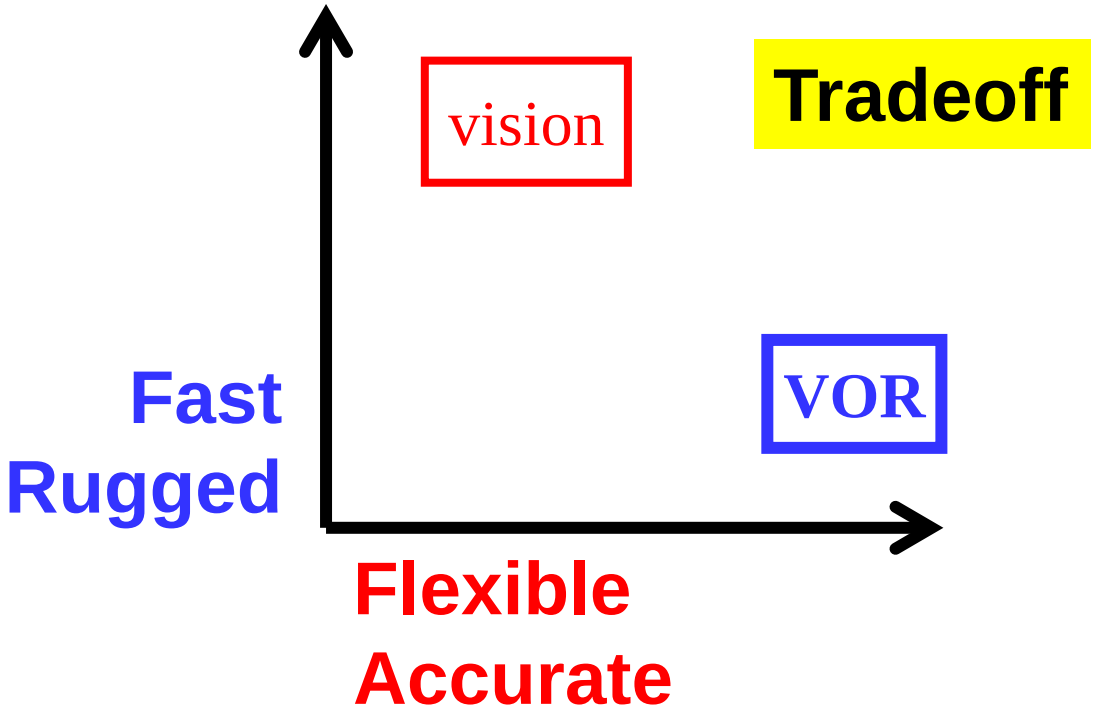
- Object motion
- Self motion



High resolution vision
robust w/motion

- Object motion
- Self motion

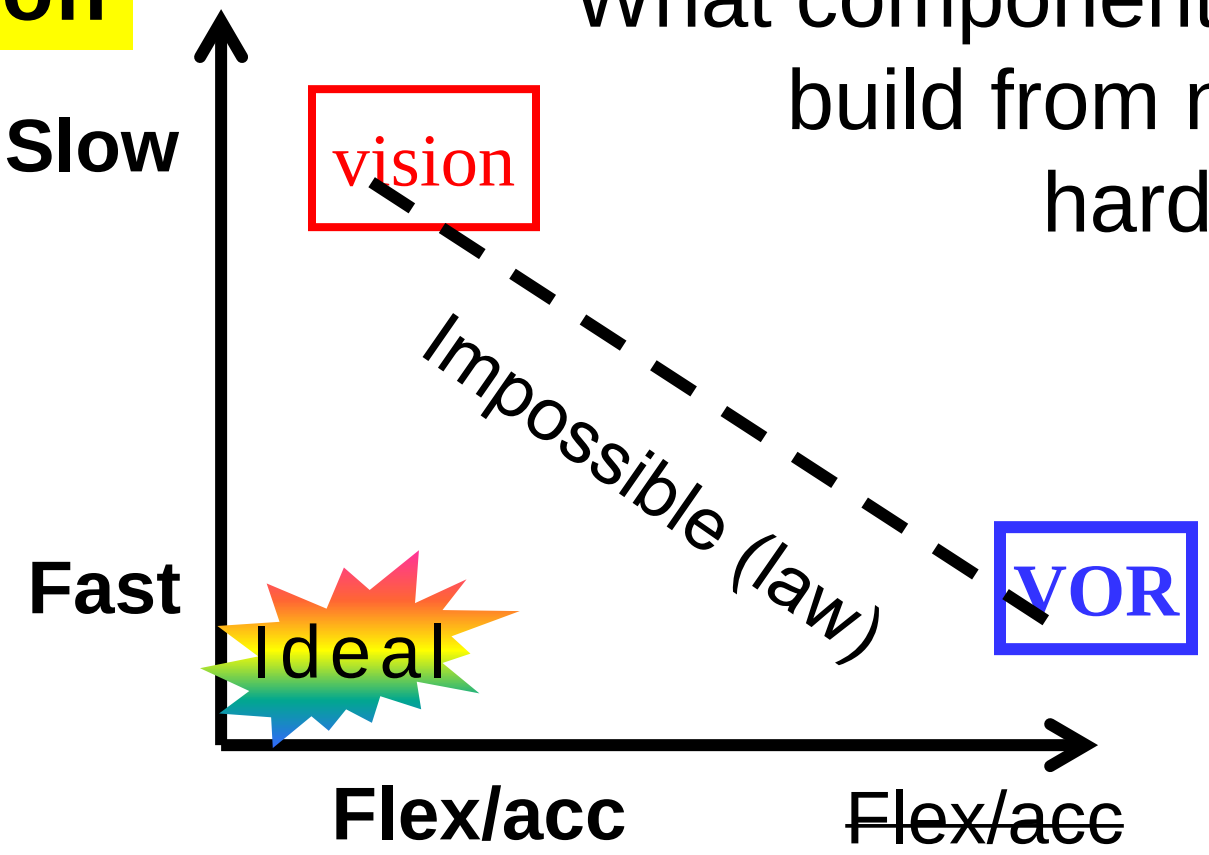



Tradeoff
vision
VOR
Fast
Rugged
Flexible
Accurate

Laws

Tradeoff

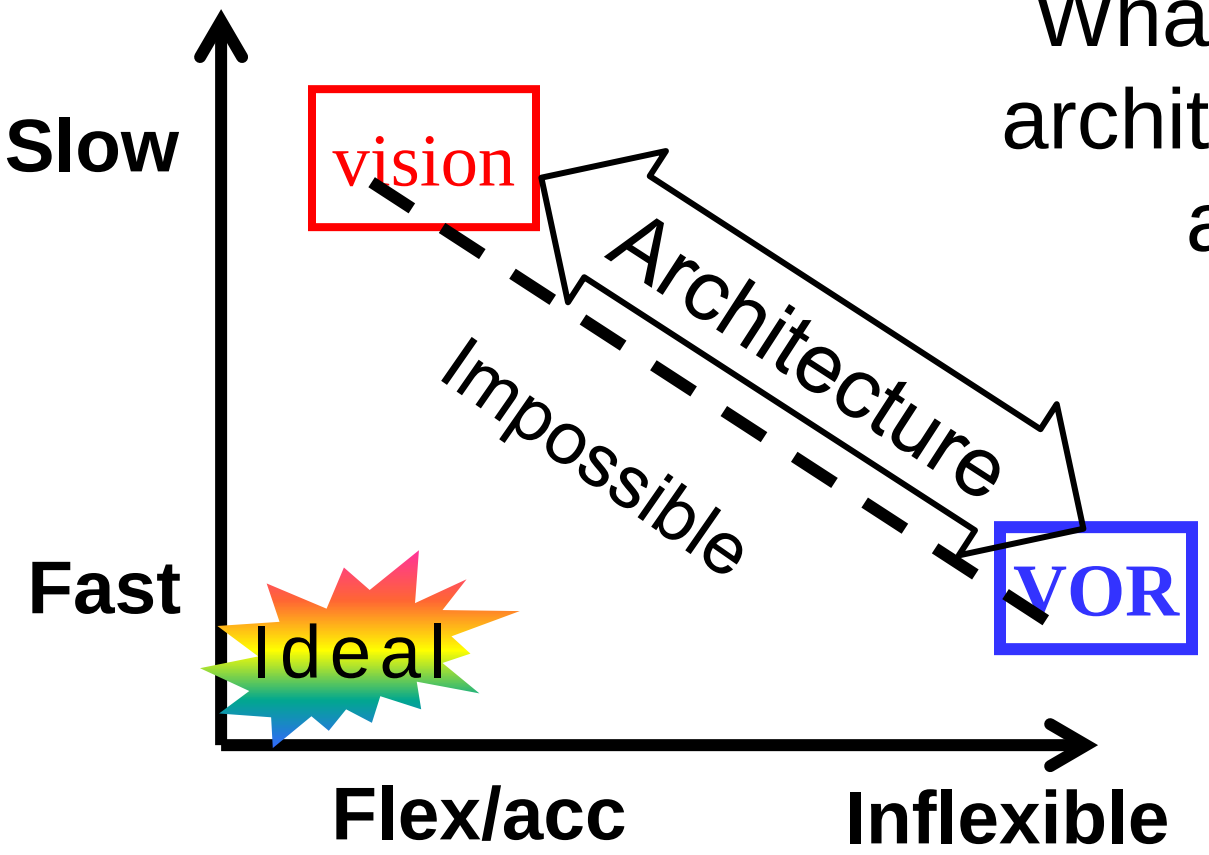
What components can build from neural hardware.

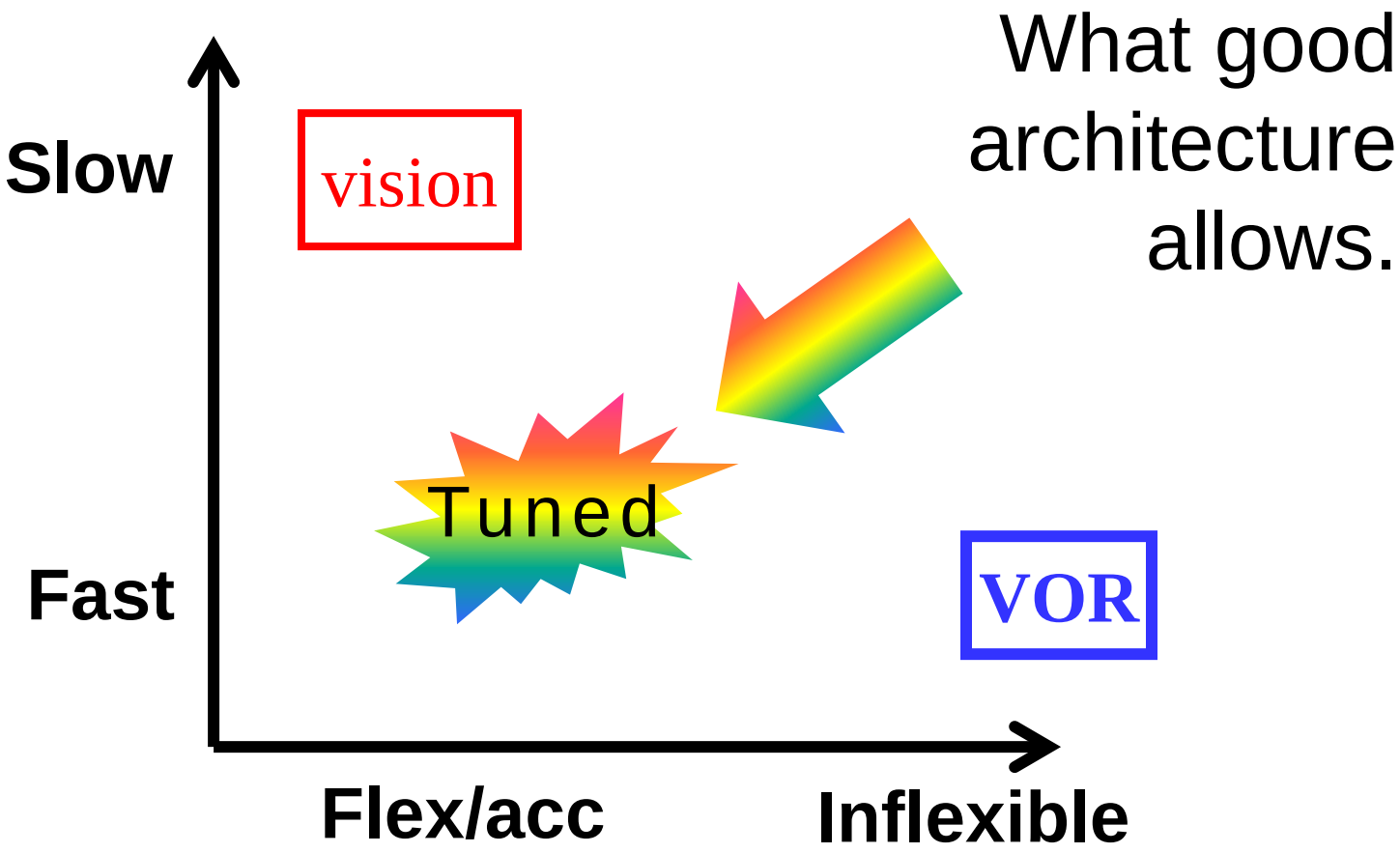


So far, just observed, no theory.

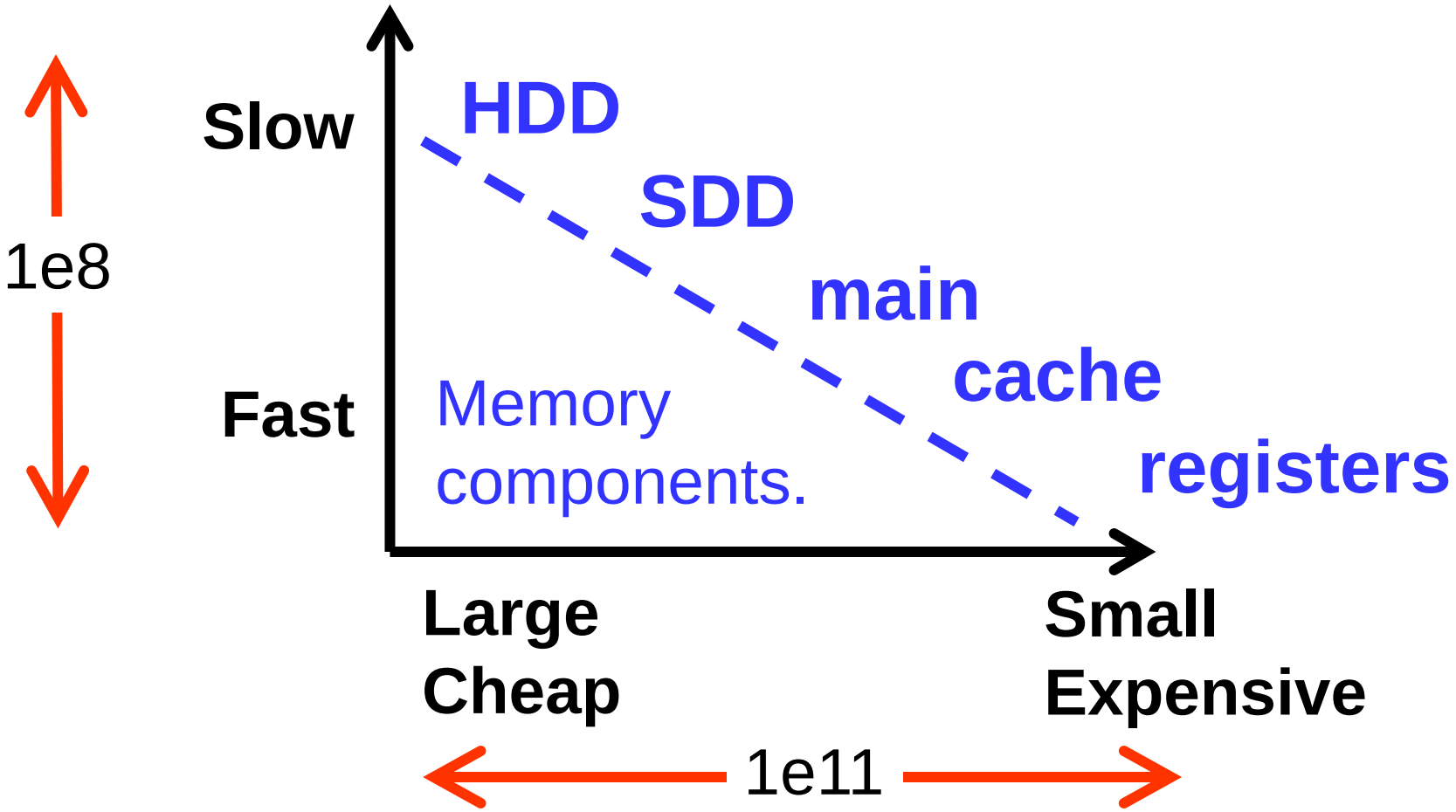
Laws and Architectures

What good architecture allows.

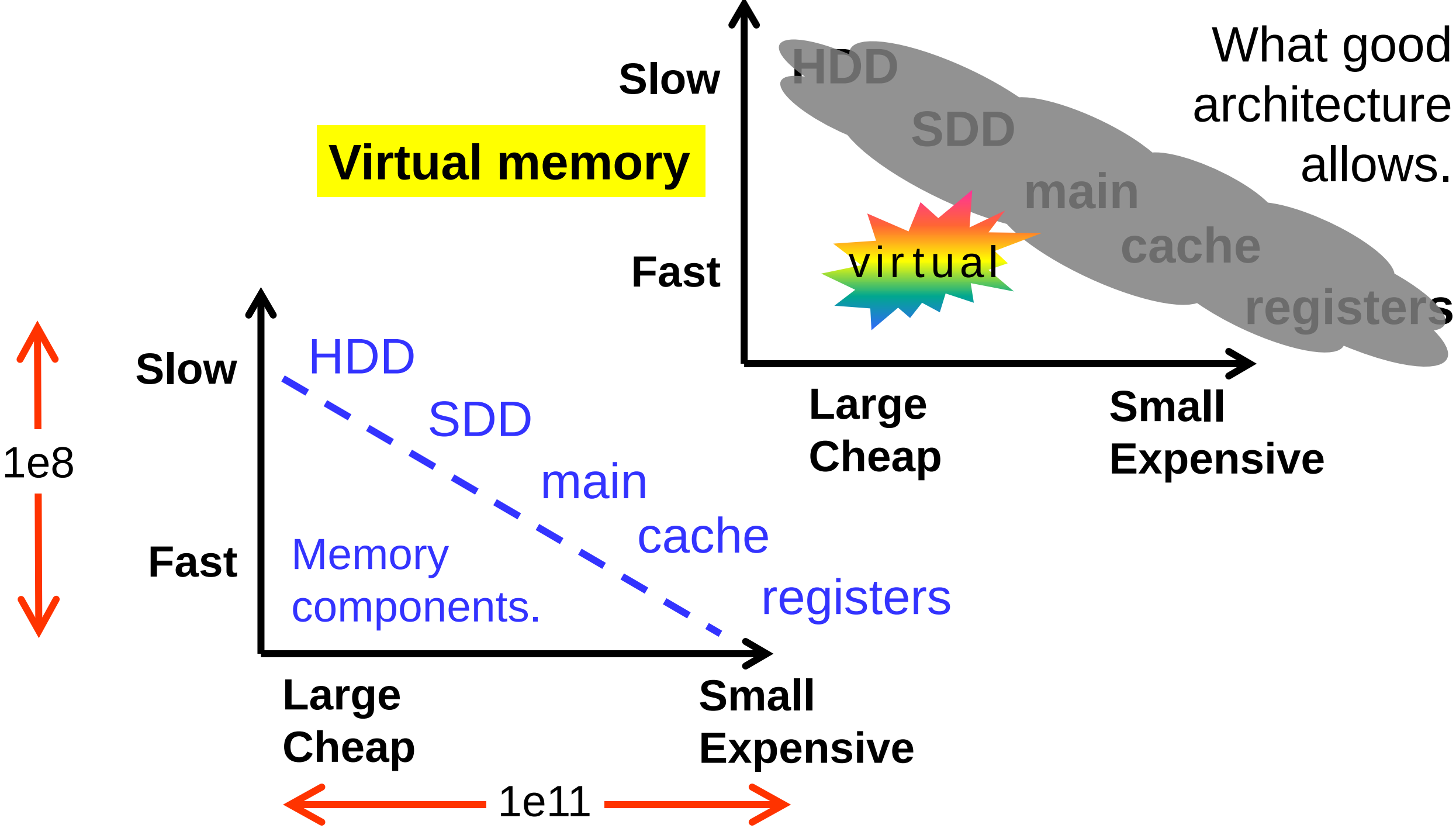



What good architecture allows.
Slow
Fast
vision
Tuned
VOR
Flex/acc
Inflexible

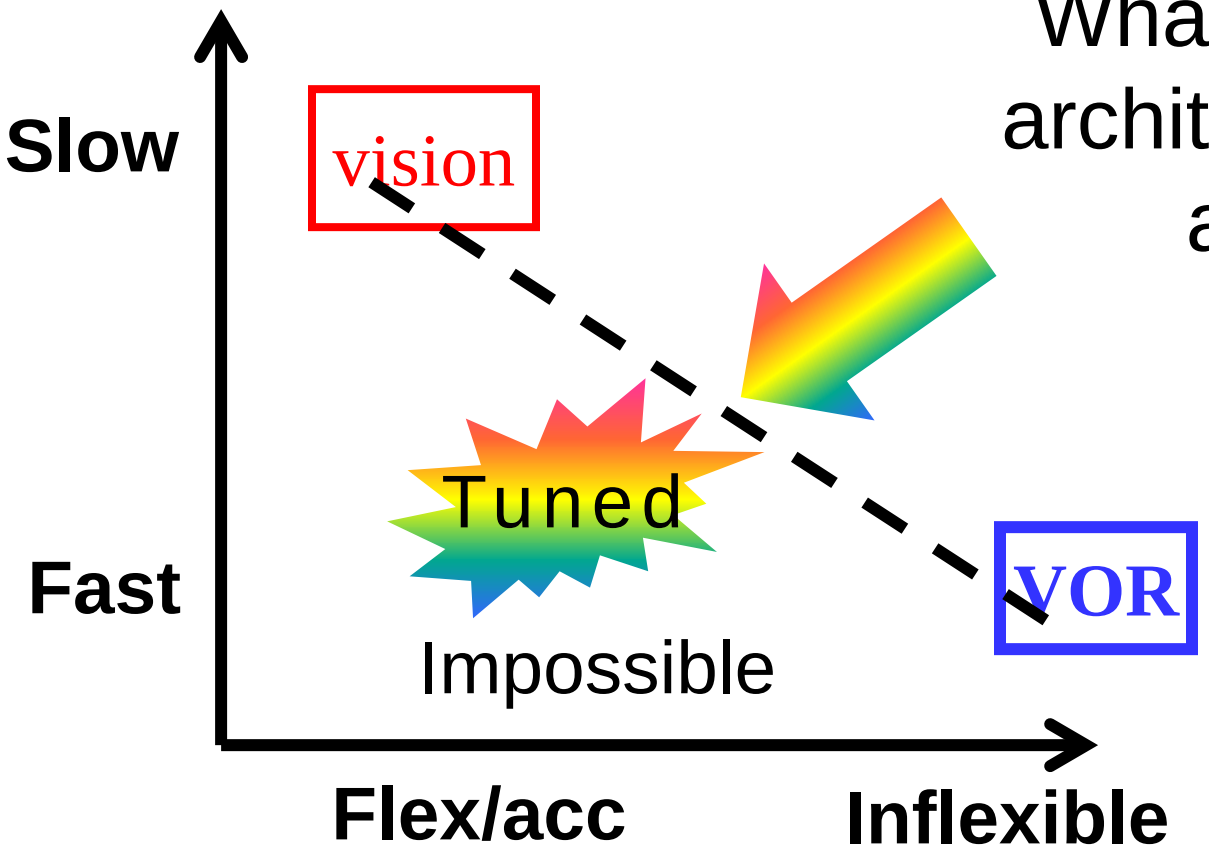
Computer memory



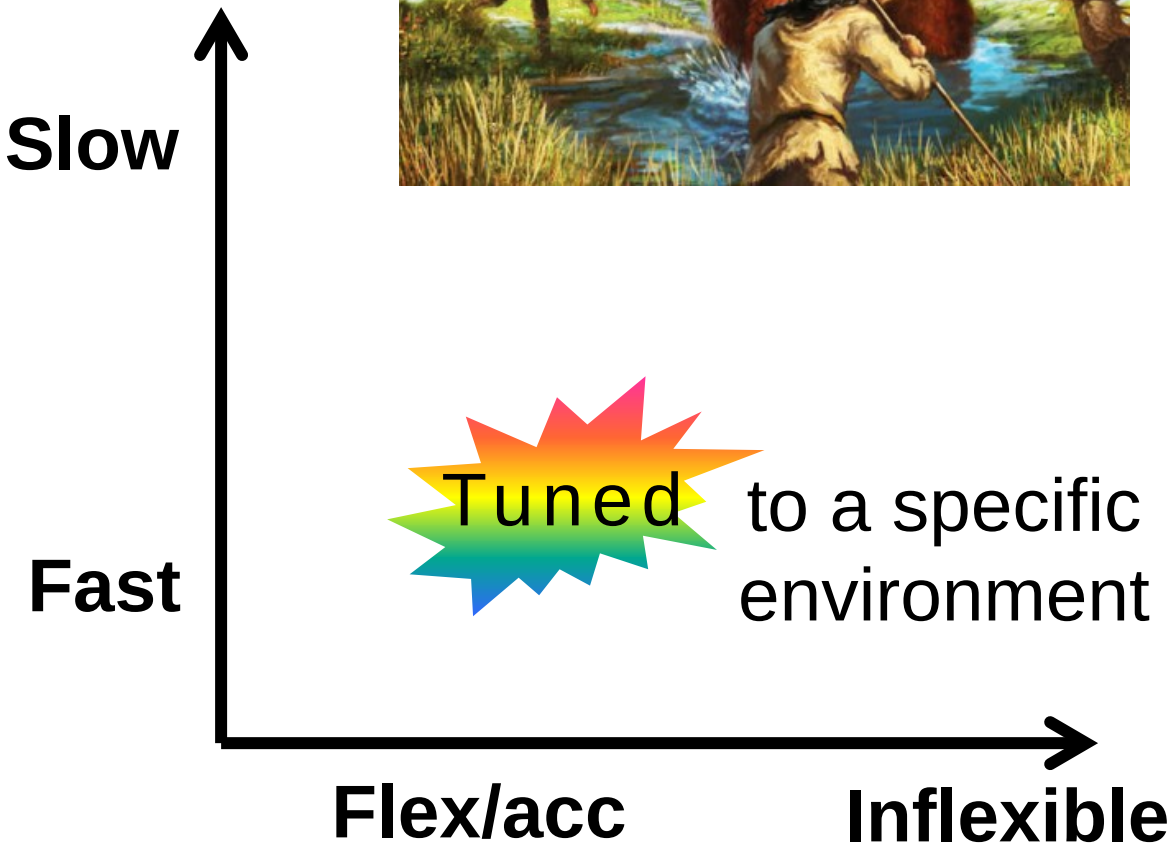
Virtual memory



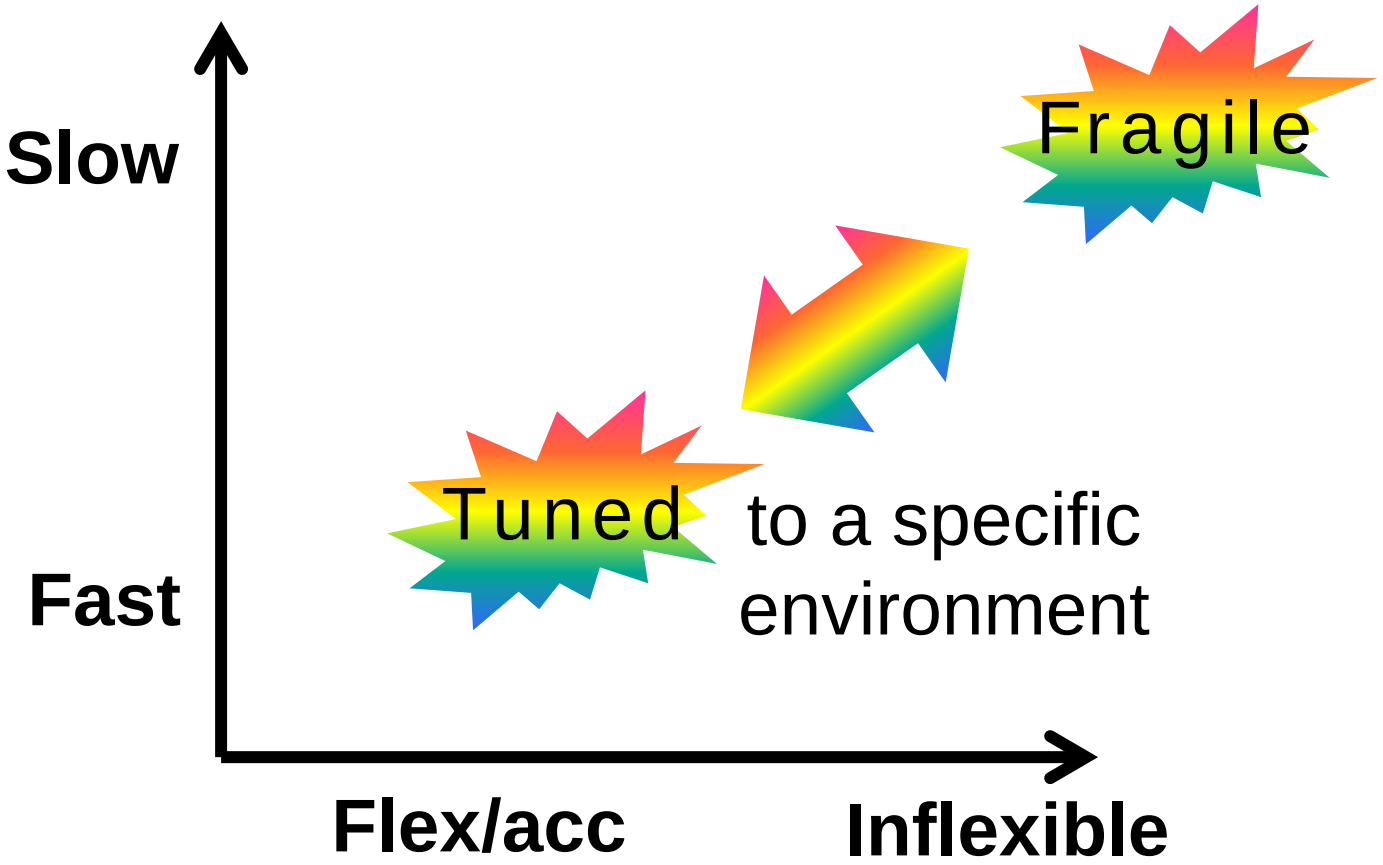
Contradiction?

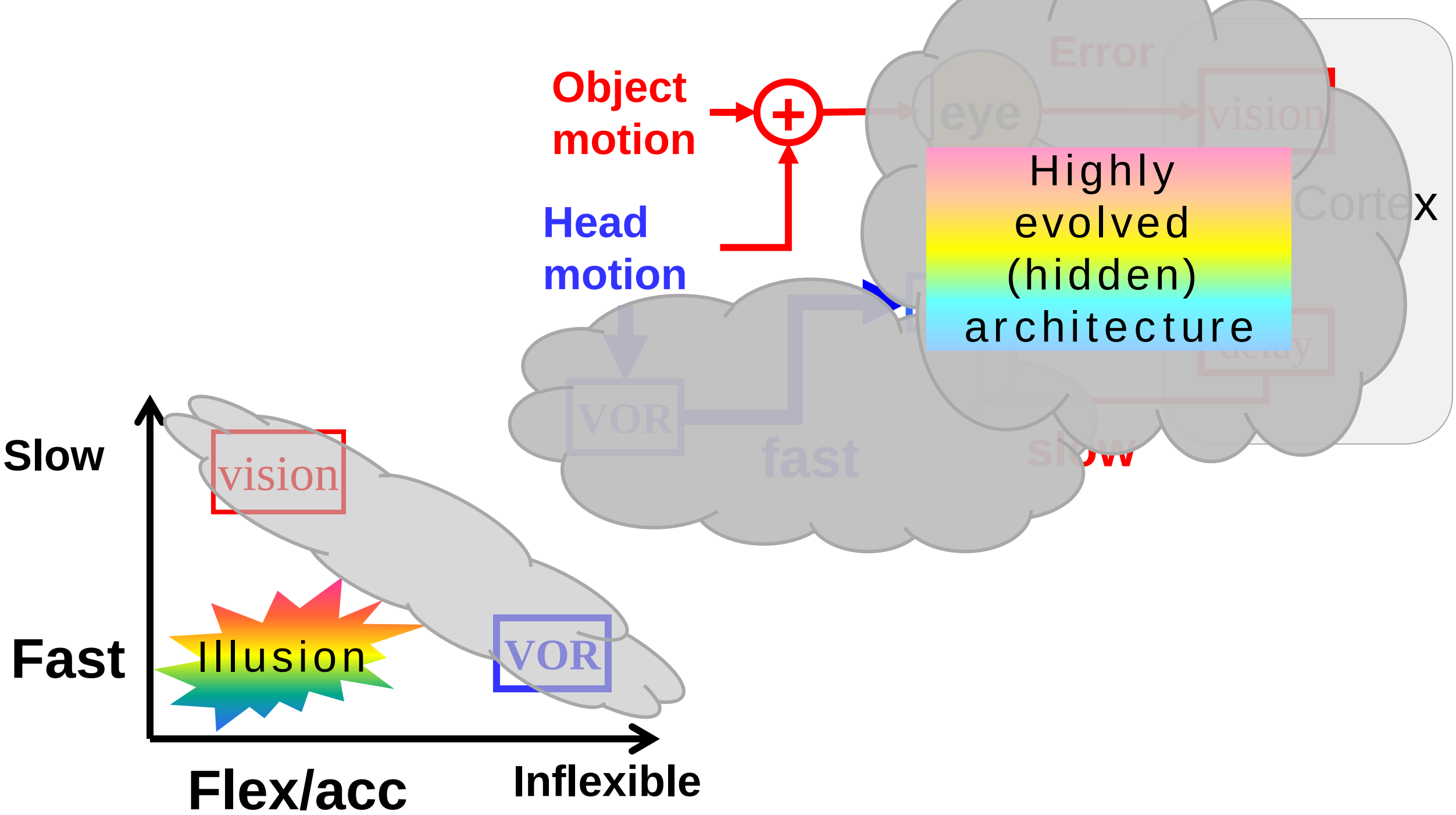


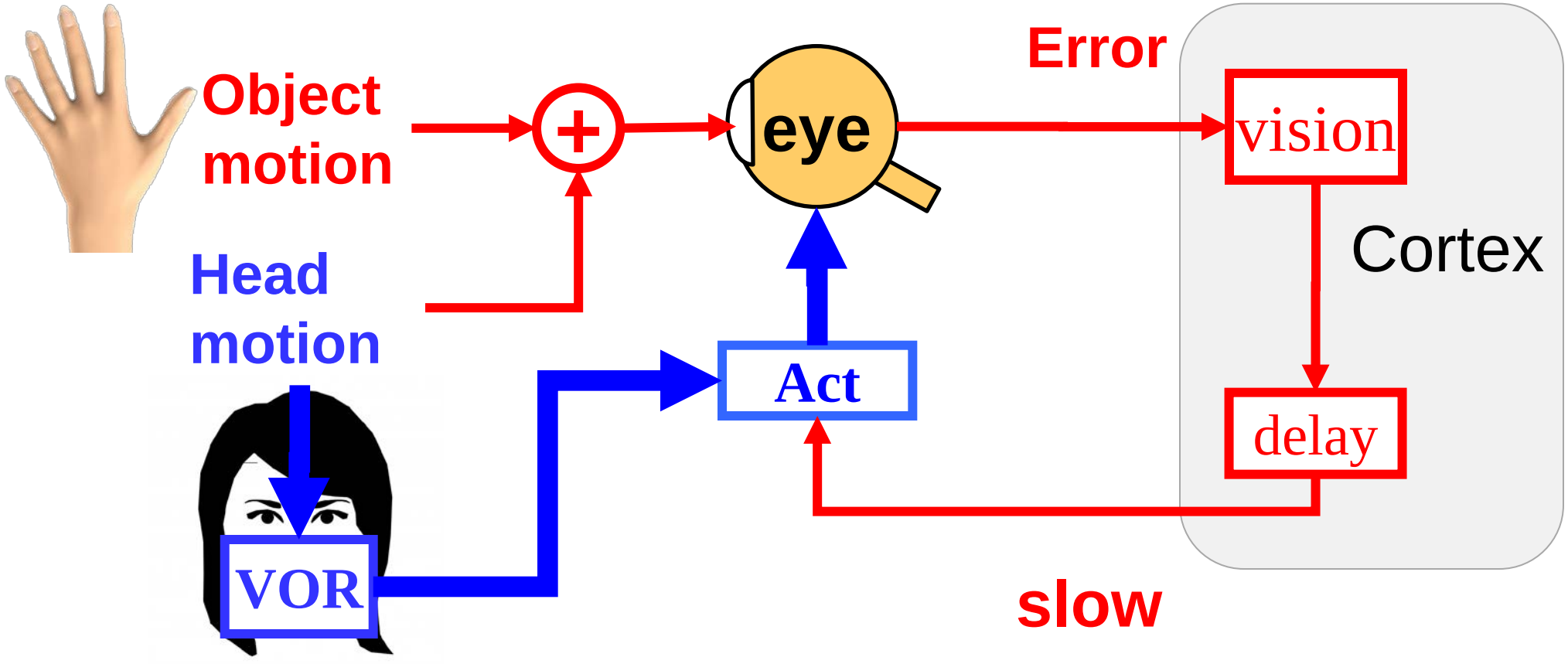
What good architecture allows.


Slow
Fast
Tuned
to a specific environment
Flex/acc
Inflexible

May be ***fragile*** in
other environments



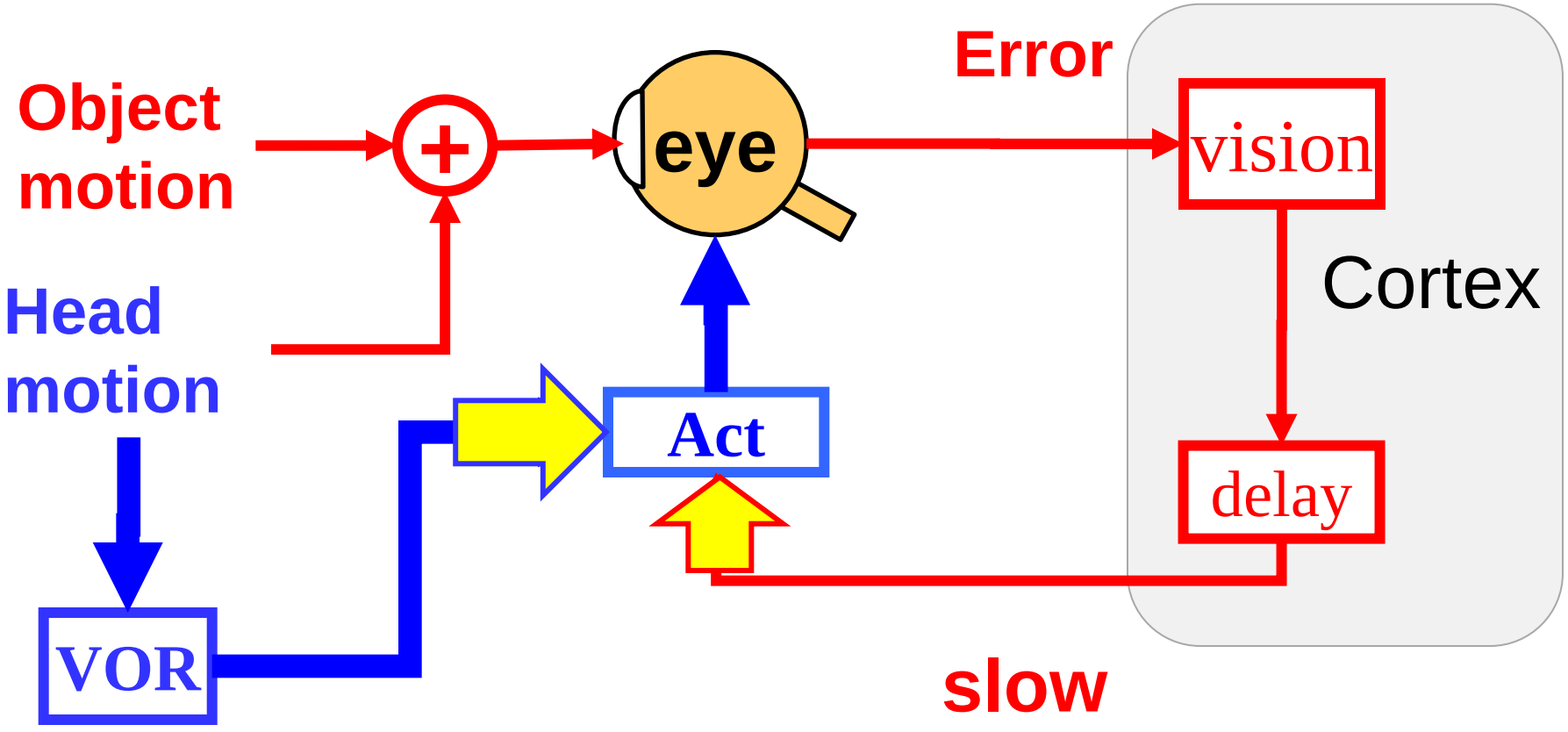

Object motion
Head motion
Error
eye
vision
Cortex
Highly evolved (hidden) architecture
VOR
fast
slow
Slow
Fast
vision
Illusion
VOR
Flex/acc
Inflexible



Summary so far.

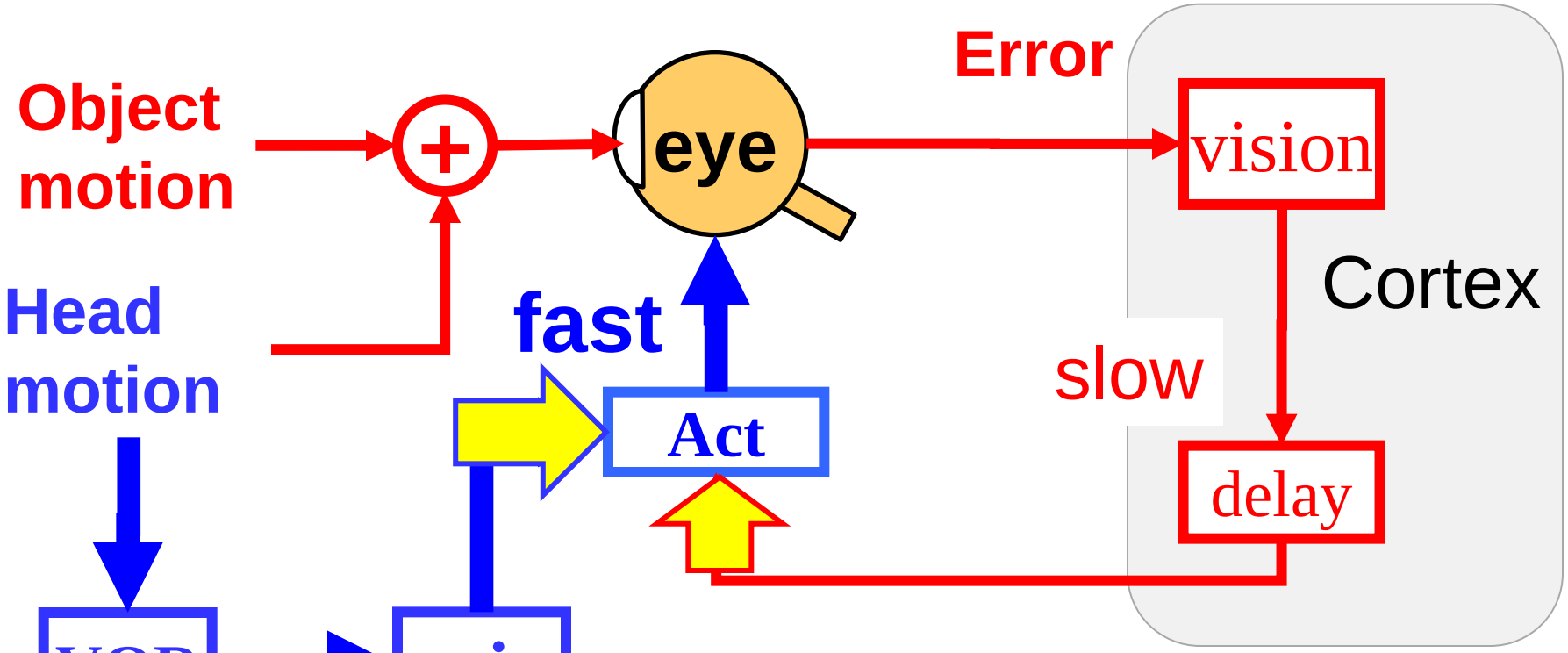
Tuned?

Next important piece?

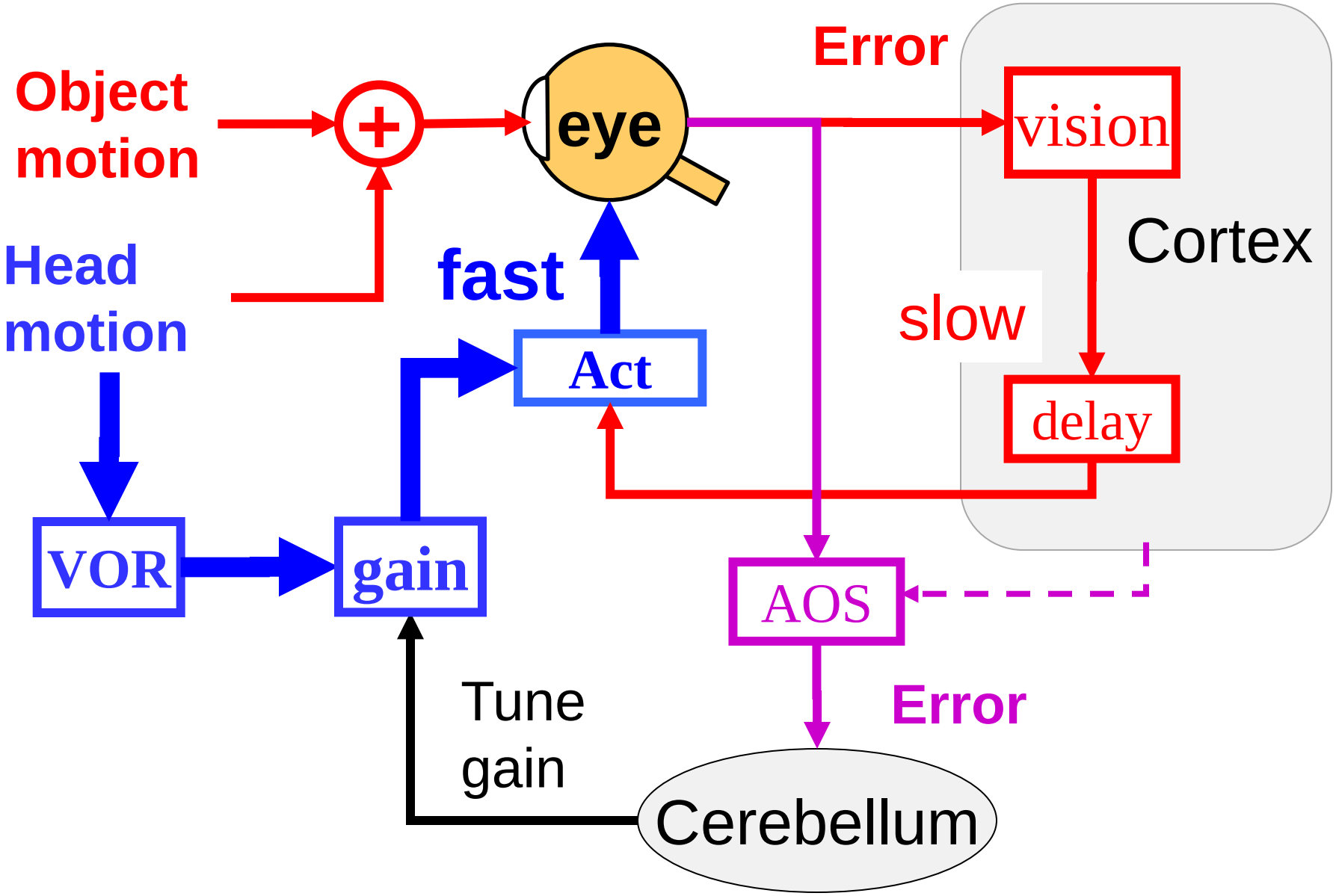


These signals must “match”

Next important piece?

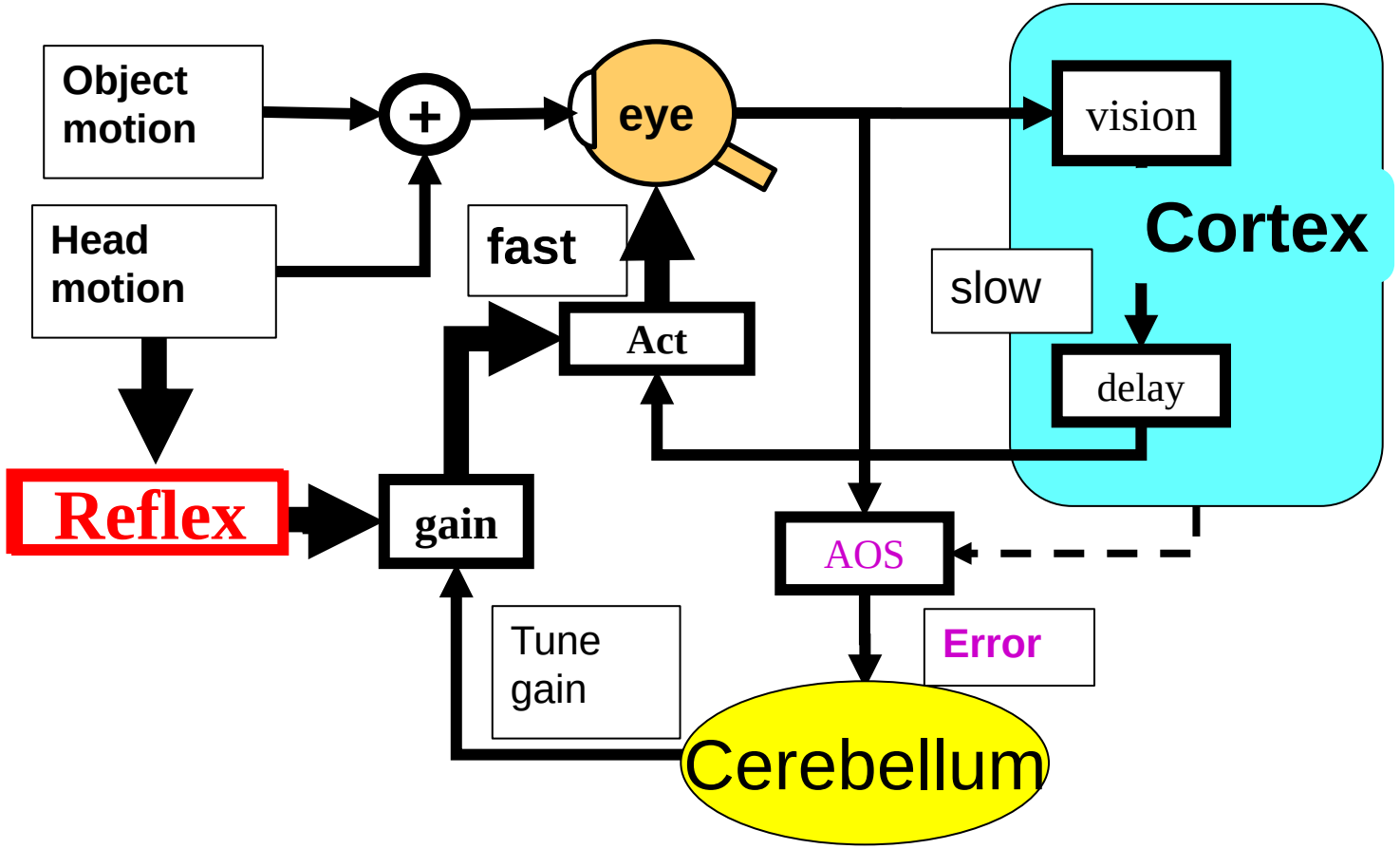


VOR and visual gain must match

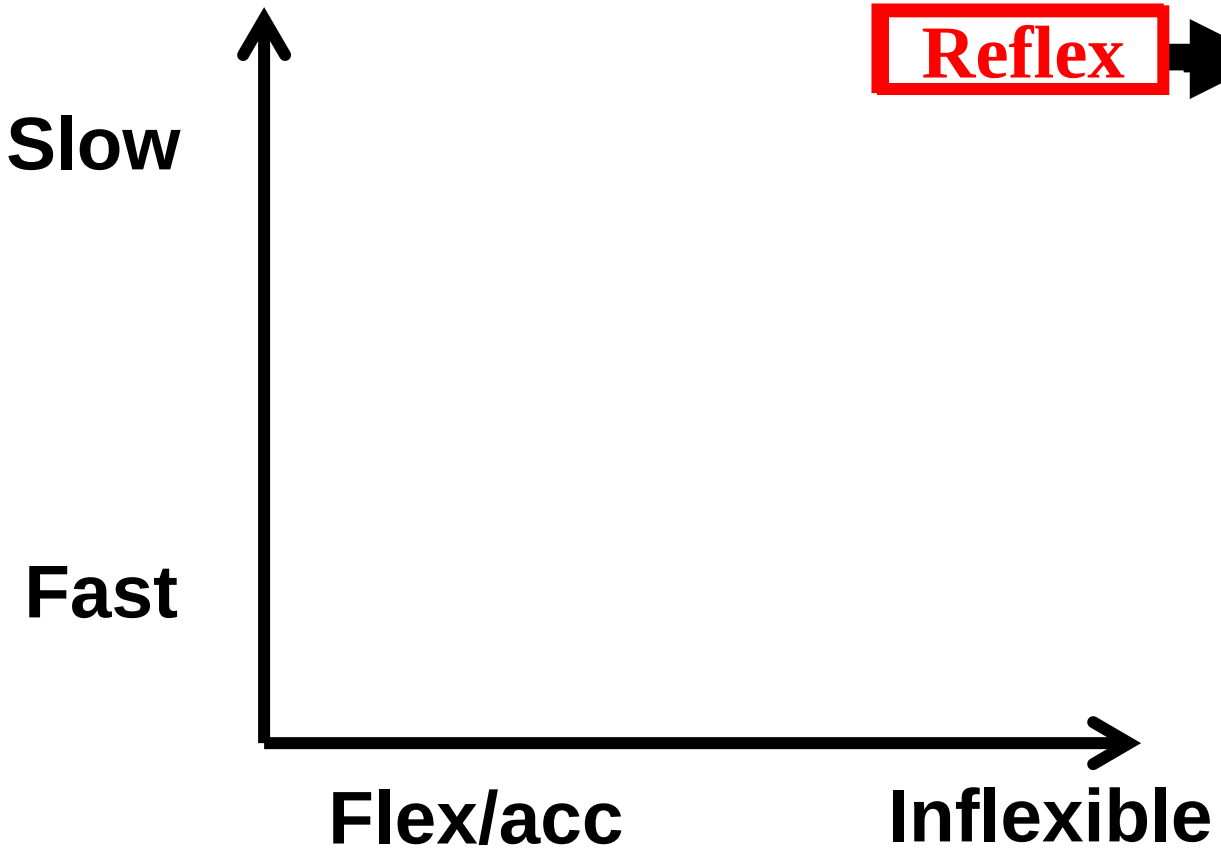


AOS = Accessory Optical system

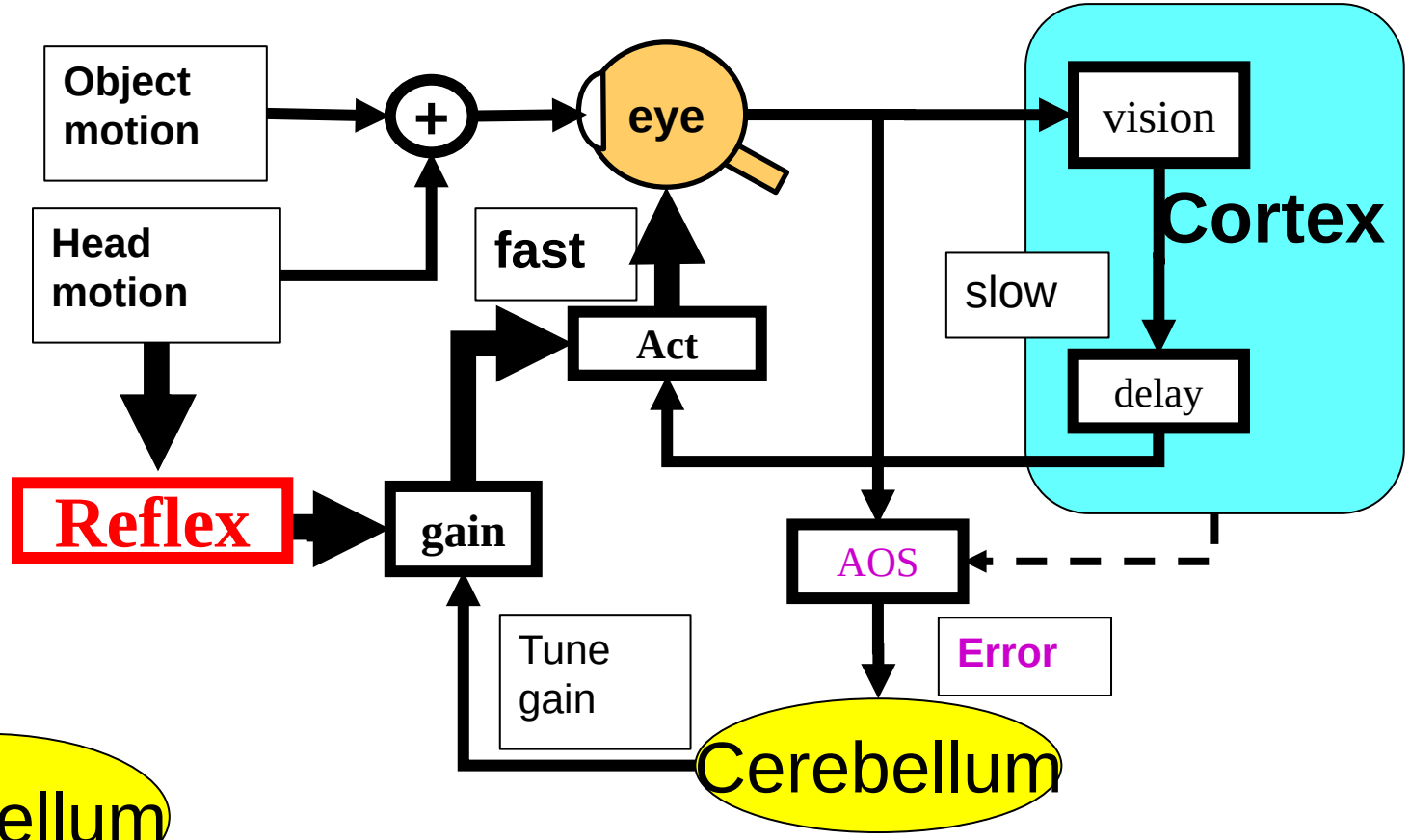
Mechanism



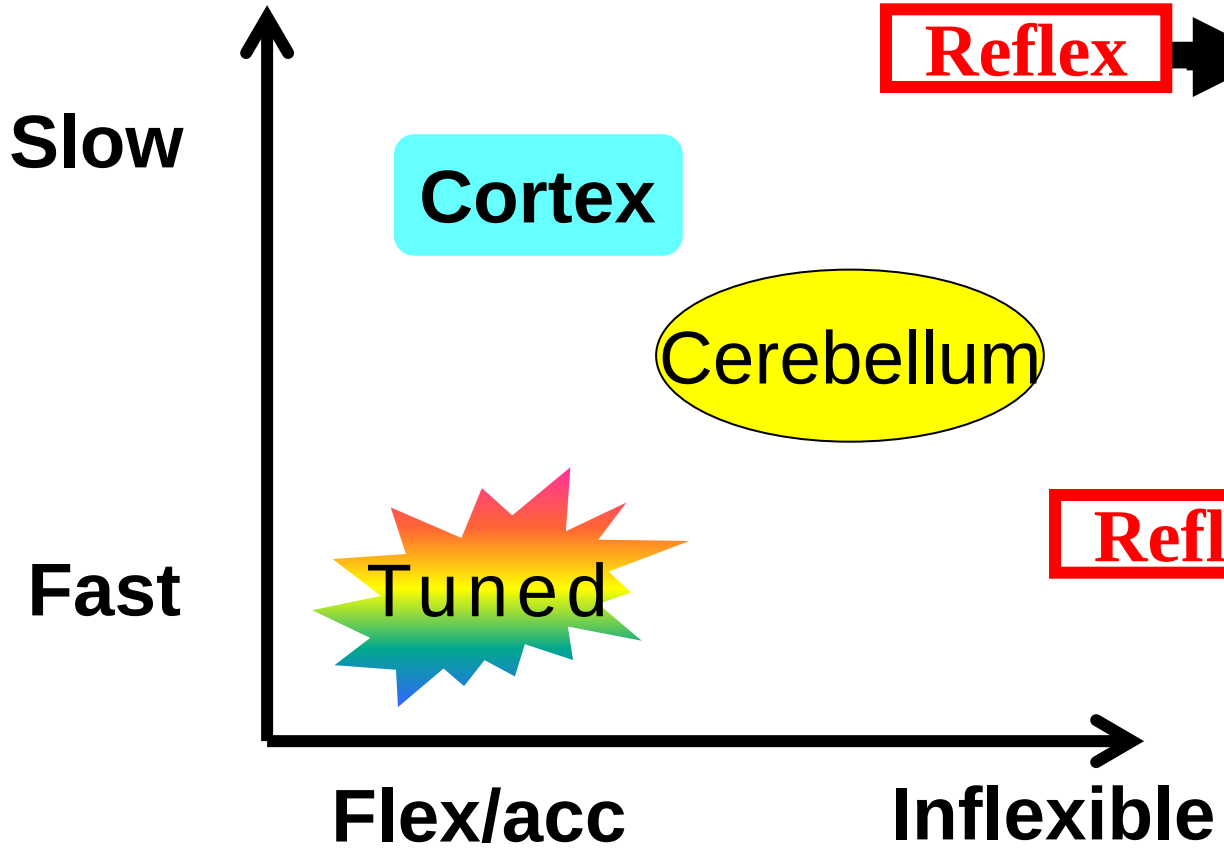
Tradeoff

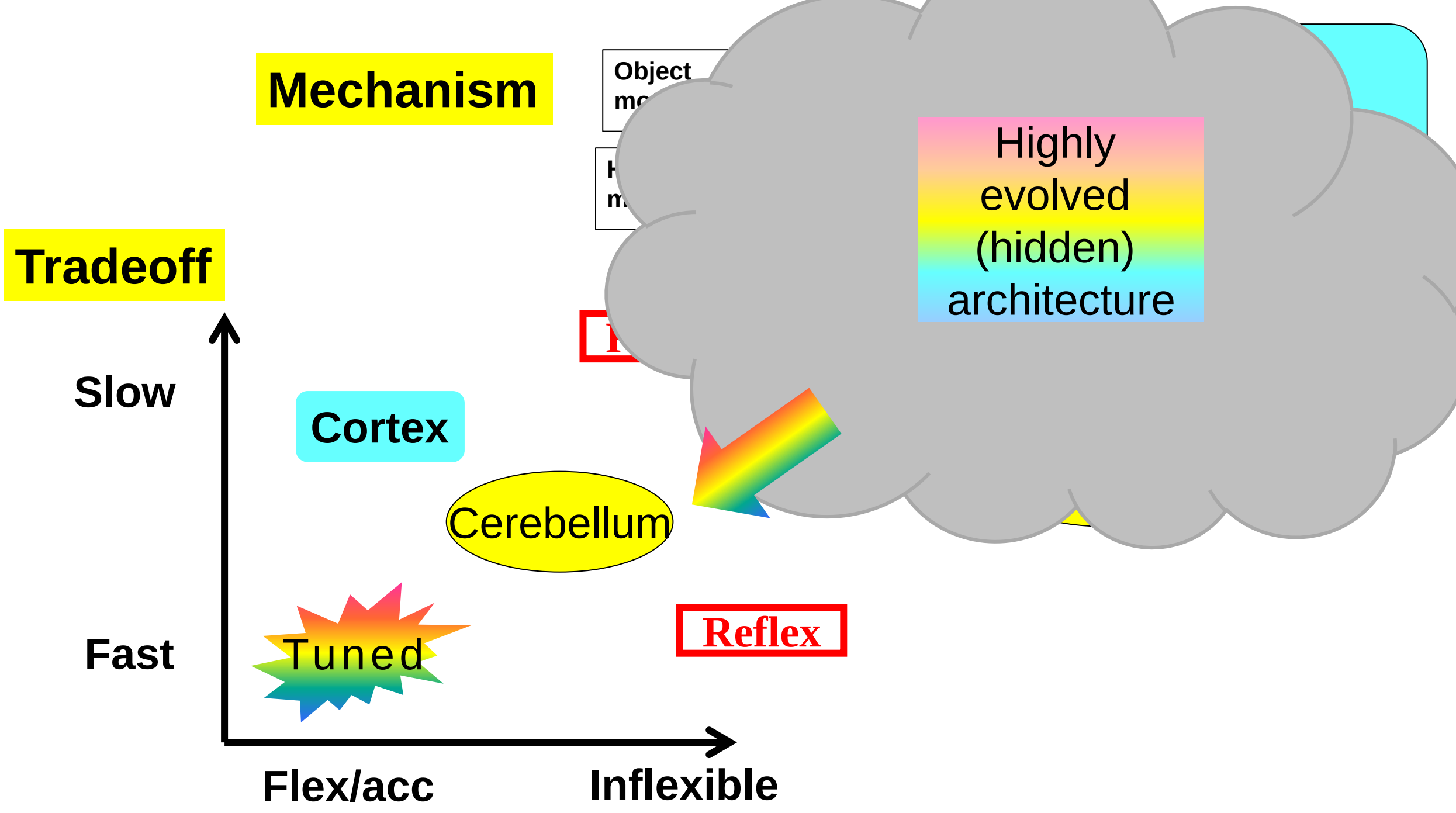


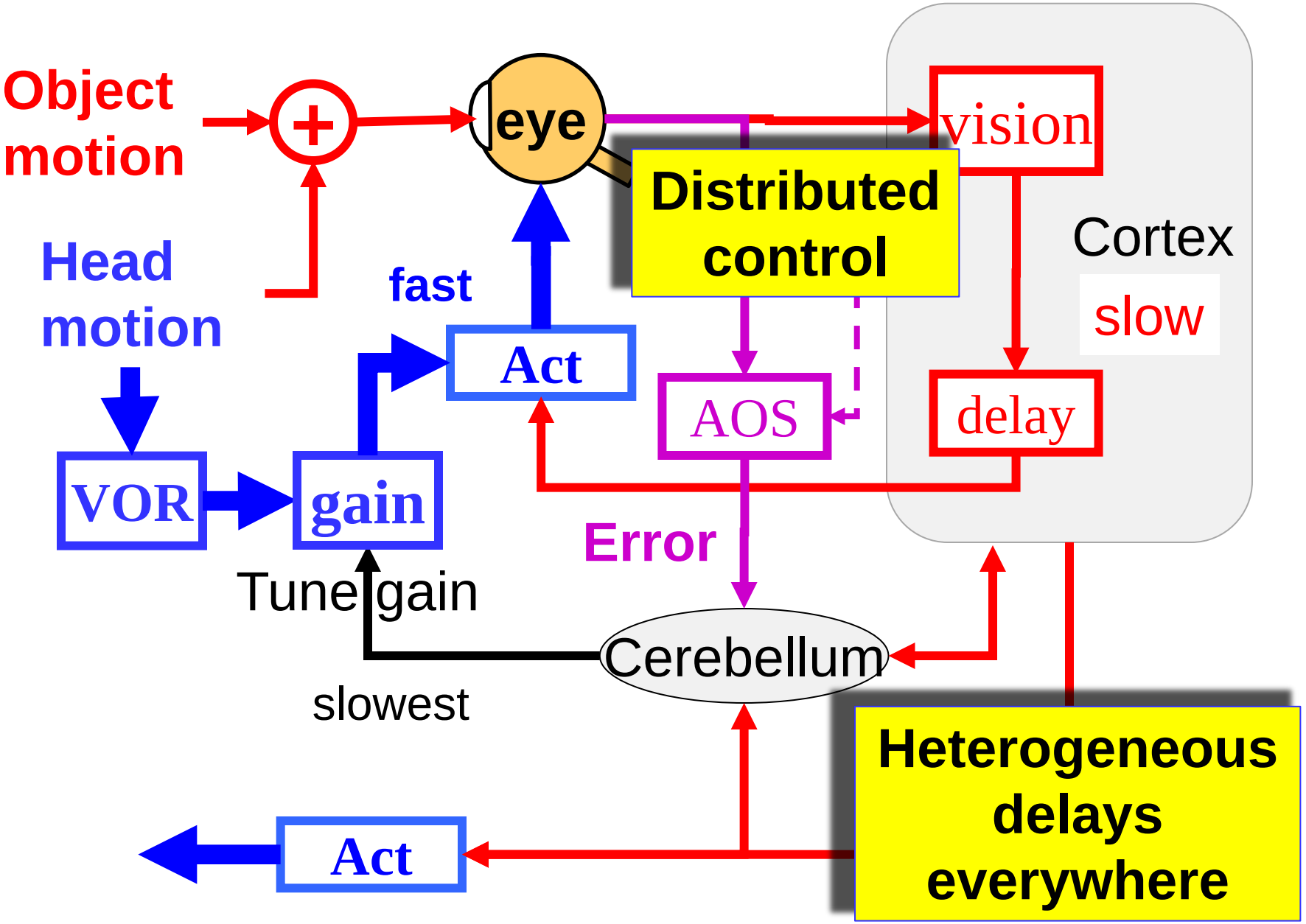
Mechanism



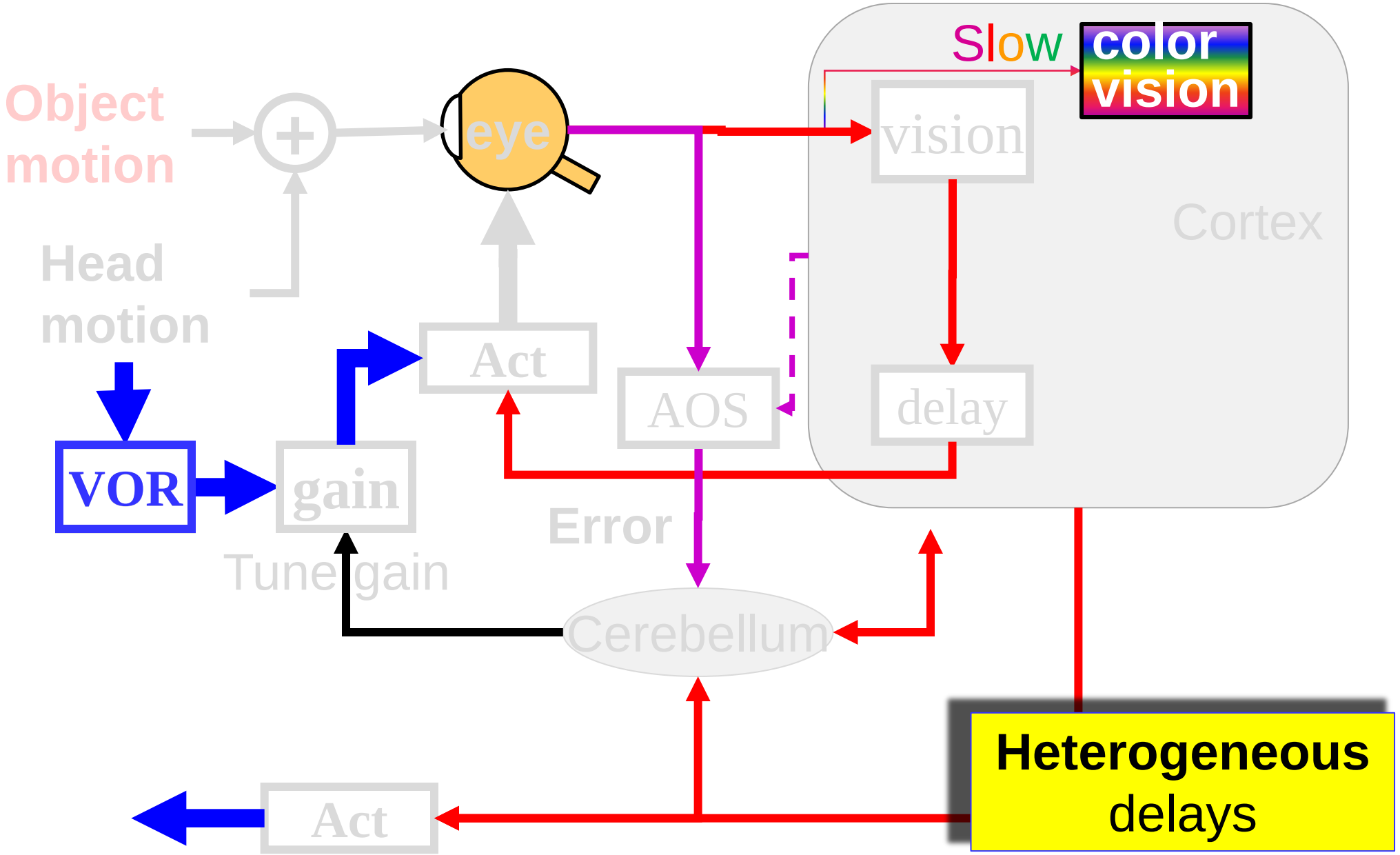
Tradeoff

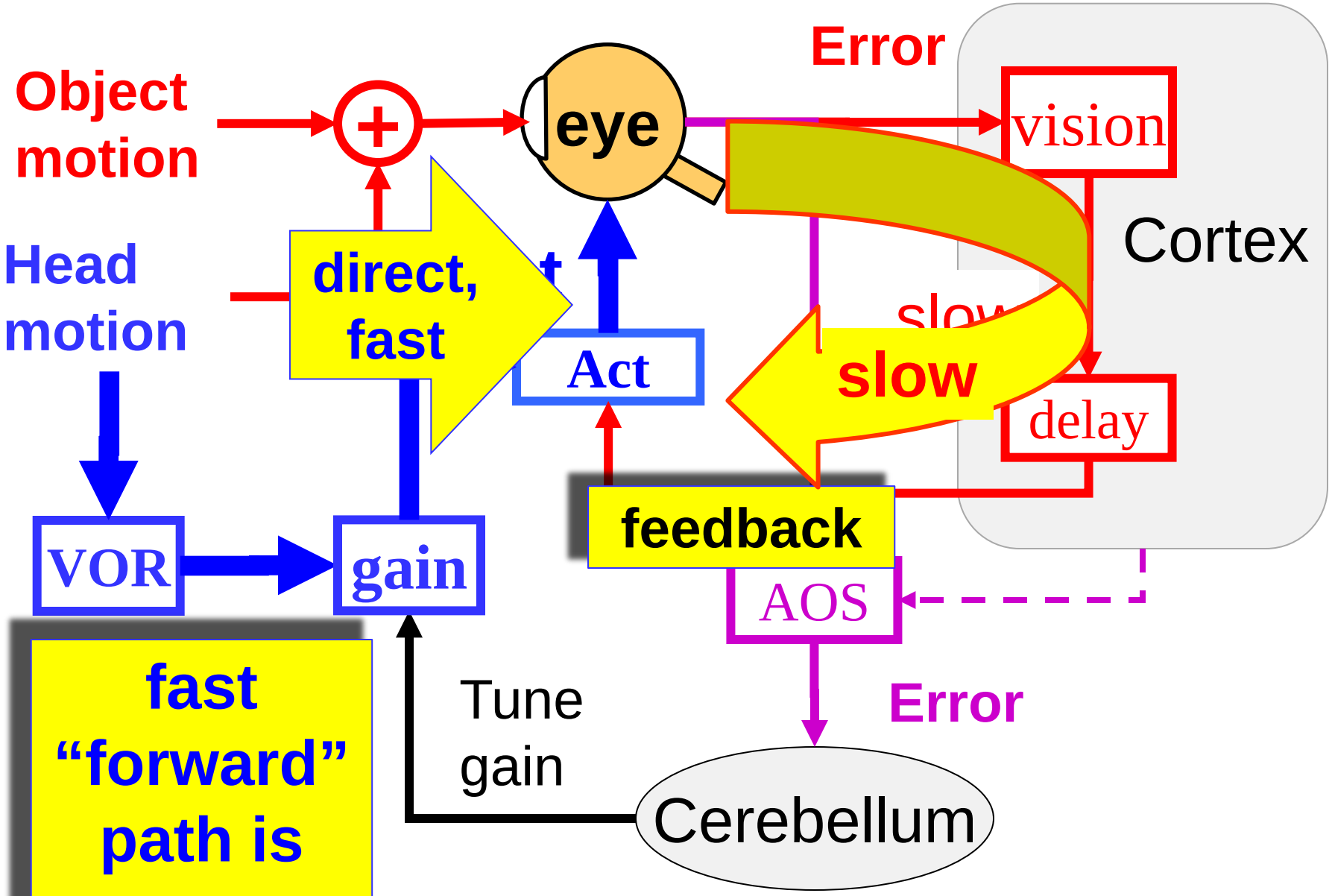



Mechanism
Object
Highly
evolved
(hidden)
architecture
Tradeoff
Slow
Cortex
Cerebellum
Fast
Tuned
Reflex
Flex/acc
Inflexible

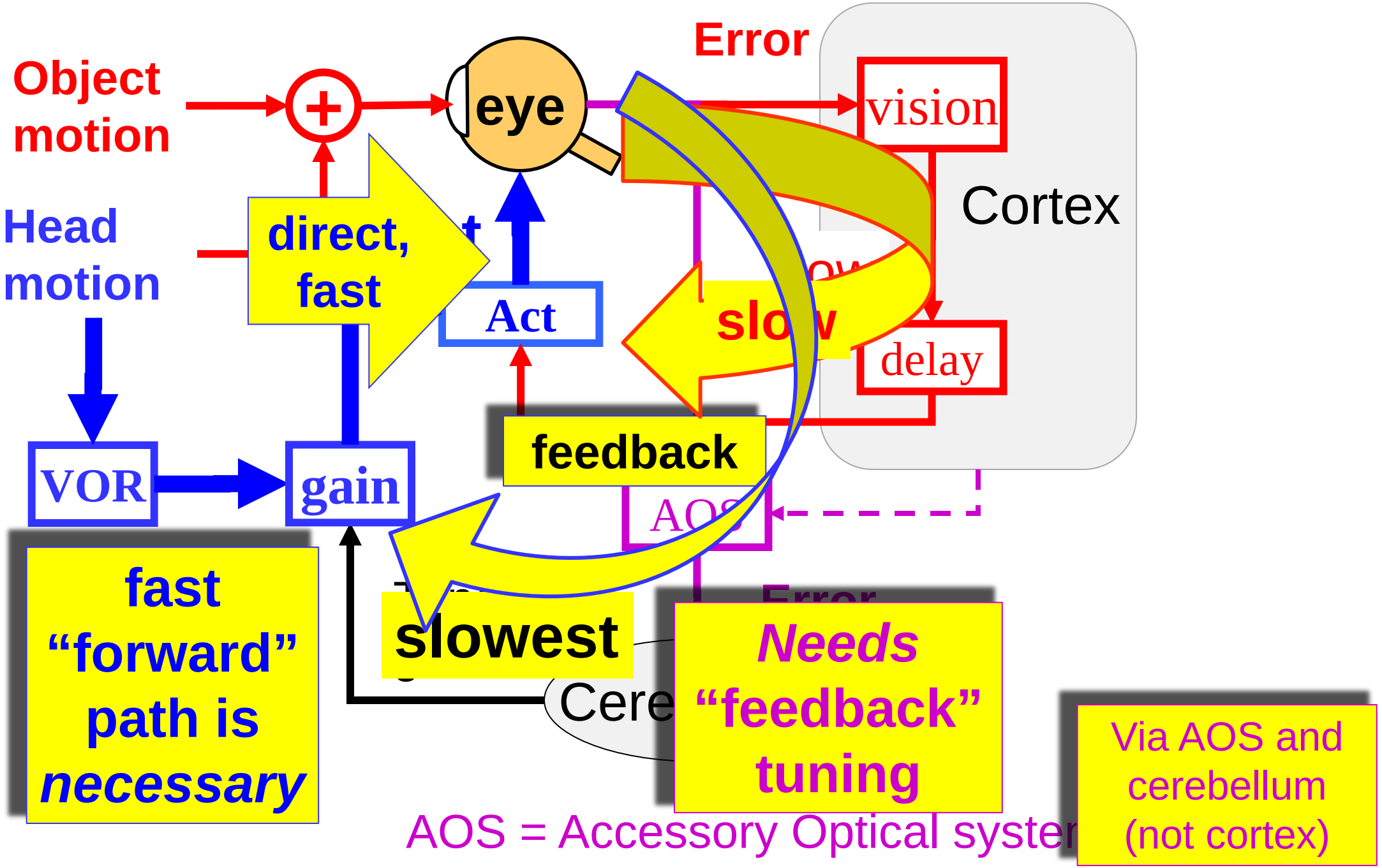


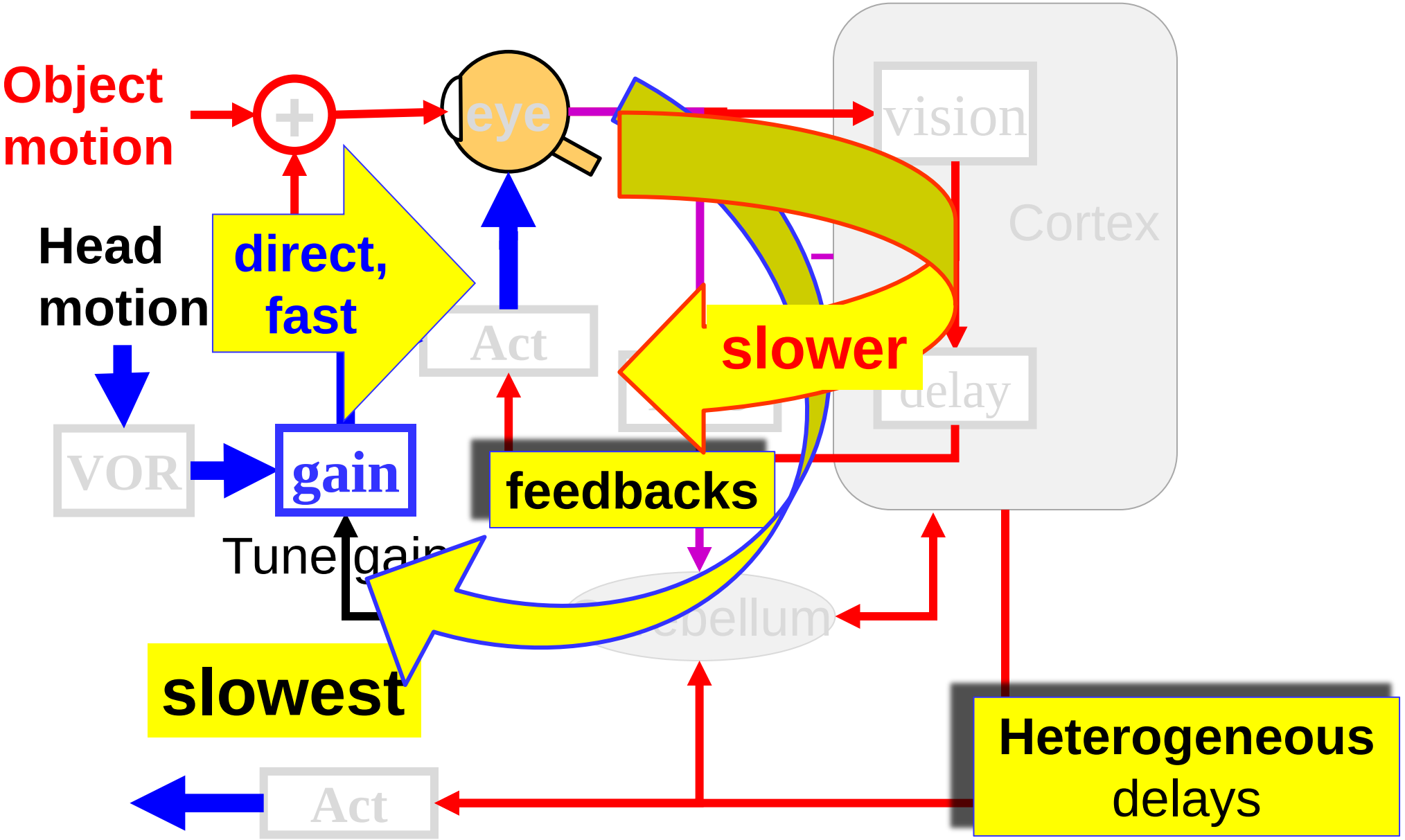
AOS = Accessory Optical system


Slow
color vision
Object motion
eye
vision
Cortex
Head motion
Act
AOS
delay
VOR
gain
Error
Tune gain
Cerebellum
Act
Heterogeneous delays

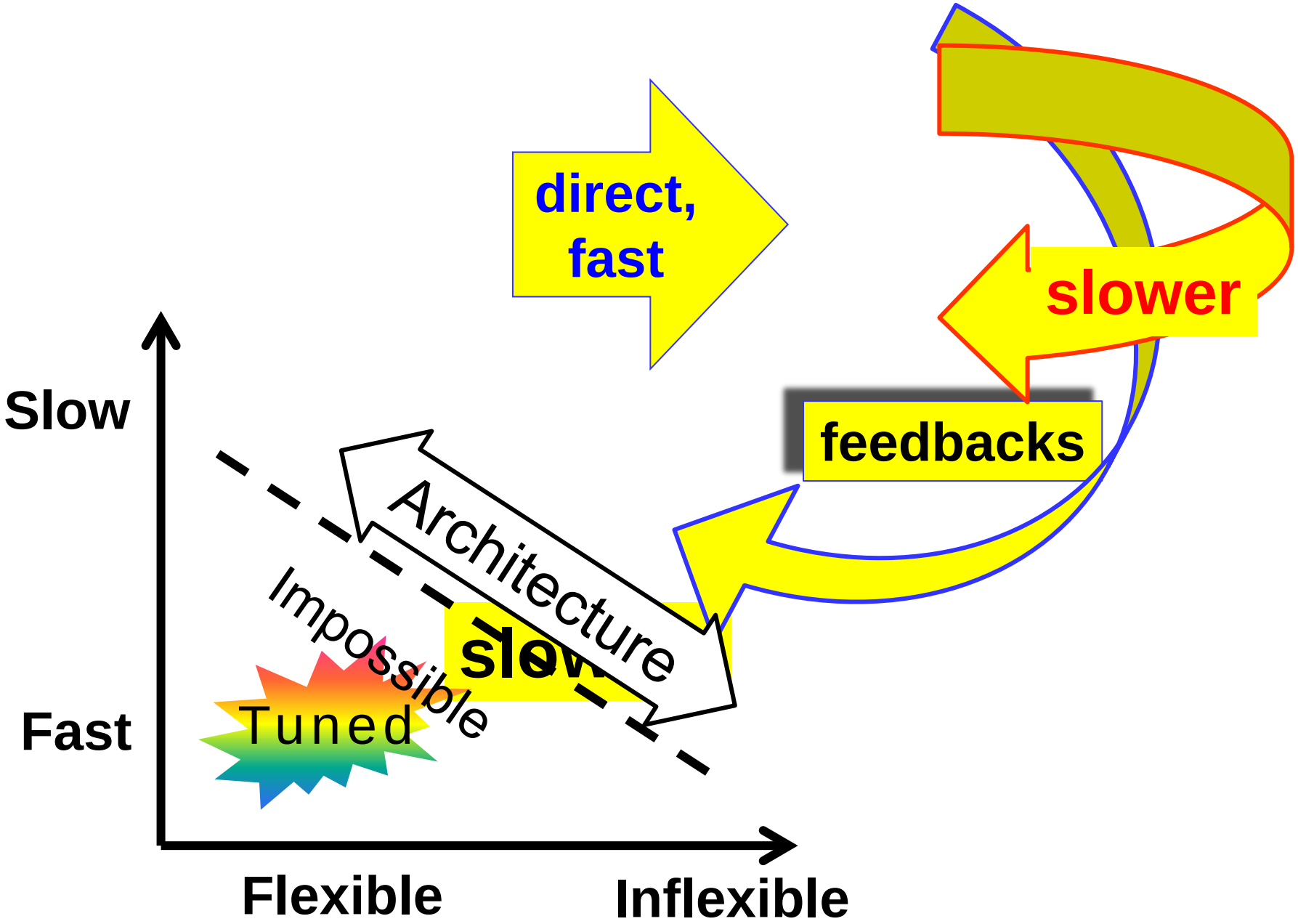


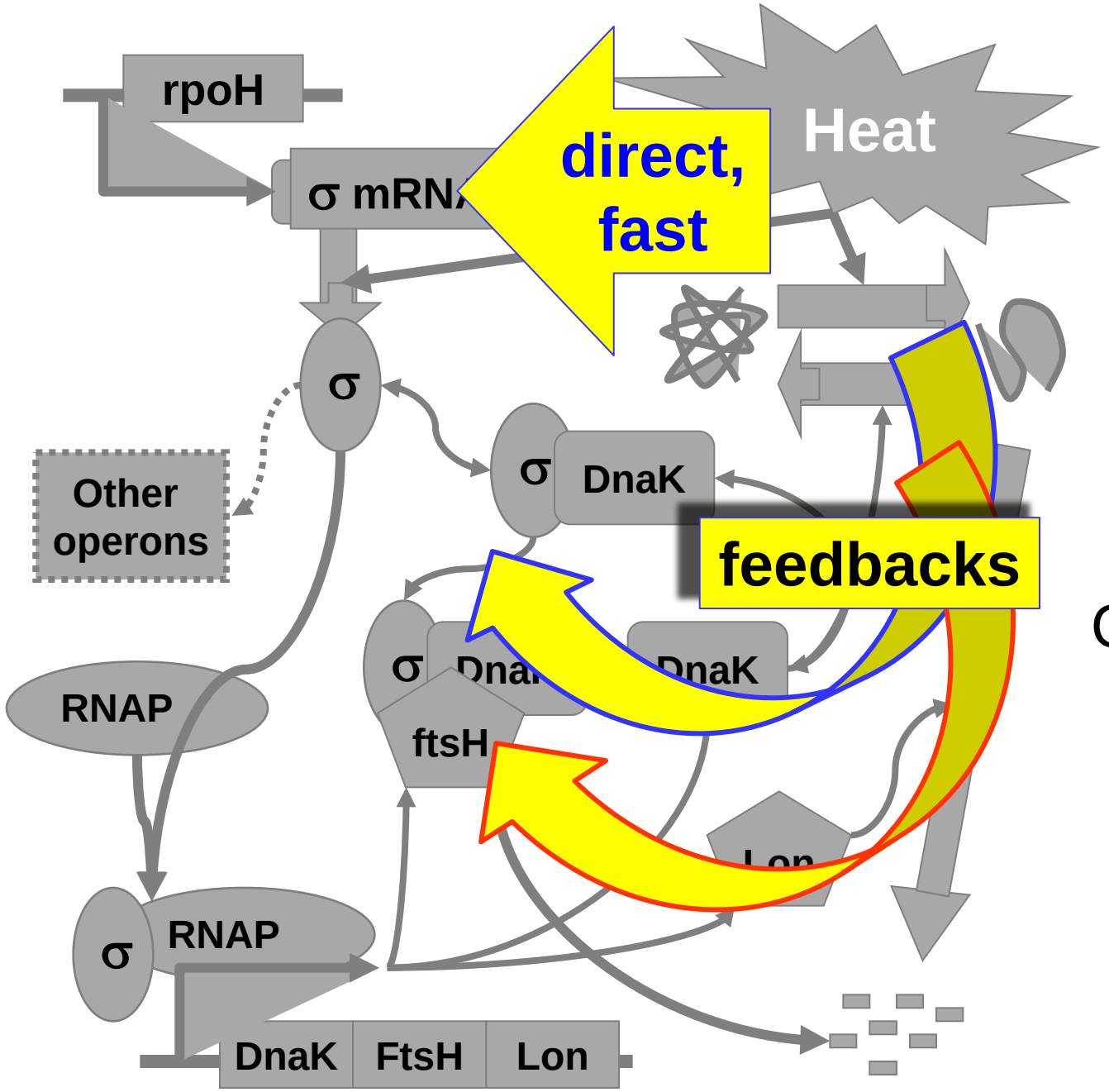
AOS = Accessory Optical system


Object motion
Head motion
eye
Error
vision
Cortex
direct, fast
Act
slow
delay
feedback
VOR
gain
AOS
slowest
fast "forward" path is *necessary*
Needs "feedback" tuning
Via AOS and cerebellum (not cortex)
AOS = Accessory Optical syster


Object motion
+
eye
vision
Cortex
Head motion
direct, fast
Act
slower
delay
VOR
gain
feedbacks
Tune gain
Cerebellum
slowest
Act
Heterogeneous delays

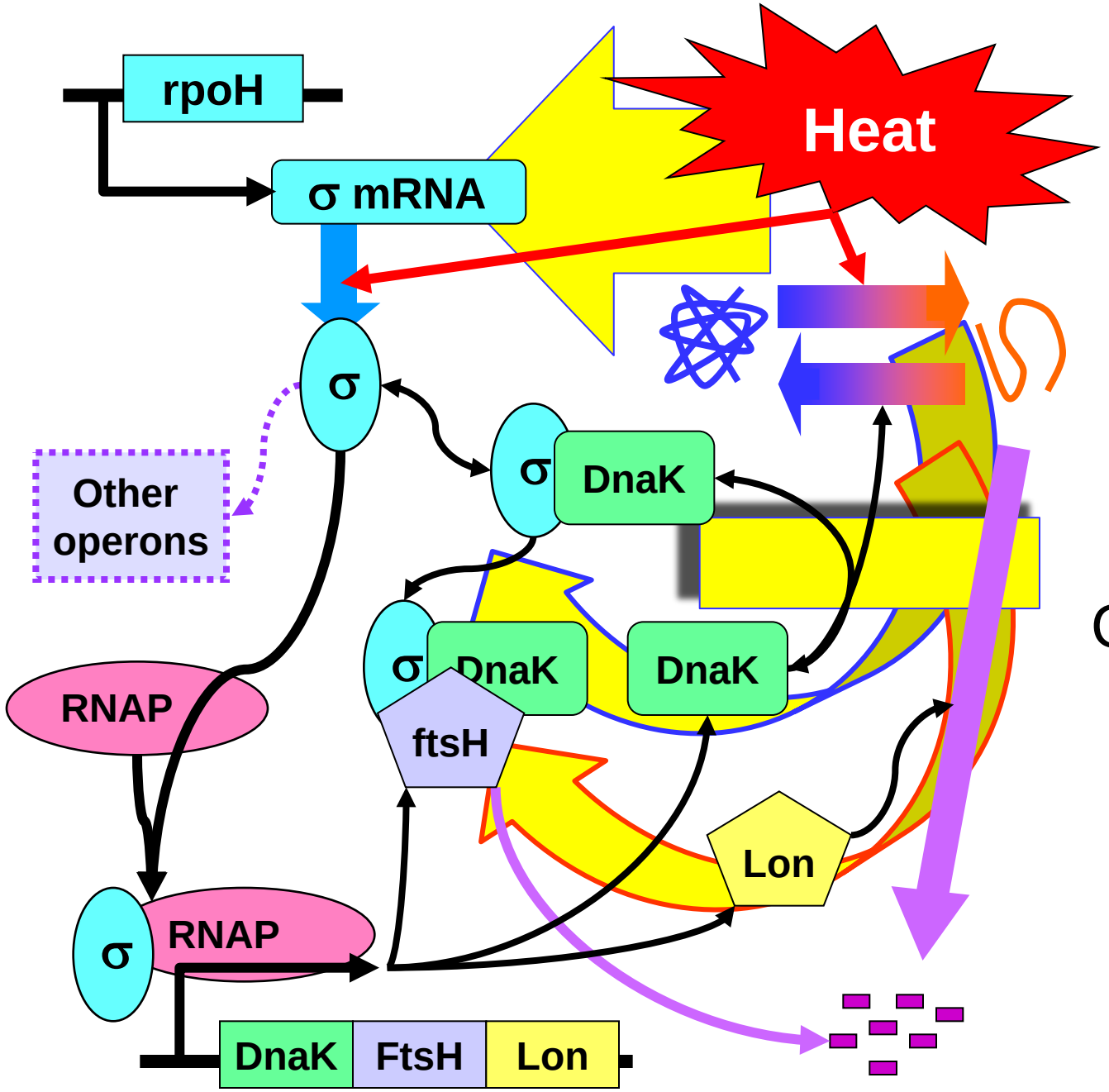
Common pattern





Control of heat shock in E. Coli

El-Samad, Kurata, Doyle, Gross, Khammash (2005), Surviving Heat Shock: Control Strategies for Robustness and Performance. *PNAS* (8): FEB 22, 2005



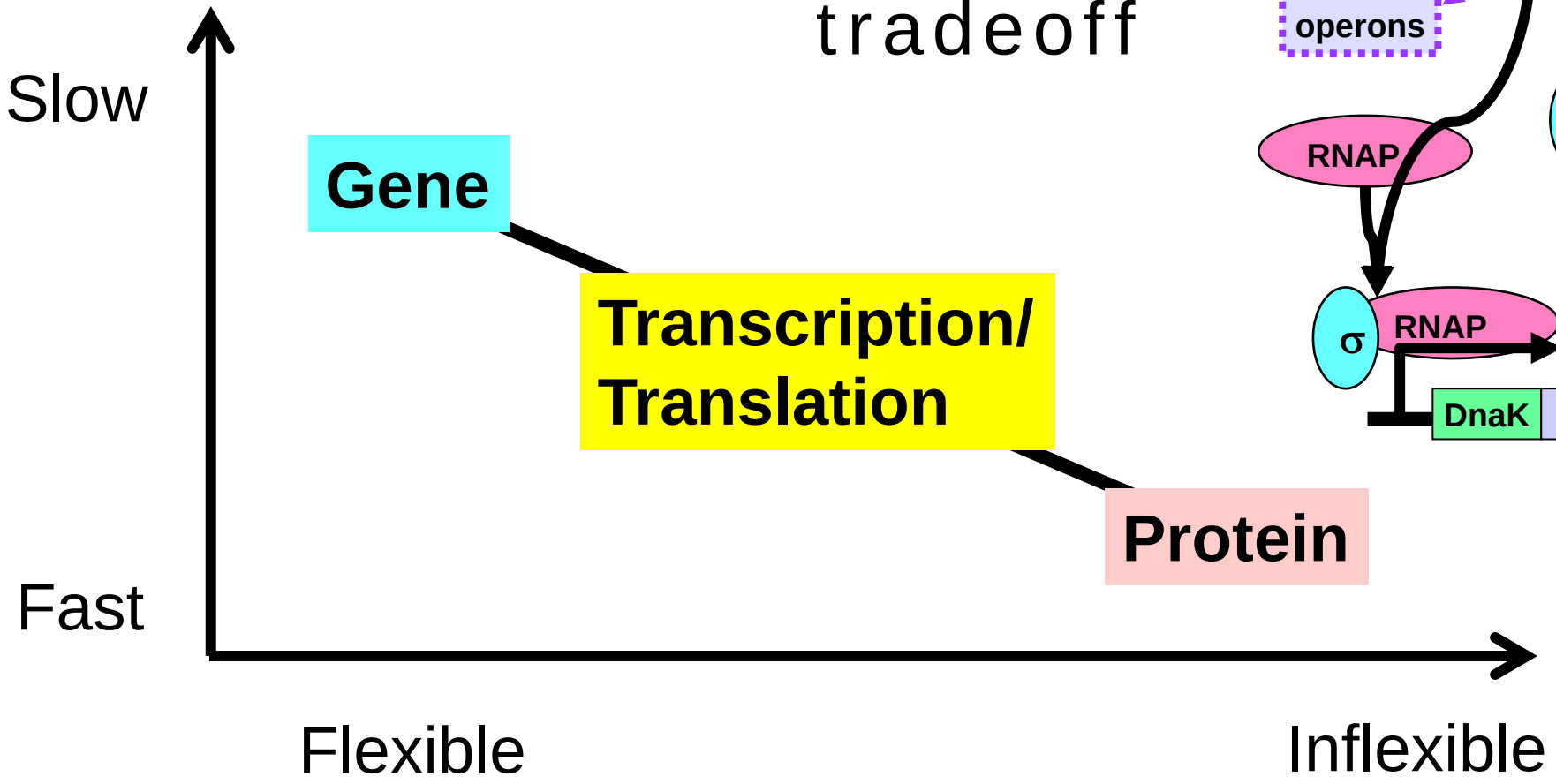
Control of heat shock in E. Coli

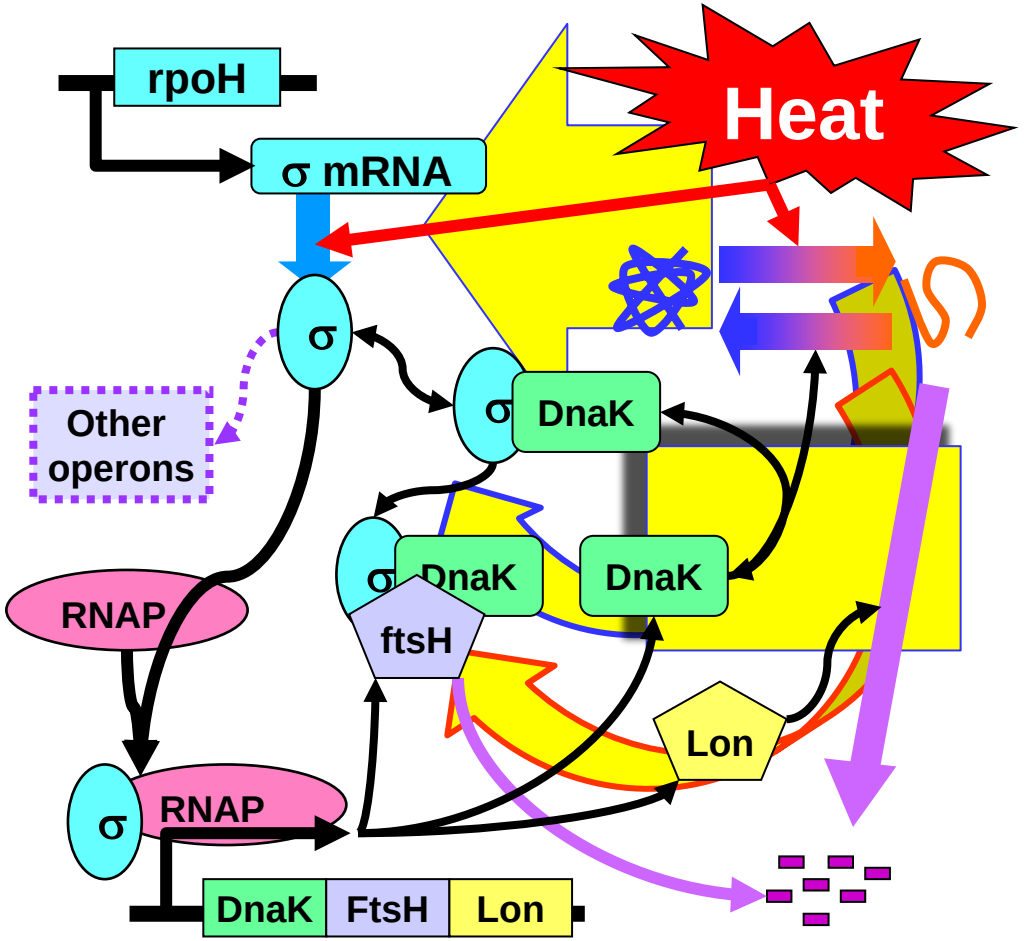
mechanism

El-Samad, Kurata, Doyle, Gross, Khammash (2005), Surviving Heat Shock: Control Strategies for Robustness and Performance. *PNAS* (8): FEB 22, 2005

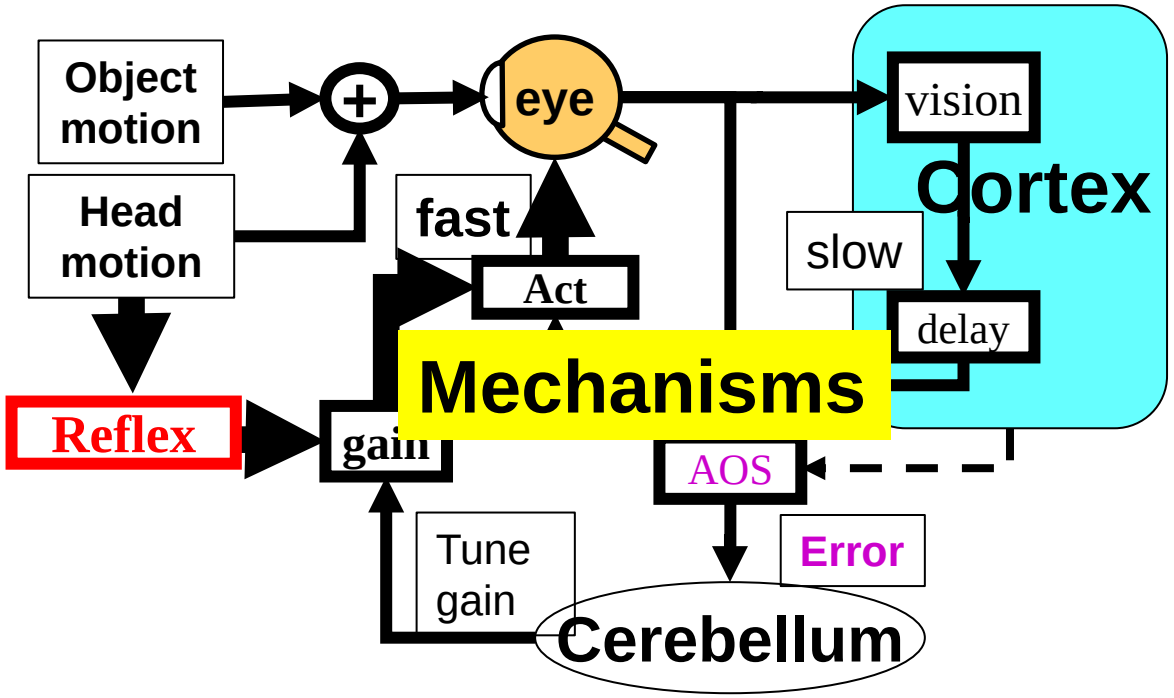
Where function is *controlled*

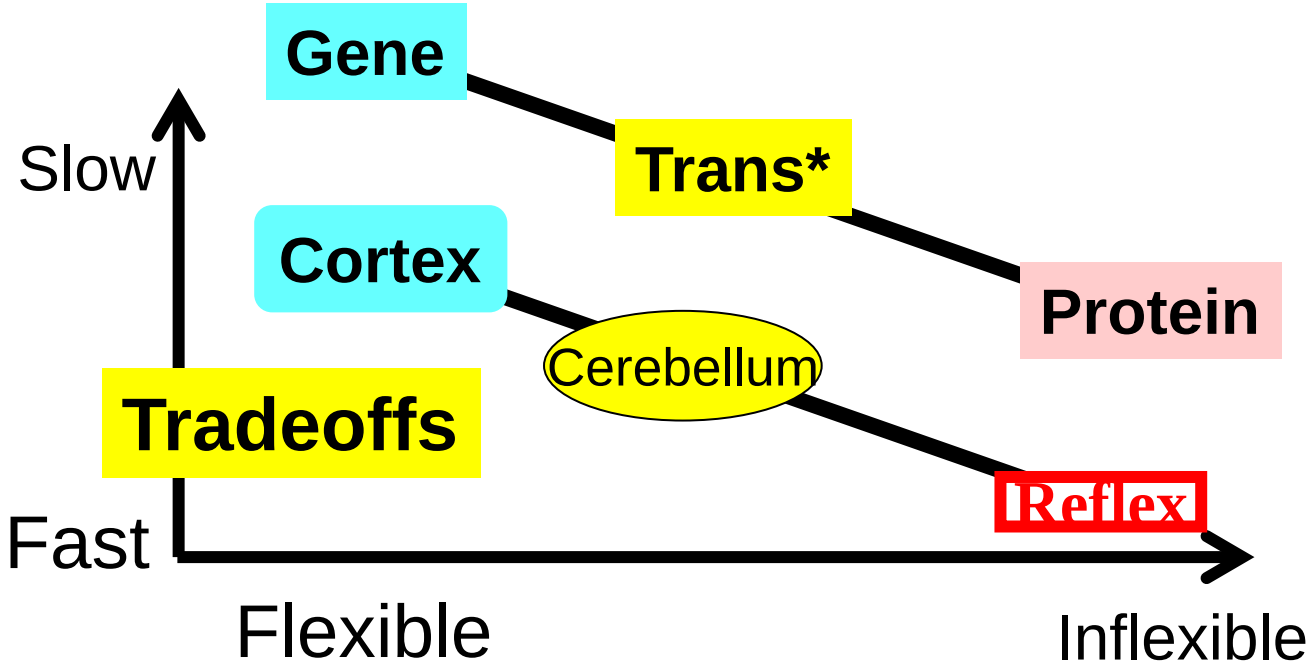
Universal tradeoff

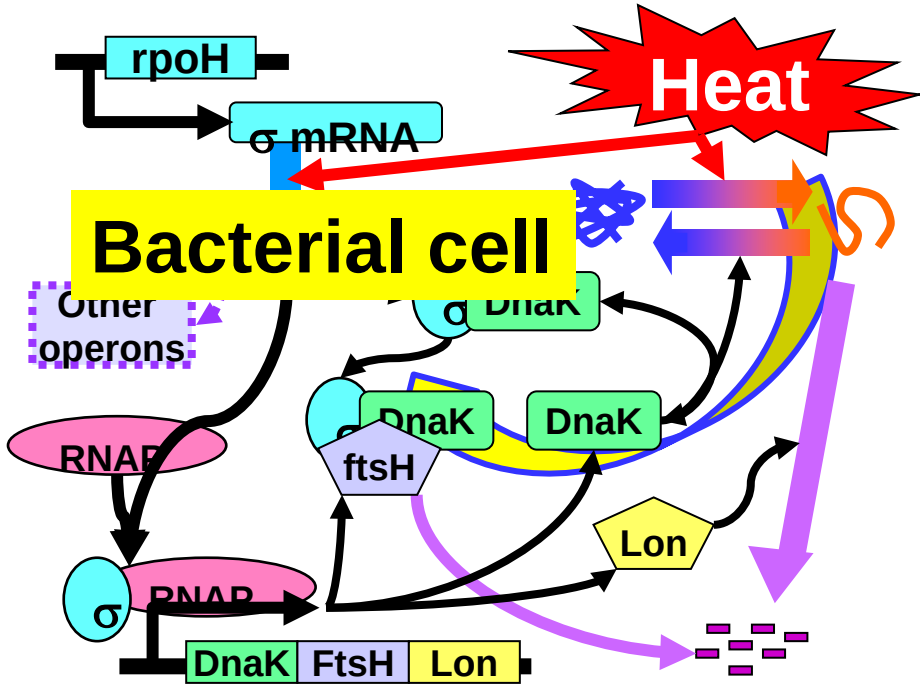




- Universal?
- Domain specific?





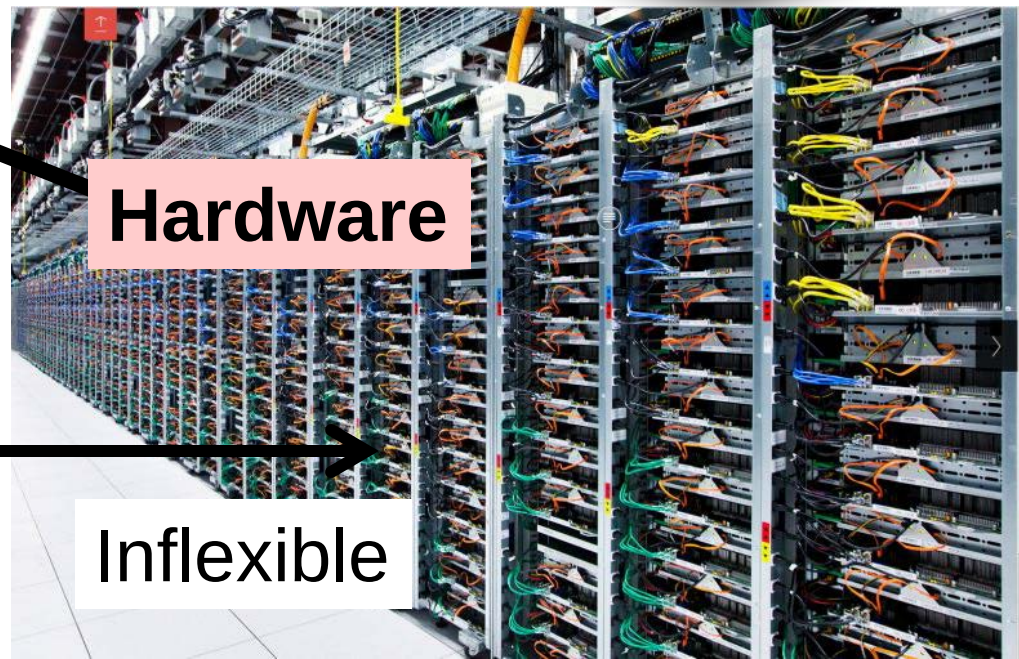


Universal tradeoff

Slow

Apps

Operating System



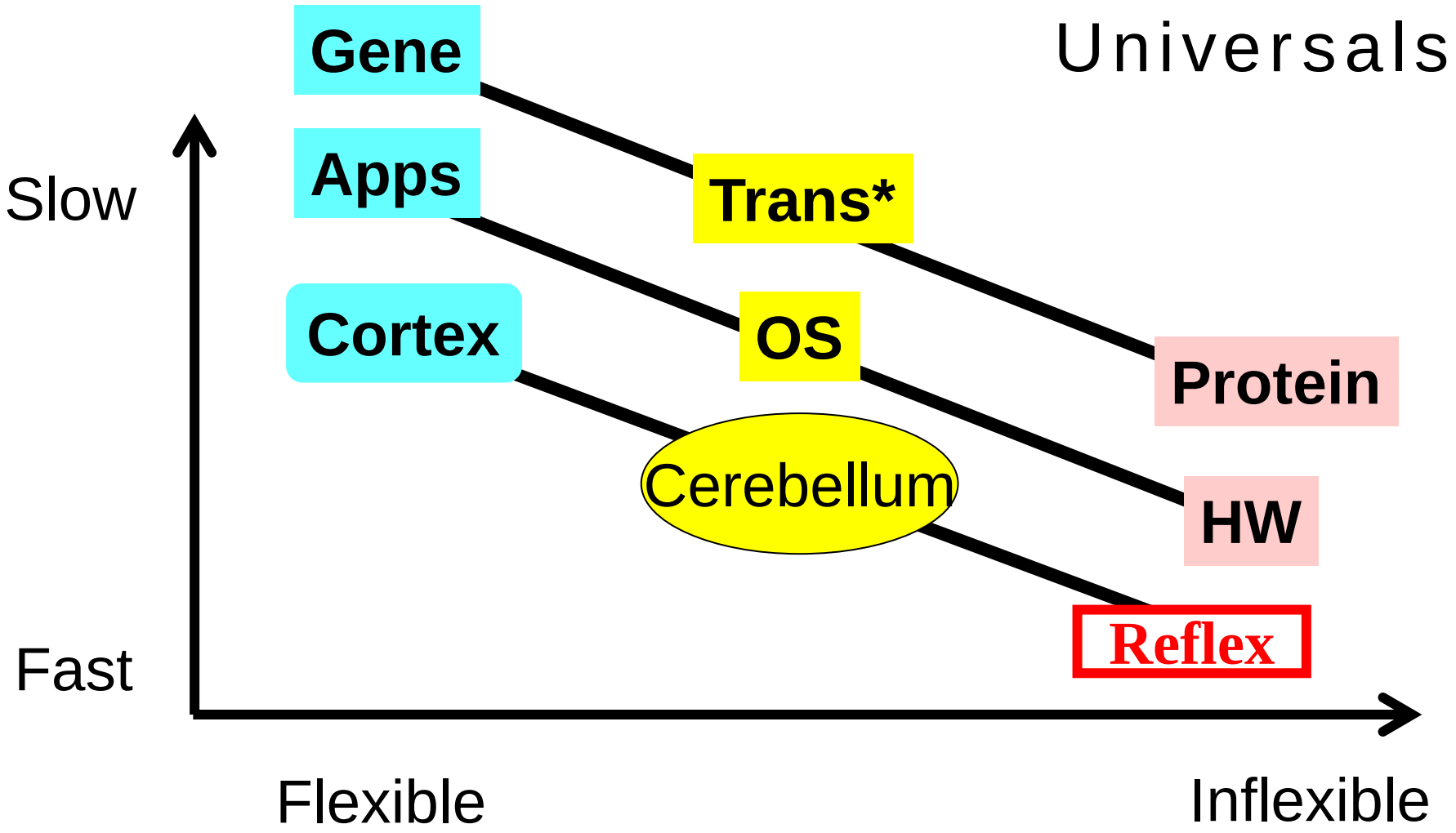
Hardware

Fast

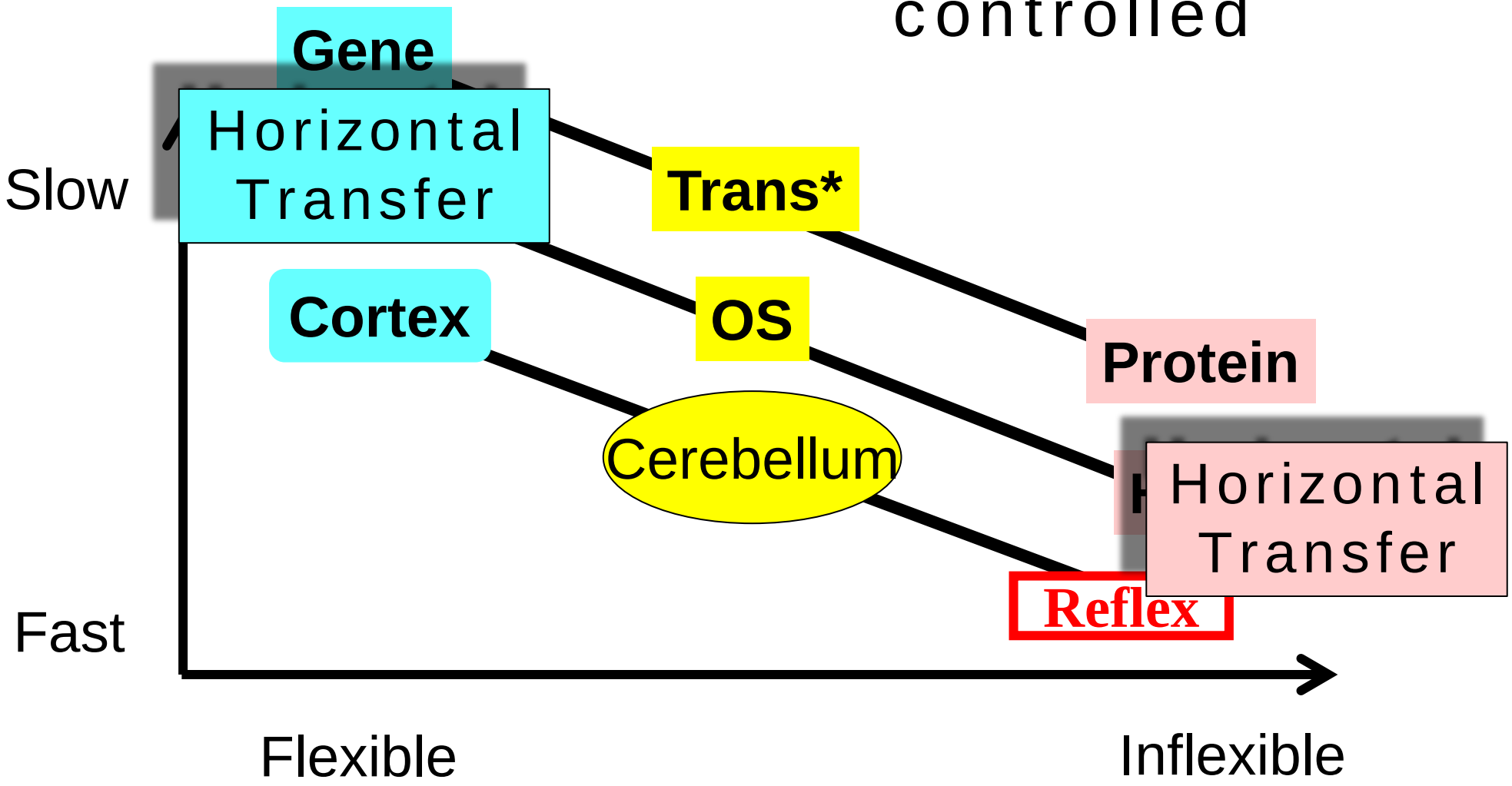
Flexible

Inflexible

Where function is *controlled*

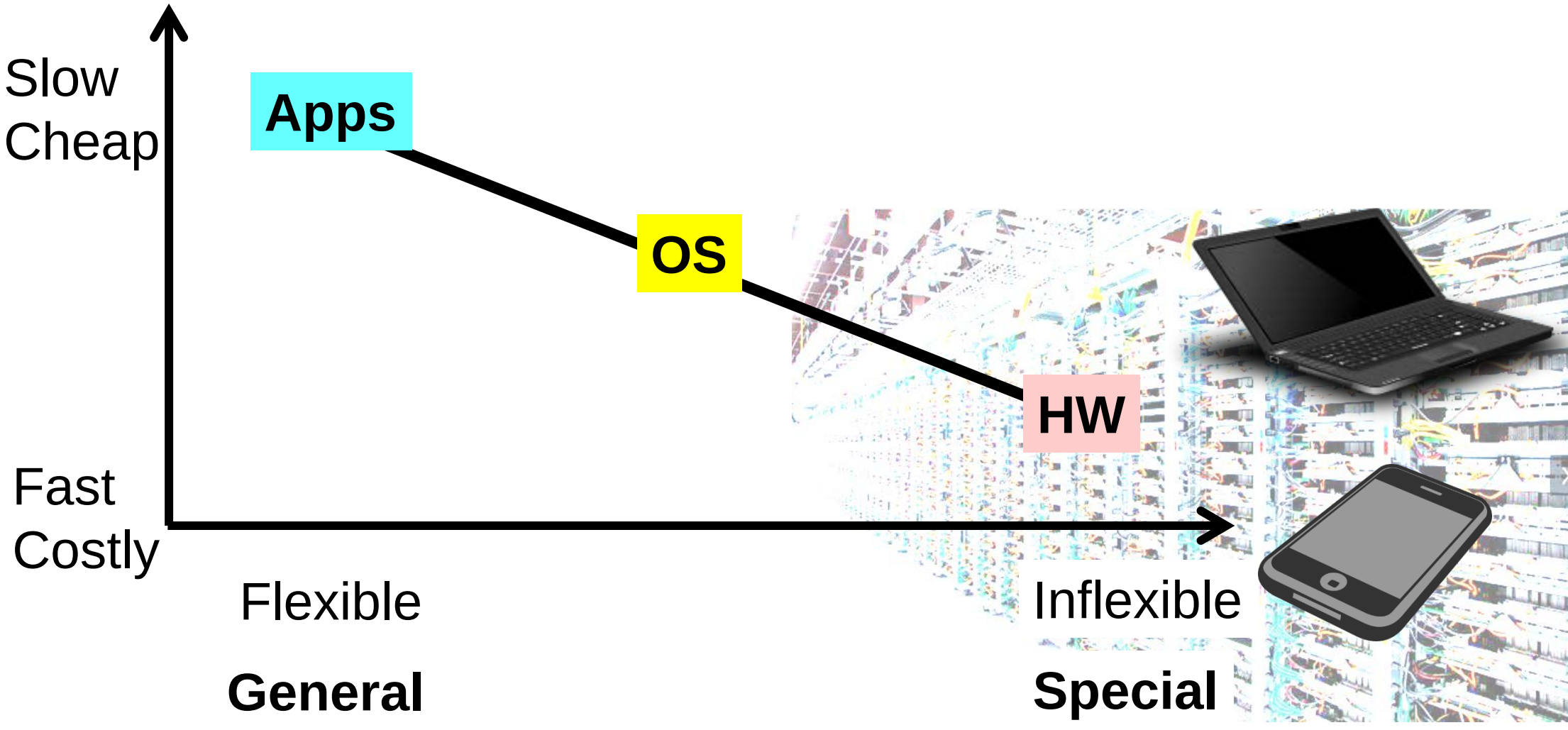


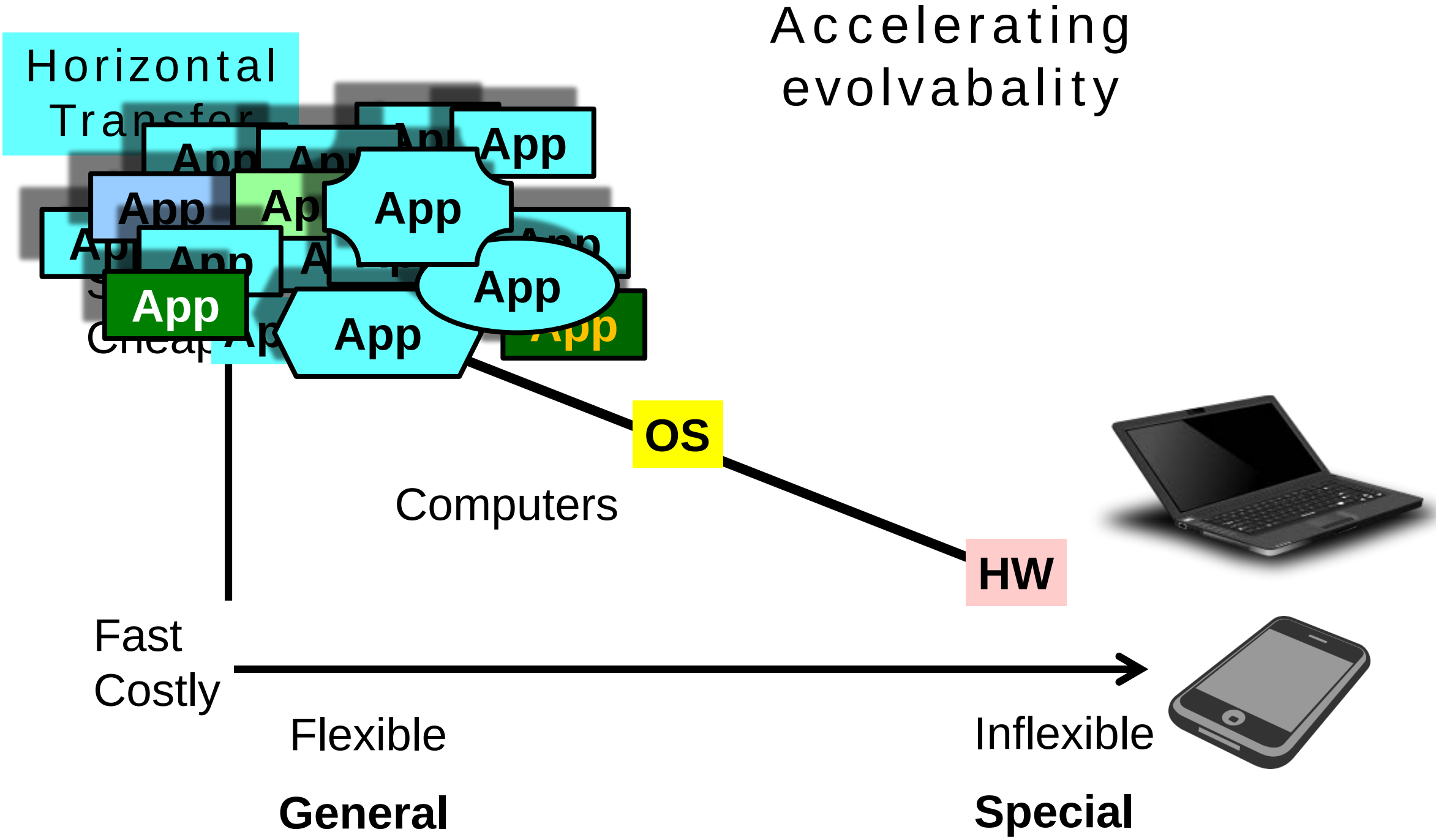
How function is *most flexibly* controlled

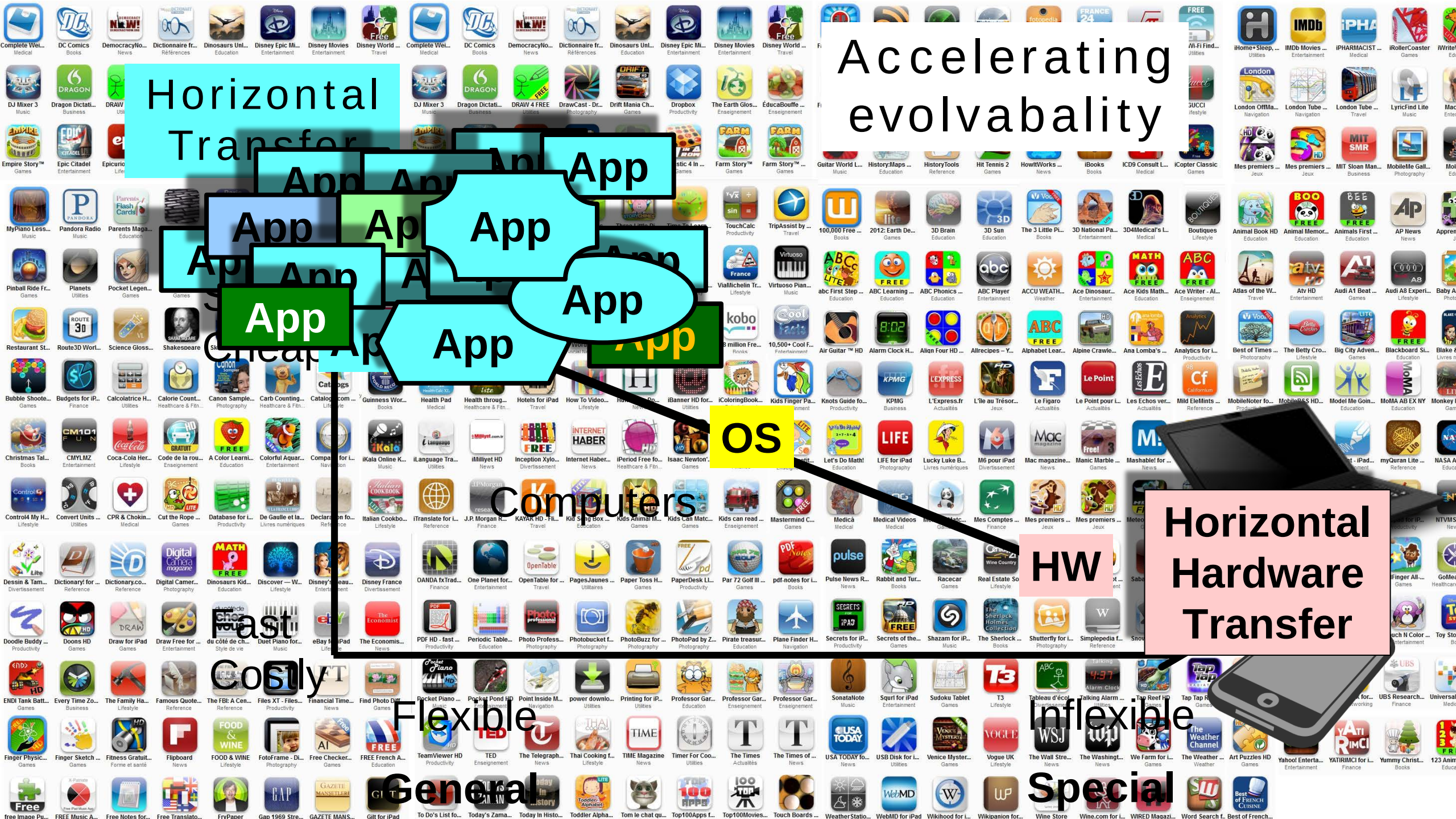


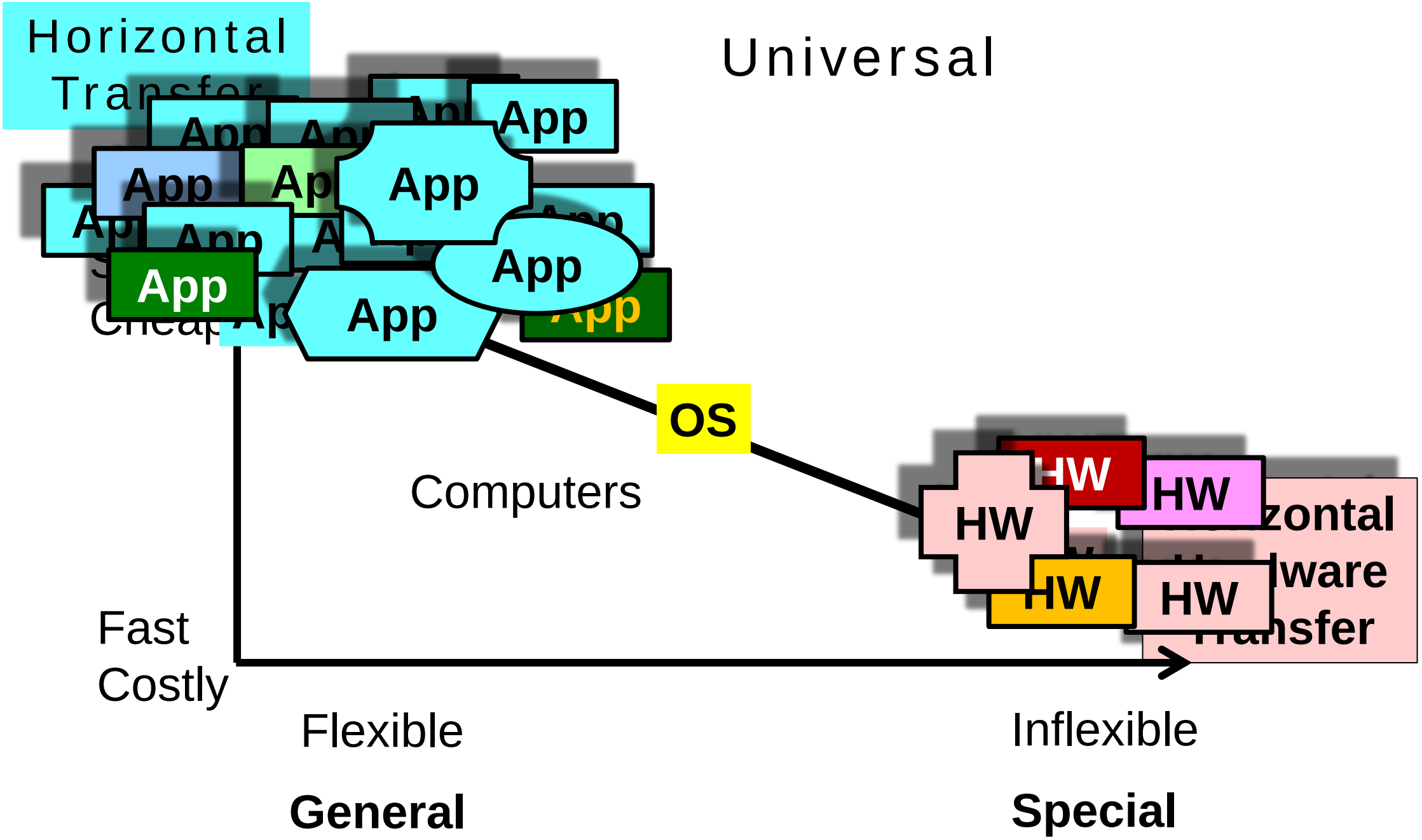
Horizontal Transfer

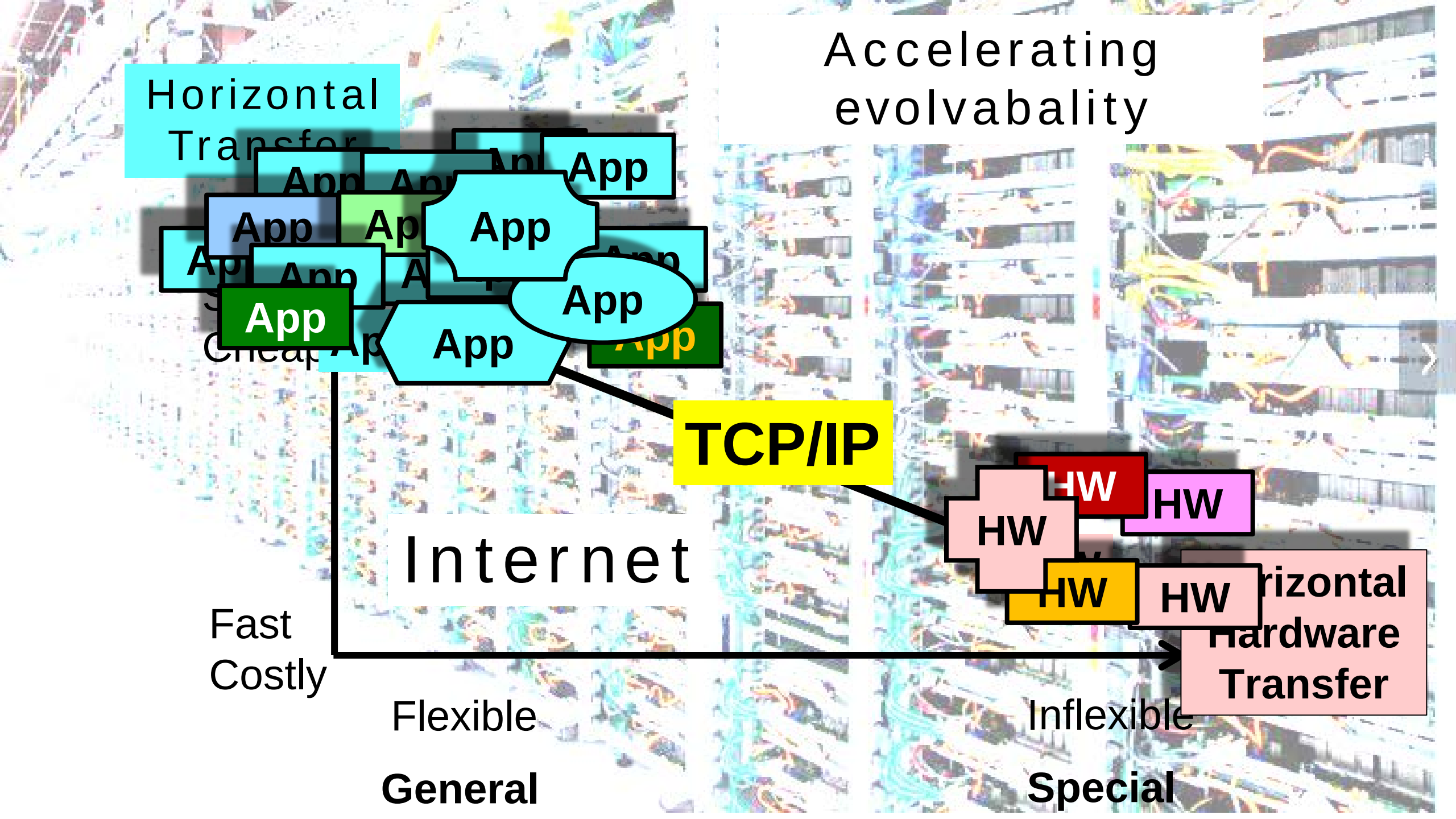
Accelerating evolvability

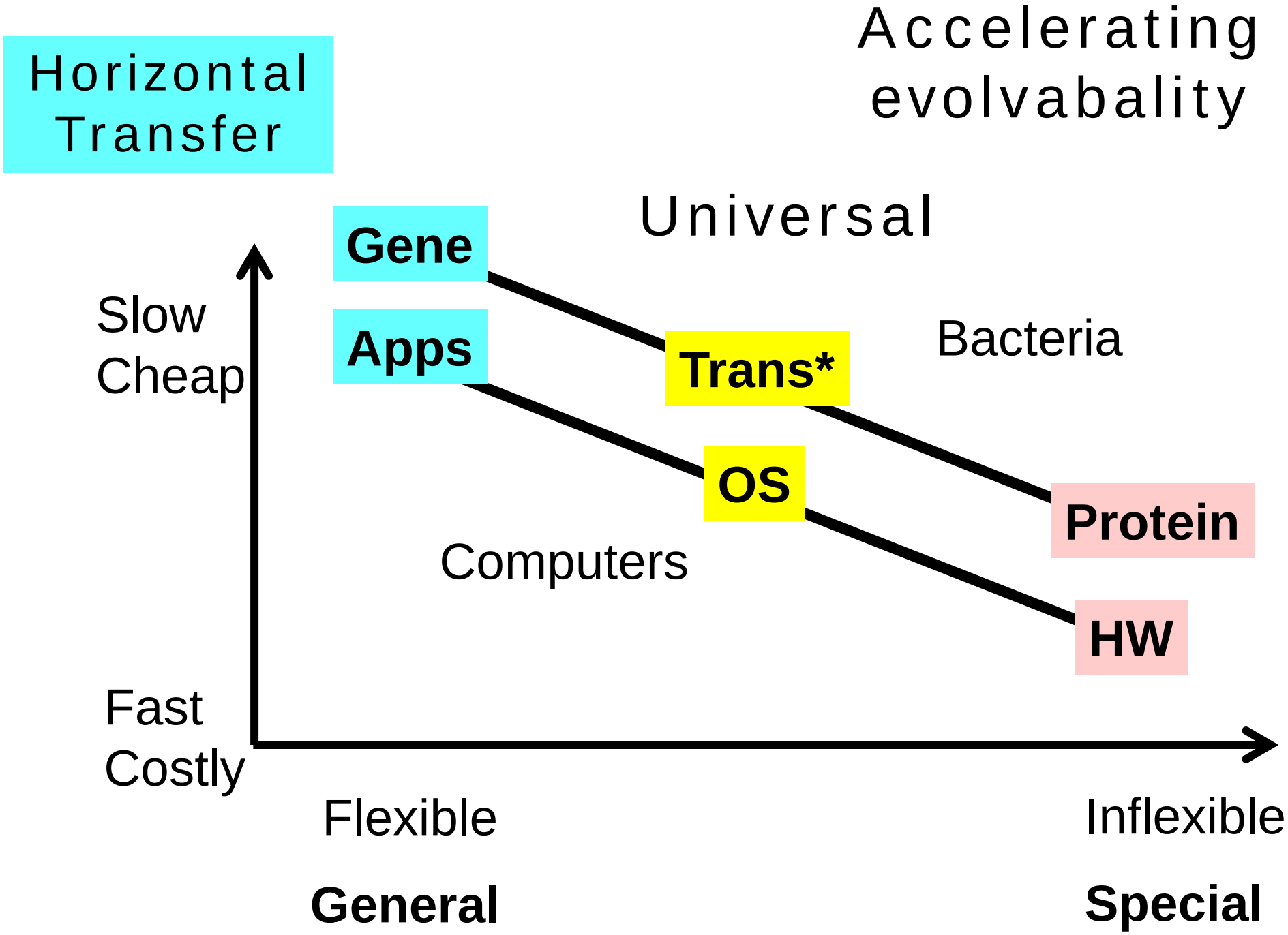



Accelerating evolvabality
Horizontal Transfer
App
App
App

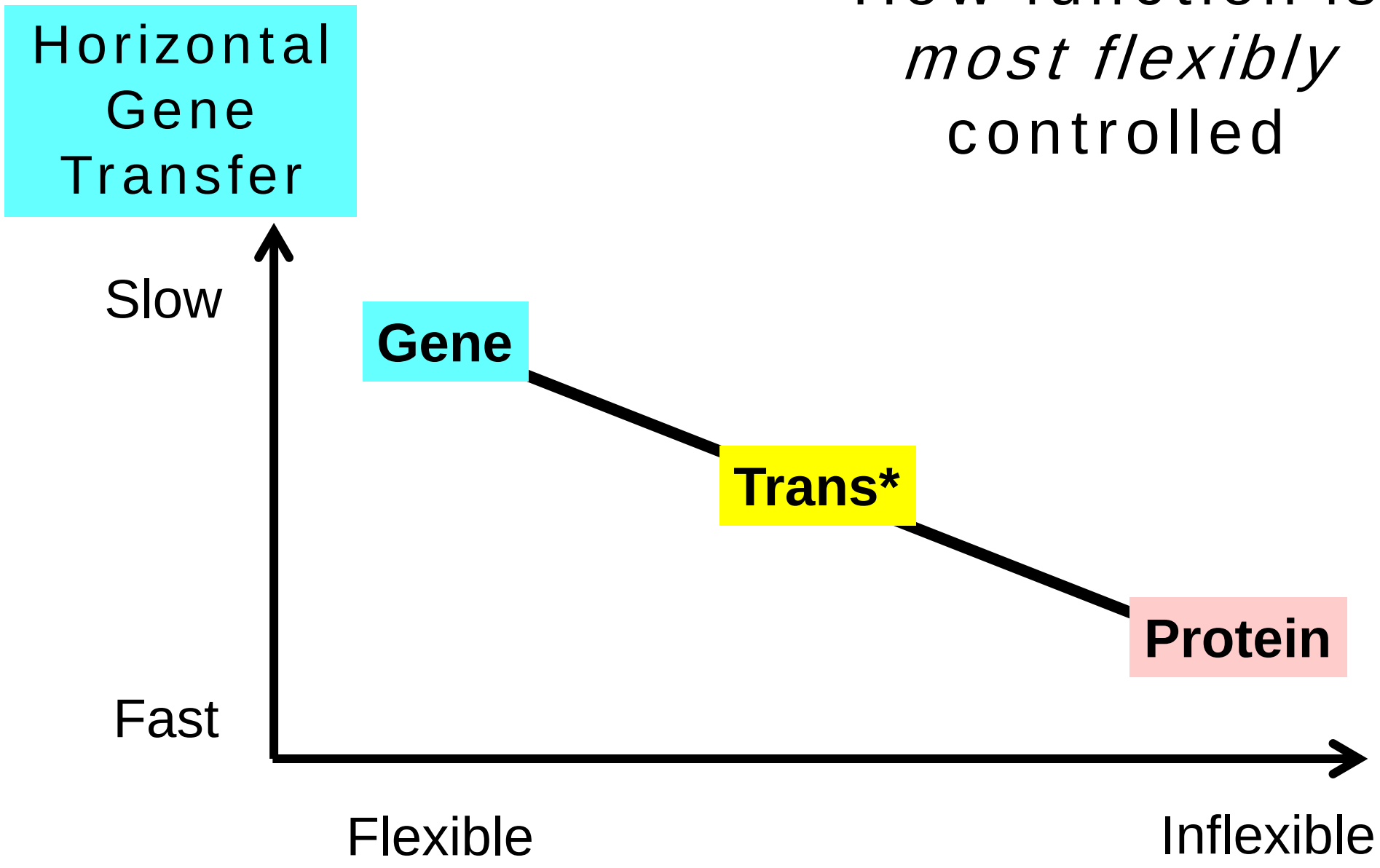

Horizontal Transfer
Accelerating evolvability
App
App
App

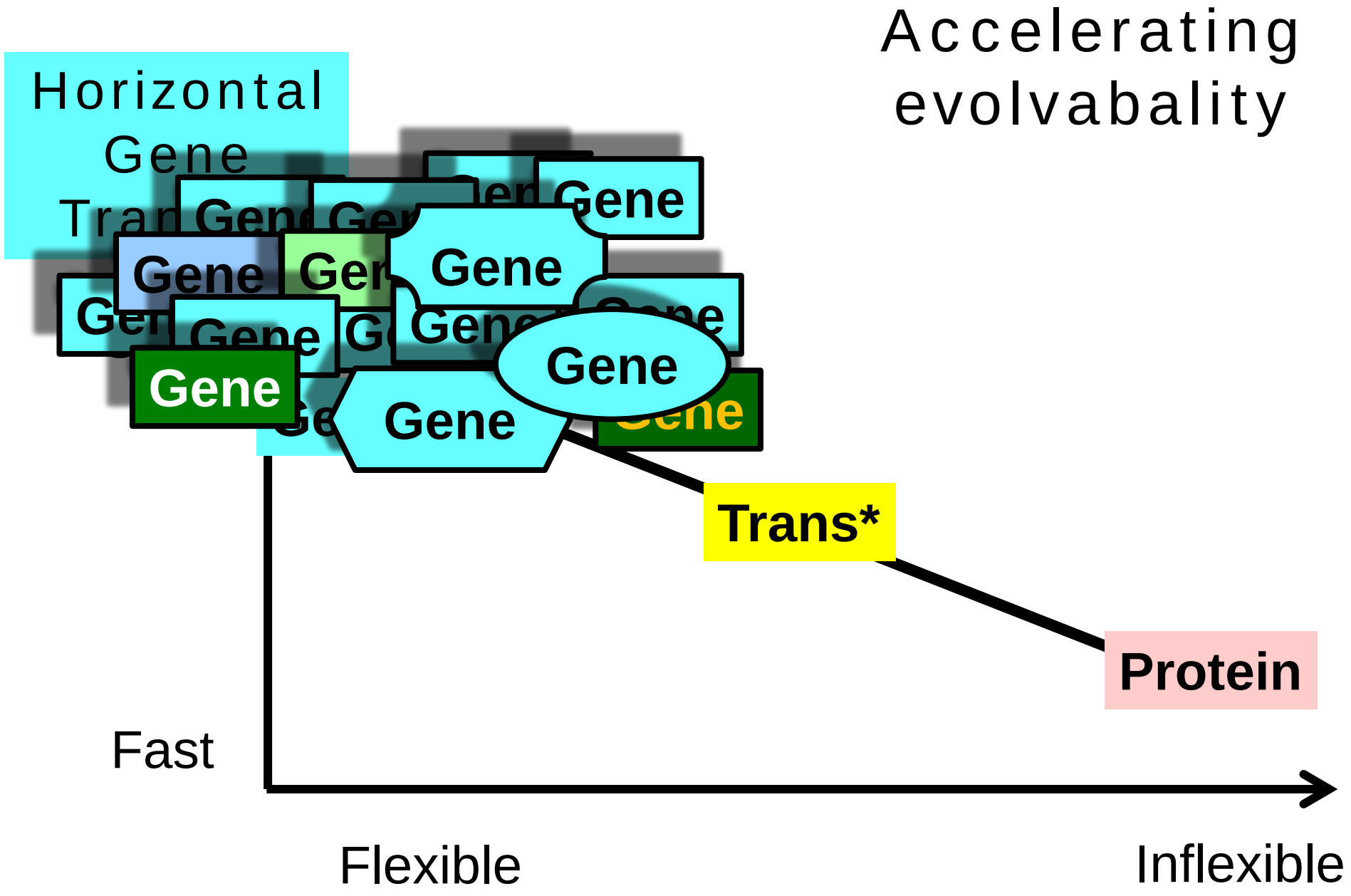

Horizontal
Universal
App
App
App


Accelerating evolvability
Horizontal Transfer
App
App
App
App
App
App
App
App
App
App
App
App
TCP/IP
Internet
HW
HW
HW
HW
HW
Horizontal Hardware Transfer
Fast
Costly
Flexible
General
Inflexible
Special

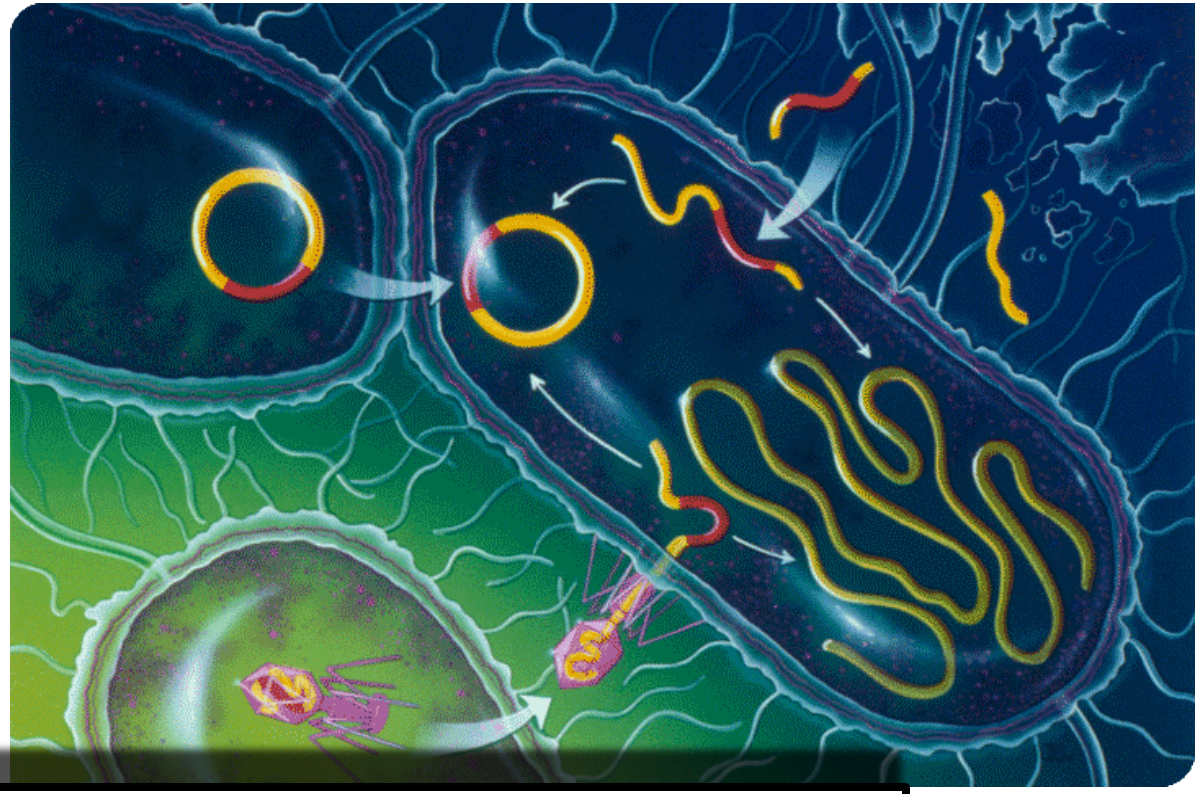

Horizontal Transfer
Accelerating evolvabality
Universal
Gene
Apps
Trans*
OS
Bacteria
Protein
Computers
HW
Slow
Cheap
Fast
Costly
Flexible
Inflexible
General
Special

How function is *most flexibly* controlled



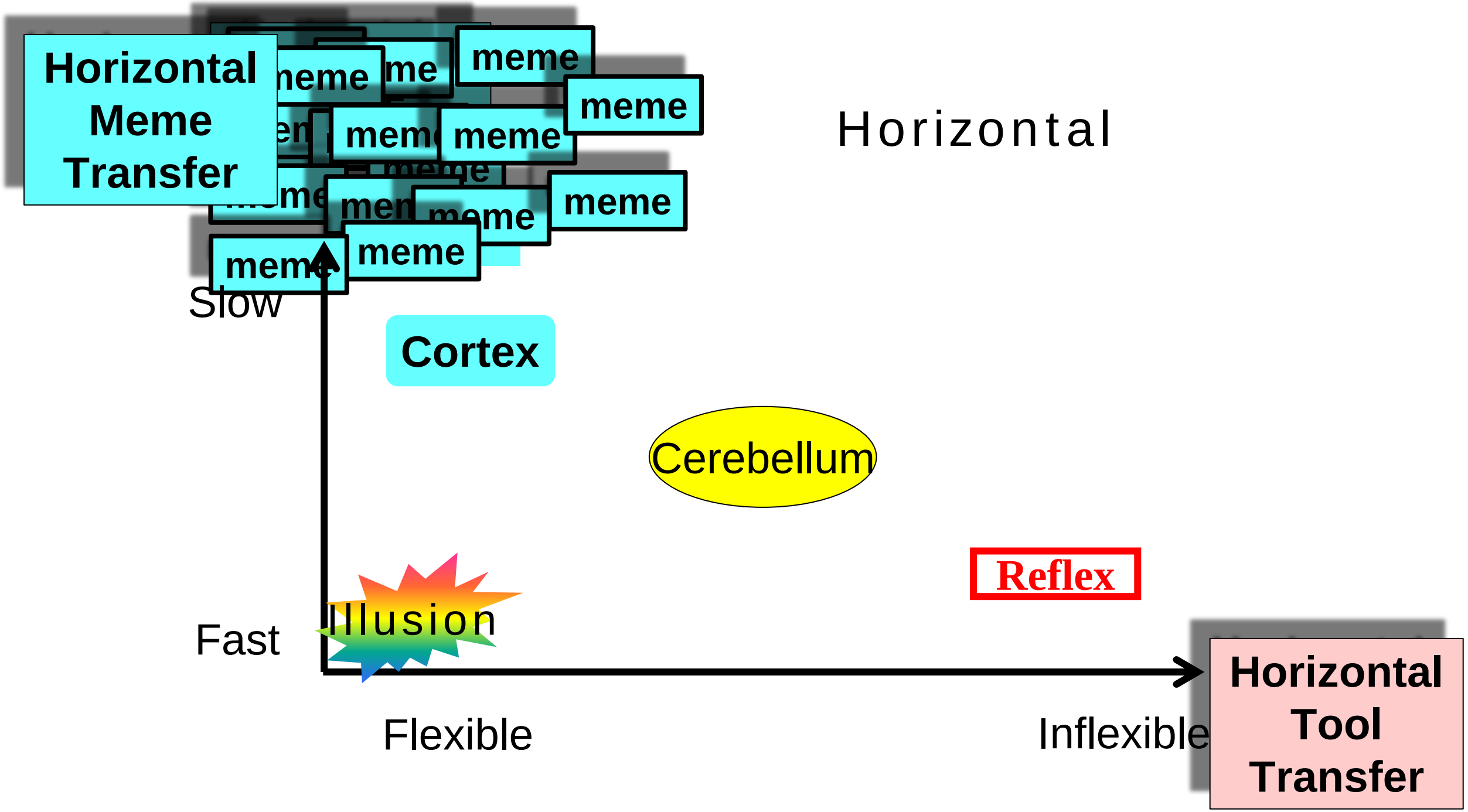

Accelerating evolvabality
Horizontal Gene Tran
Gene
Trans*
Protein
Fast
Flexible
Inflexible

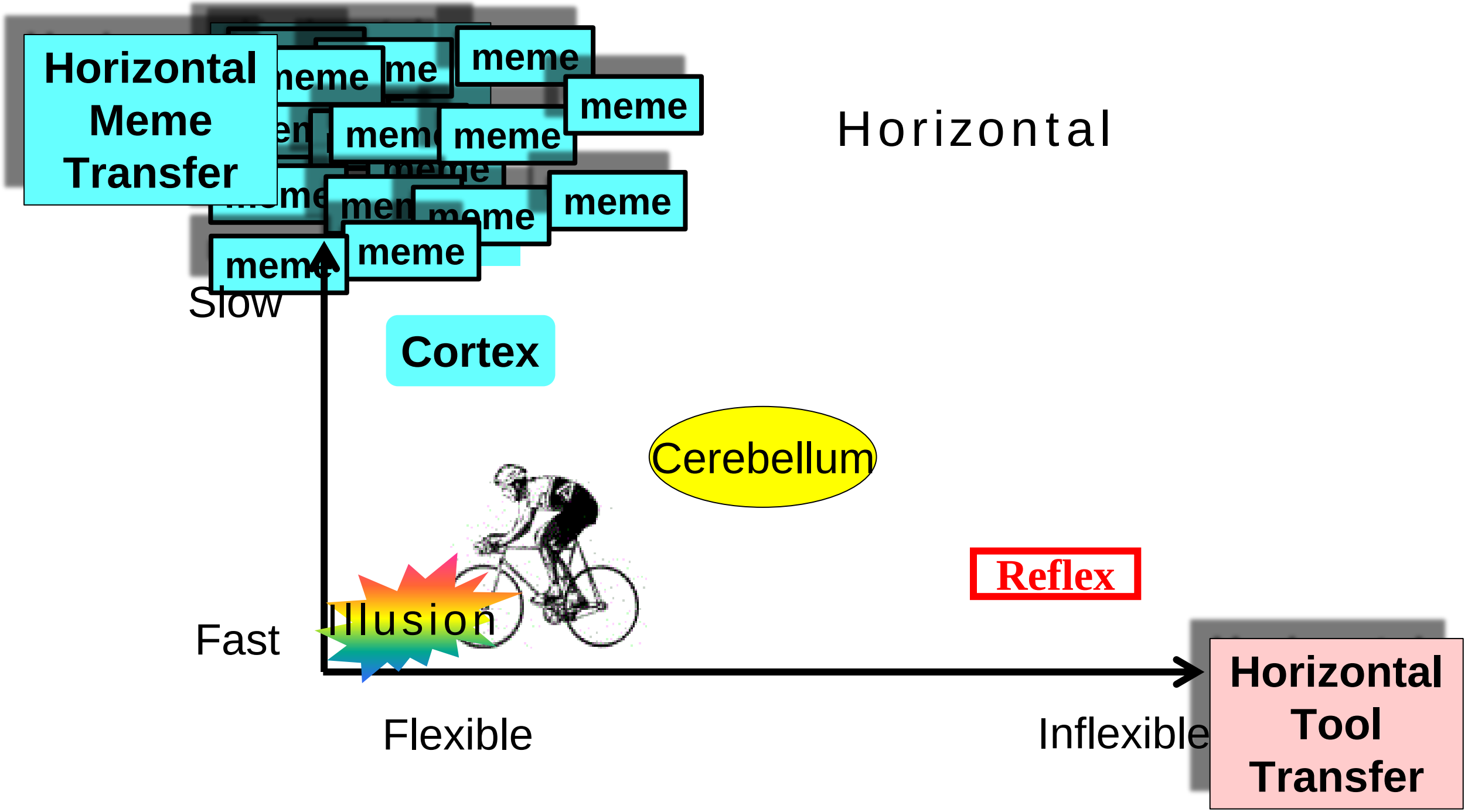
Horizontal Gene Transfer

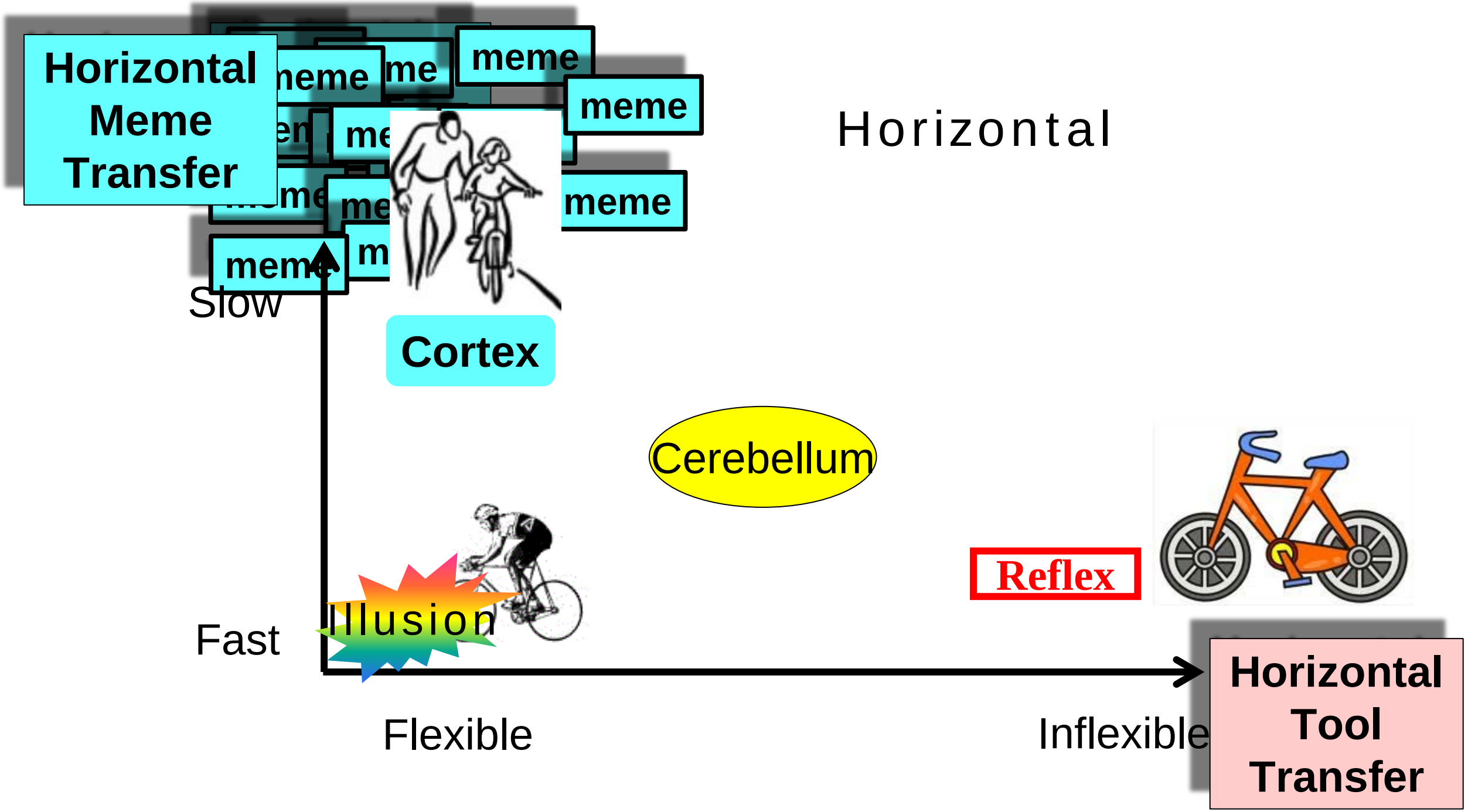


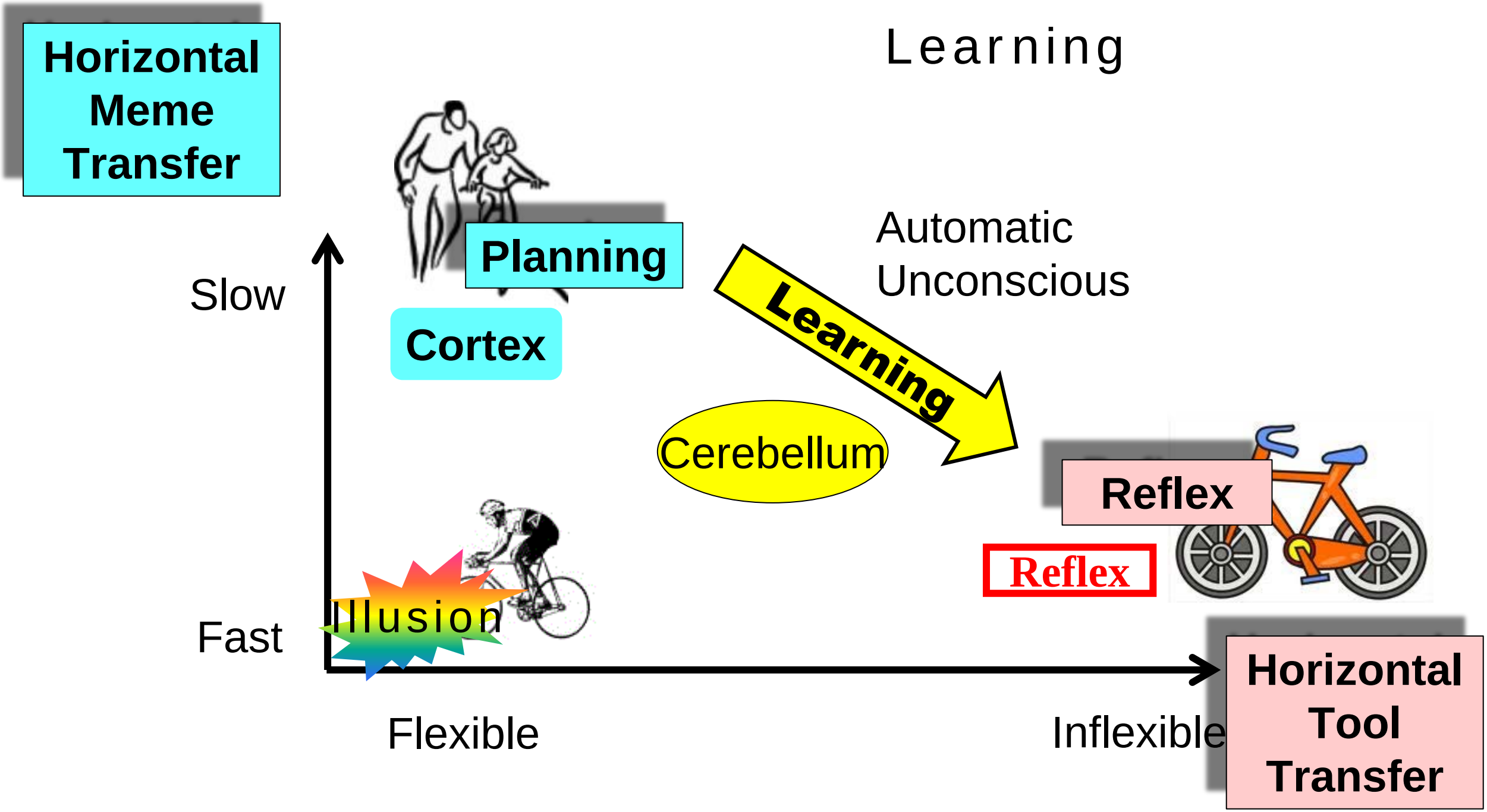
Sequence ~100 E Coli (*not* chosen randomly)

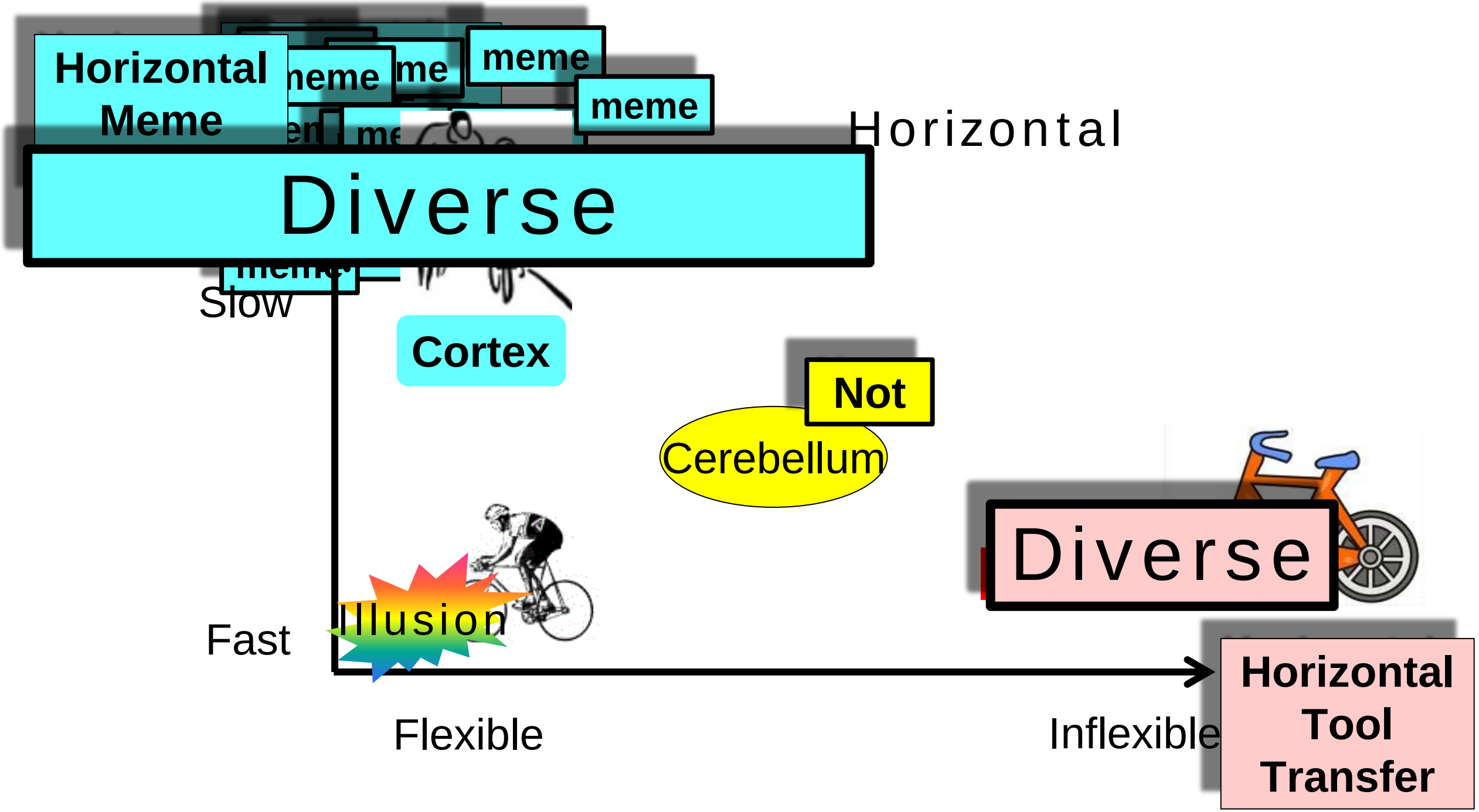
- ~ 4K genes per cell
- >20K *different* in total (pangenome)
- < 1K universally shared
- ~ 300 essential (minimal)

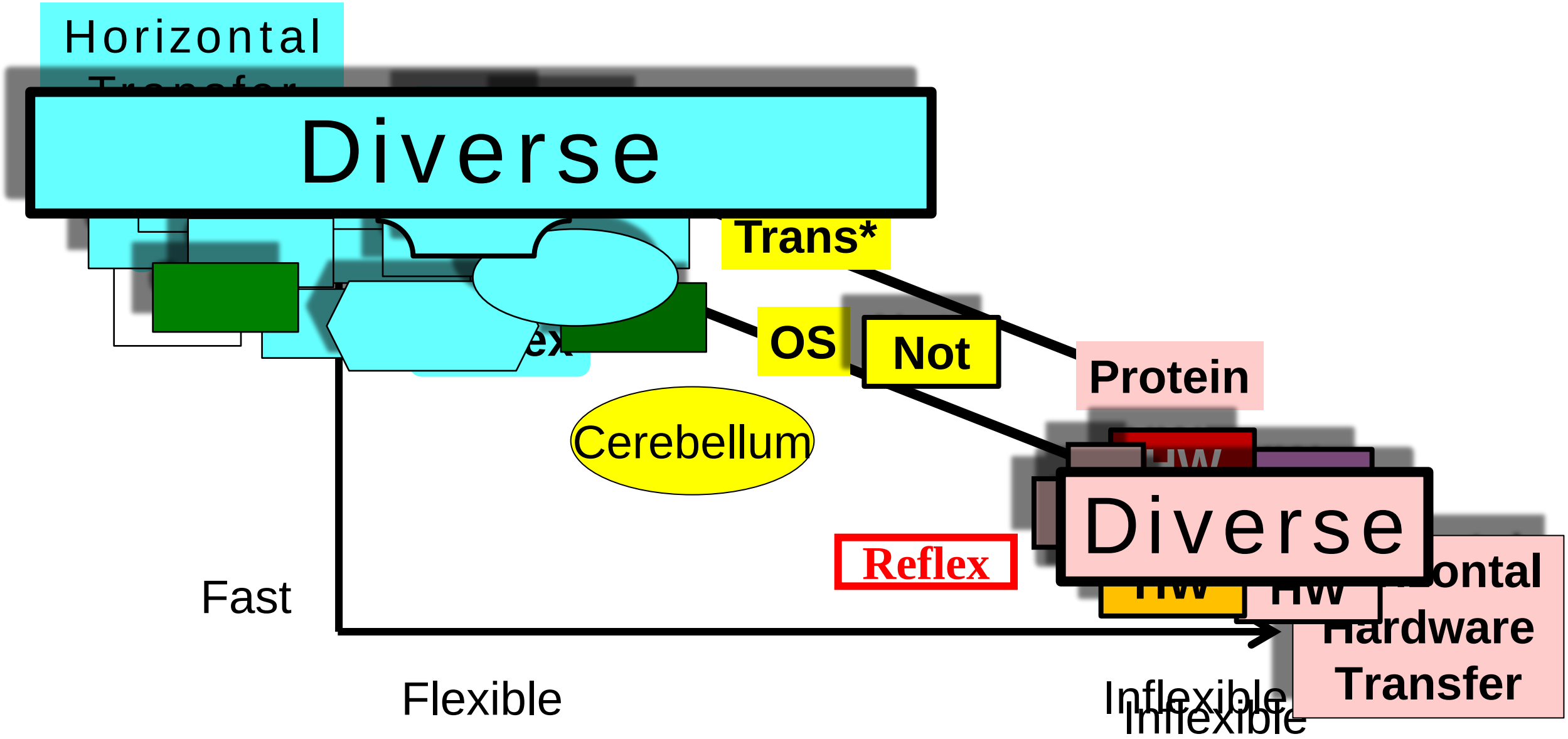

Horizontal Meme Transfer
meme
meme
meme
meme
meme
meme
meme
meme
meme
meme
Horizontal
Slow
Cortex
Cerebellum
Reflex
Illusion
Fast
Flexible
Inflexible
Horizontal Tool Transfer

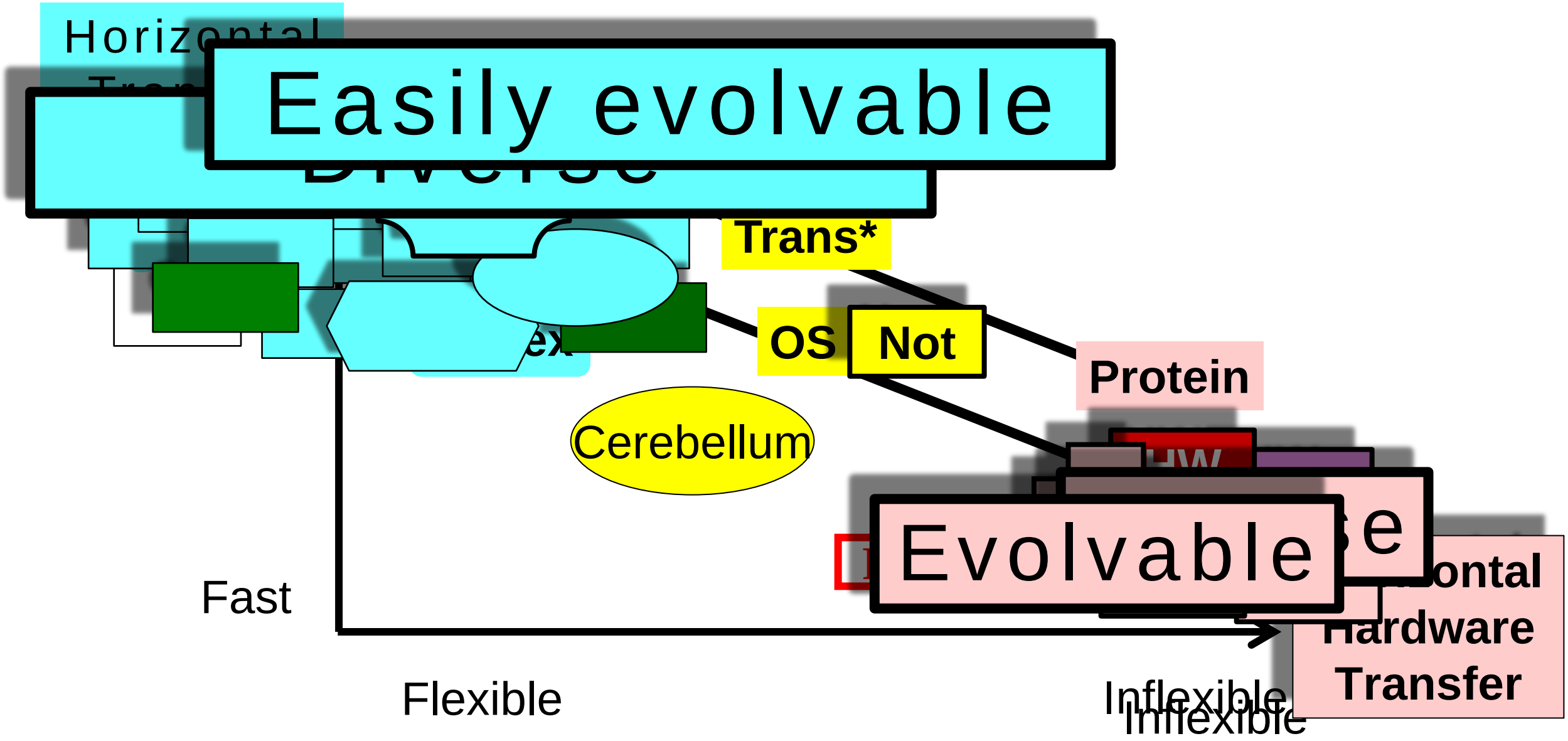

Horizontal Meme Transfer
meme
meme
meme
meme
meme
meme
meme
meme
meme
Horizontal
Slow
Cortex
Cerebellum
Reflex
Illusion
Fast
Flexible
Inflexible
Horizontal Tool Transfer

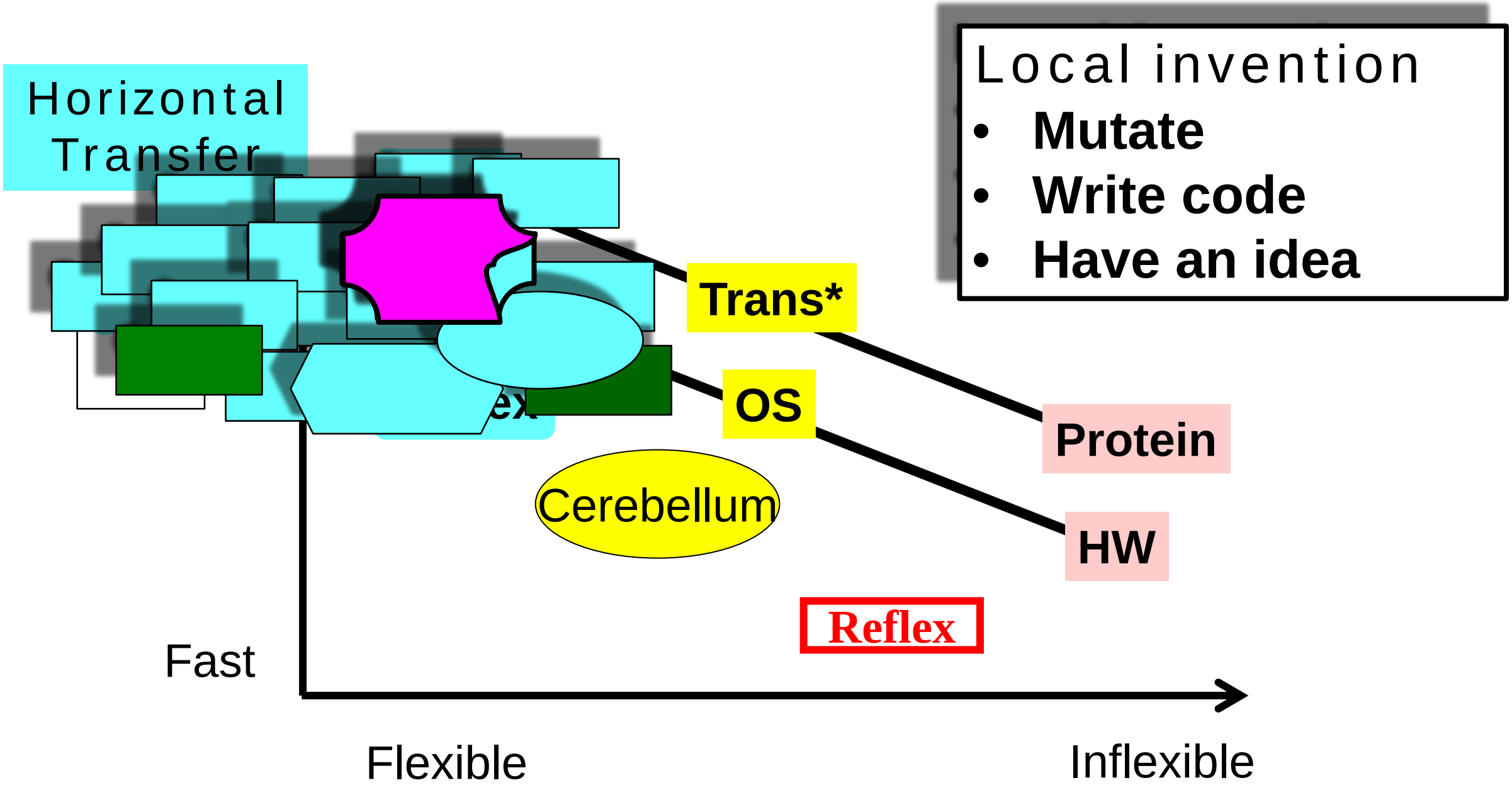

Horizontal Meme Transfer
meme
meme
meme
meme
meme
meme
Horizontal
Slow
Cortex
Cerebellum
Reflex
Illusion
Fast
Flexible
Inflexible
Horizontal Tool Transfer

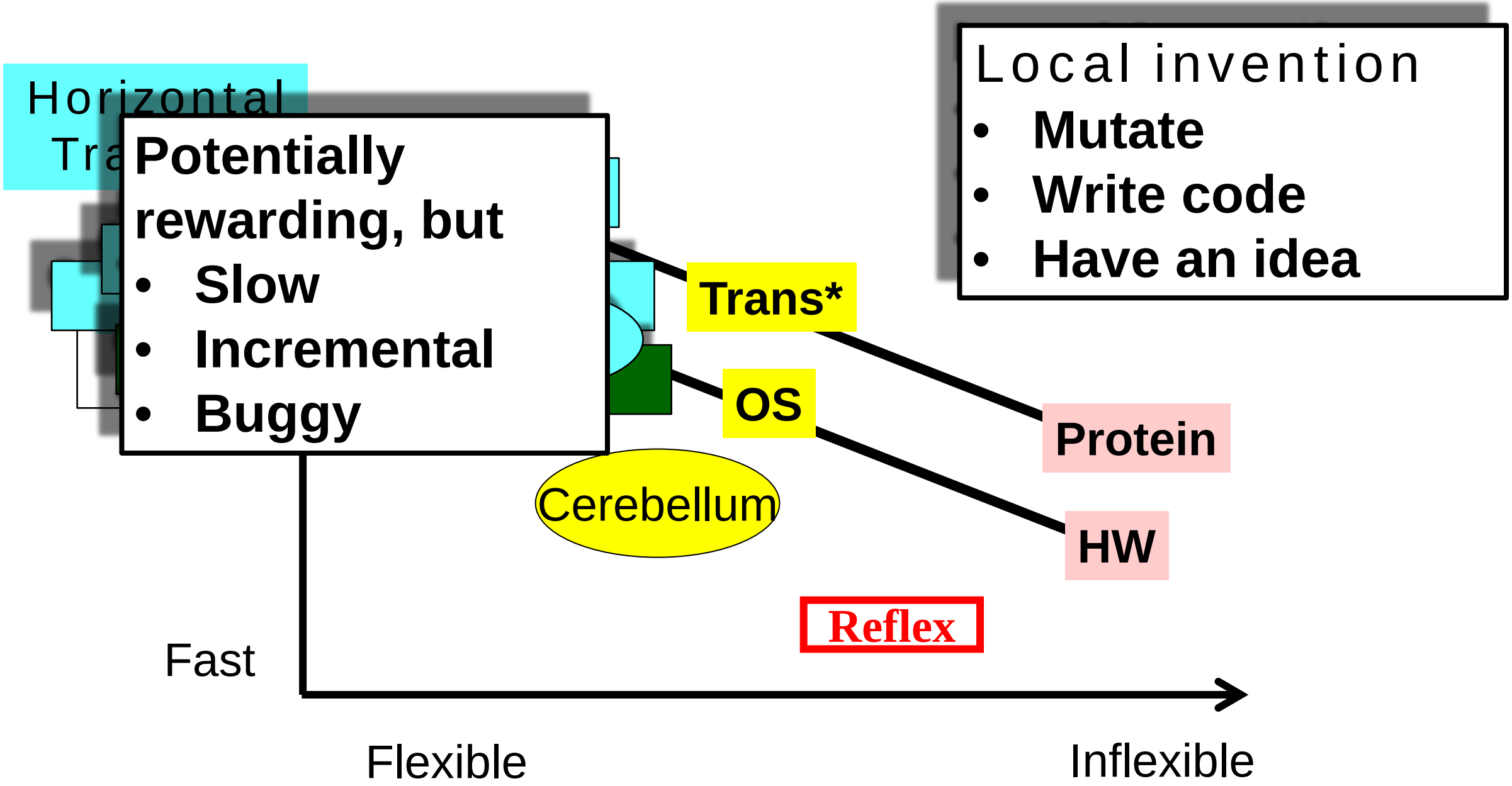

Horizontal Meme Transfer
Learning
Planning
Cortex
Slow
Automatic
Unconscious
Learning
Cerebellum
Reflex
Reflex
Illusion
Fast
Flexible
Inflexible
Horizontal Tool Transfer

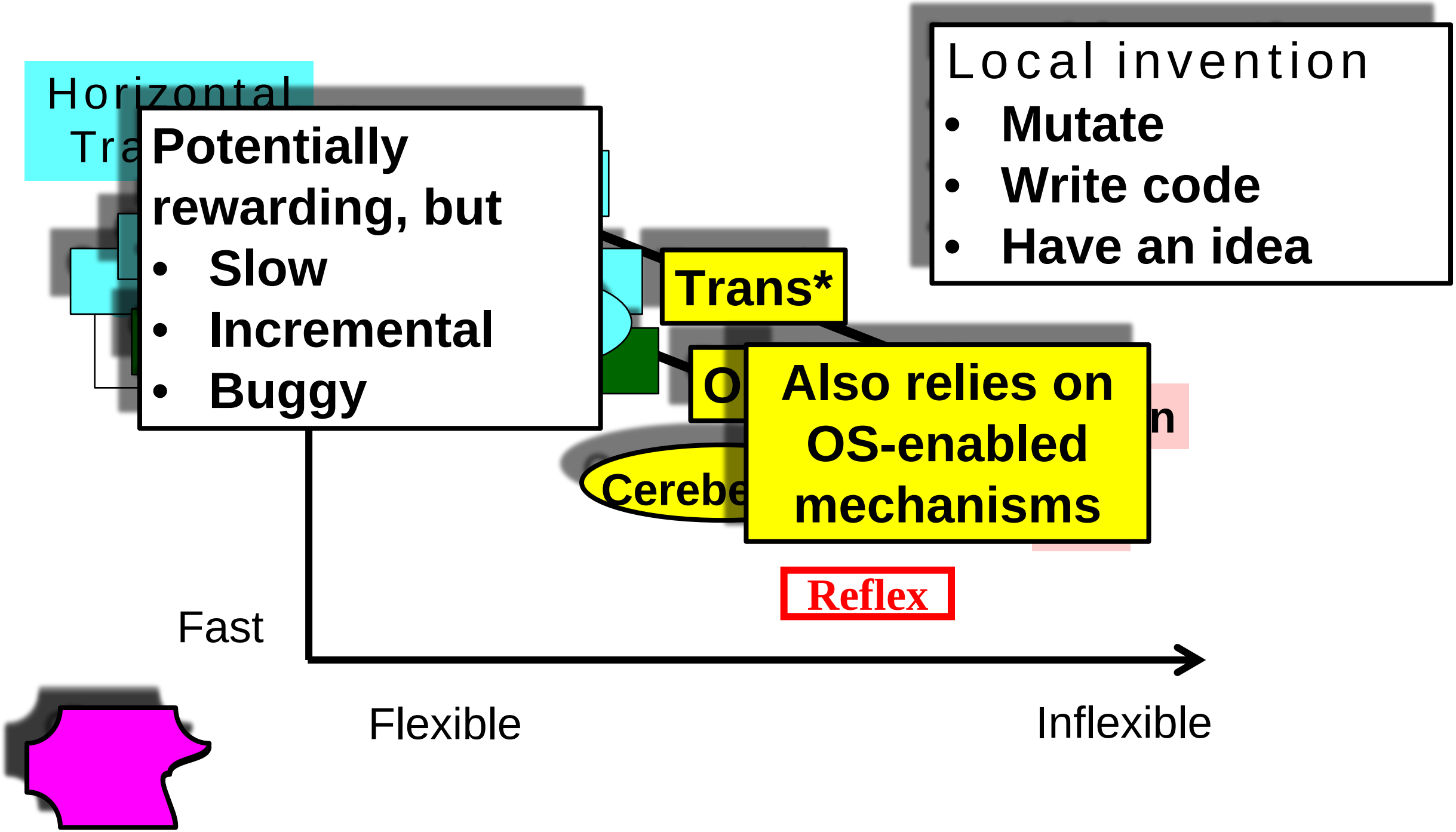

Horizontal Meme
meme
meme
meme
Horizontal
Diverse
Slow
Cortex
Not
Cerebellum
Diverse
Illusion
Fast
Flexible
Inflexible
Horizontal Tool Transfer

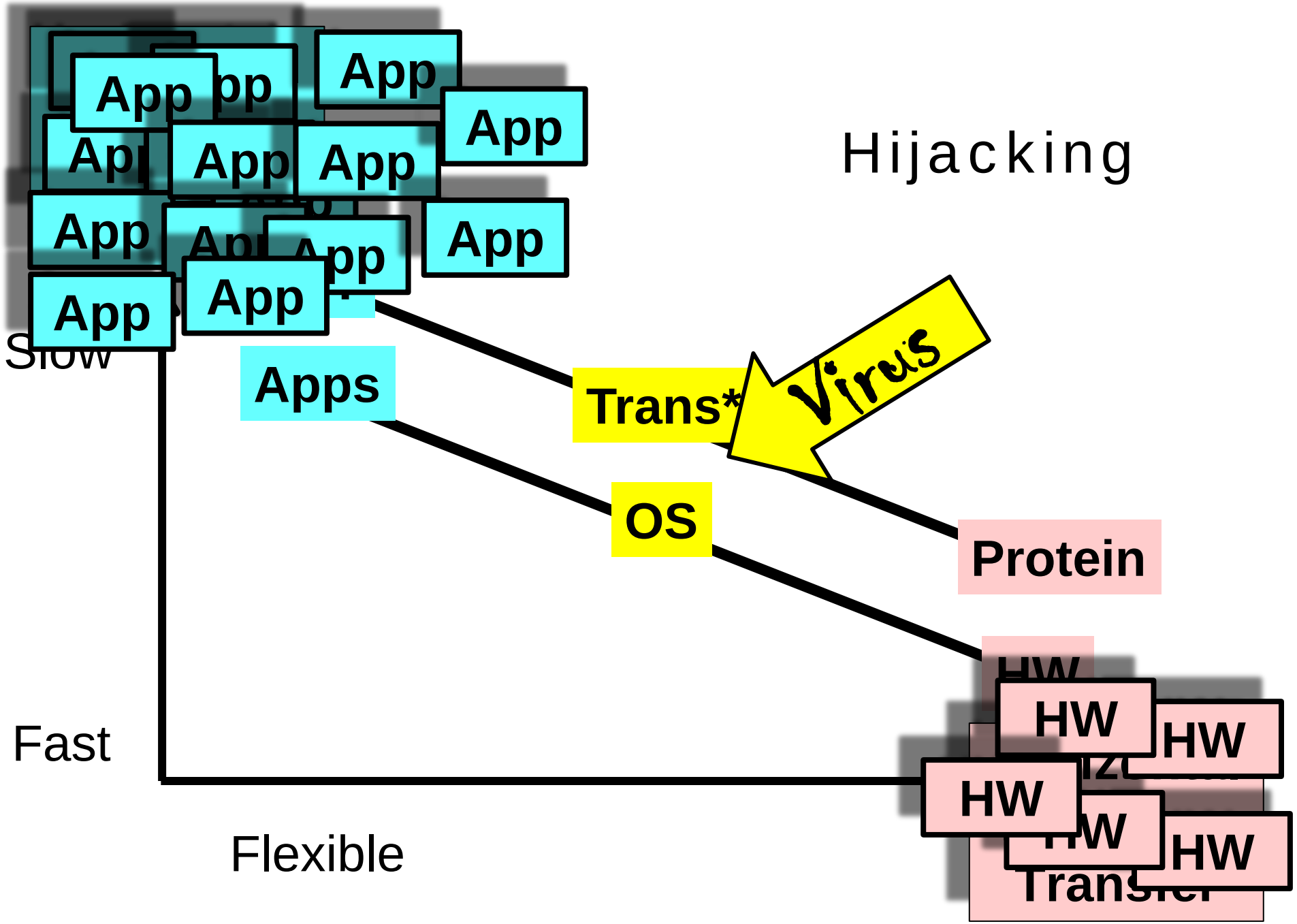

Horizontal
Diverse
Trans*
OS
Not
Protein
Cerebellum
Diverse
Reflex
Fast
Flexible
Inflexible
Hardware
Transfer


Easily evolvable
Trans*
OS
Not
Protein
Cerebellum
Evolvable
Fast
Flexible
Inflexible
Hardware
Transfer

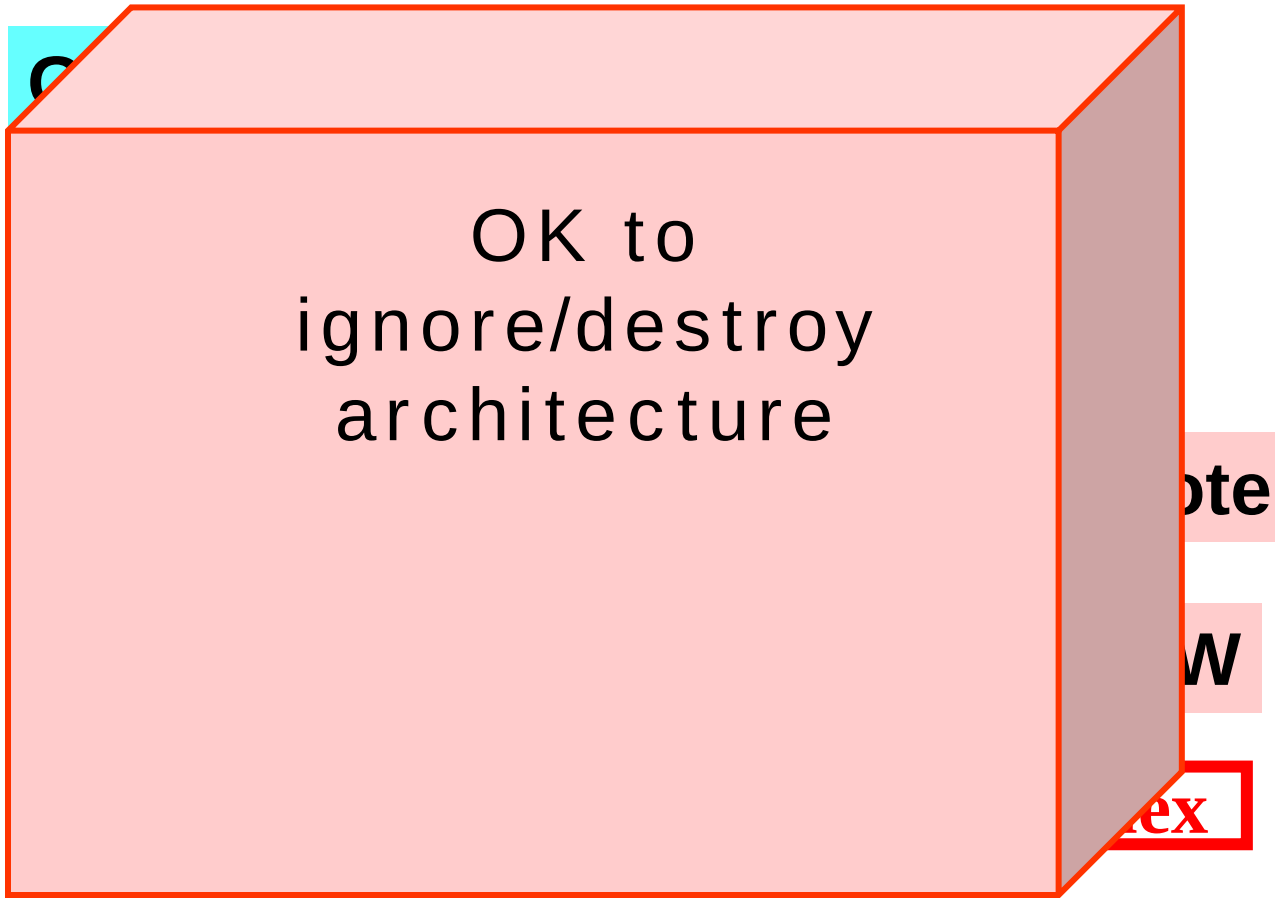

Horizontal Transfer
Local invention
• Mutate
• Write code
• Have an idea
Trans*
OS
Protein
Cerebellum
HW
Reflex
Fast
Flexible
Inflexible

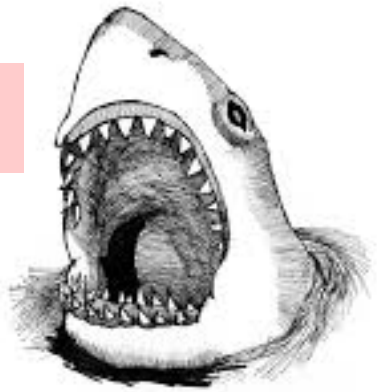

Horizontal
Tra
Potentially rewarding, but
• Slow
• Incremental
• Buggy
Local invention
• Mutate
• Write code
• Have an idea
Trans*
OS
Protein
Cerebellum
HW
Reflex
Fast
Flexible
Inflexible


Horizontal
Tra
Potentially rewarding, but
• Slow
• Incremental
• Buggy
Local invention
• Mutate
• Write code
• Have an idea
Trans*
O
Also relies on OS-enabled mechanisms
n
Cerebe
Reflex
Fast
Flexible
Inflexible


Hijacking
App
App
App
App
App
App
App
App
App
App
App
Slow
Apps
Trans*
Virus
OS
Protein
HW
HW
HW
HW
HW
Fast
Flexible

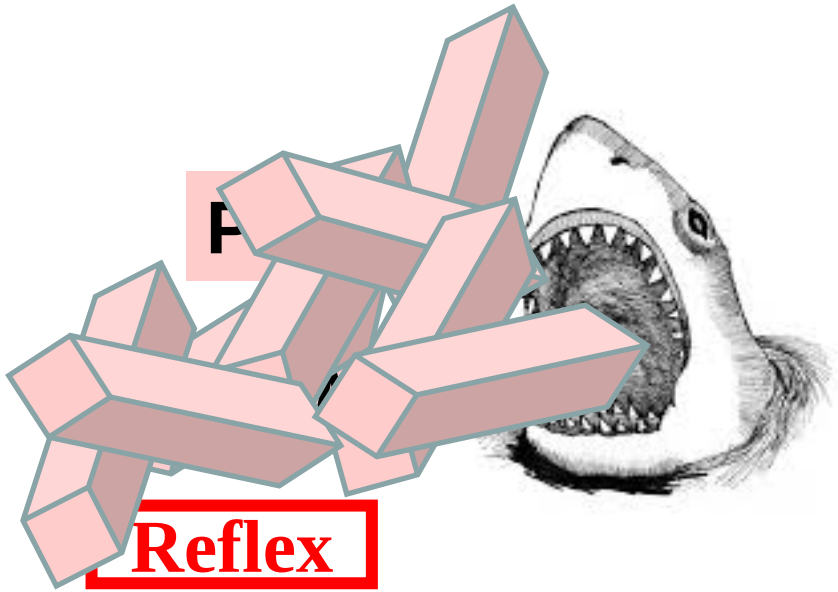
Predators don’t care about architecture





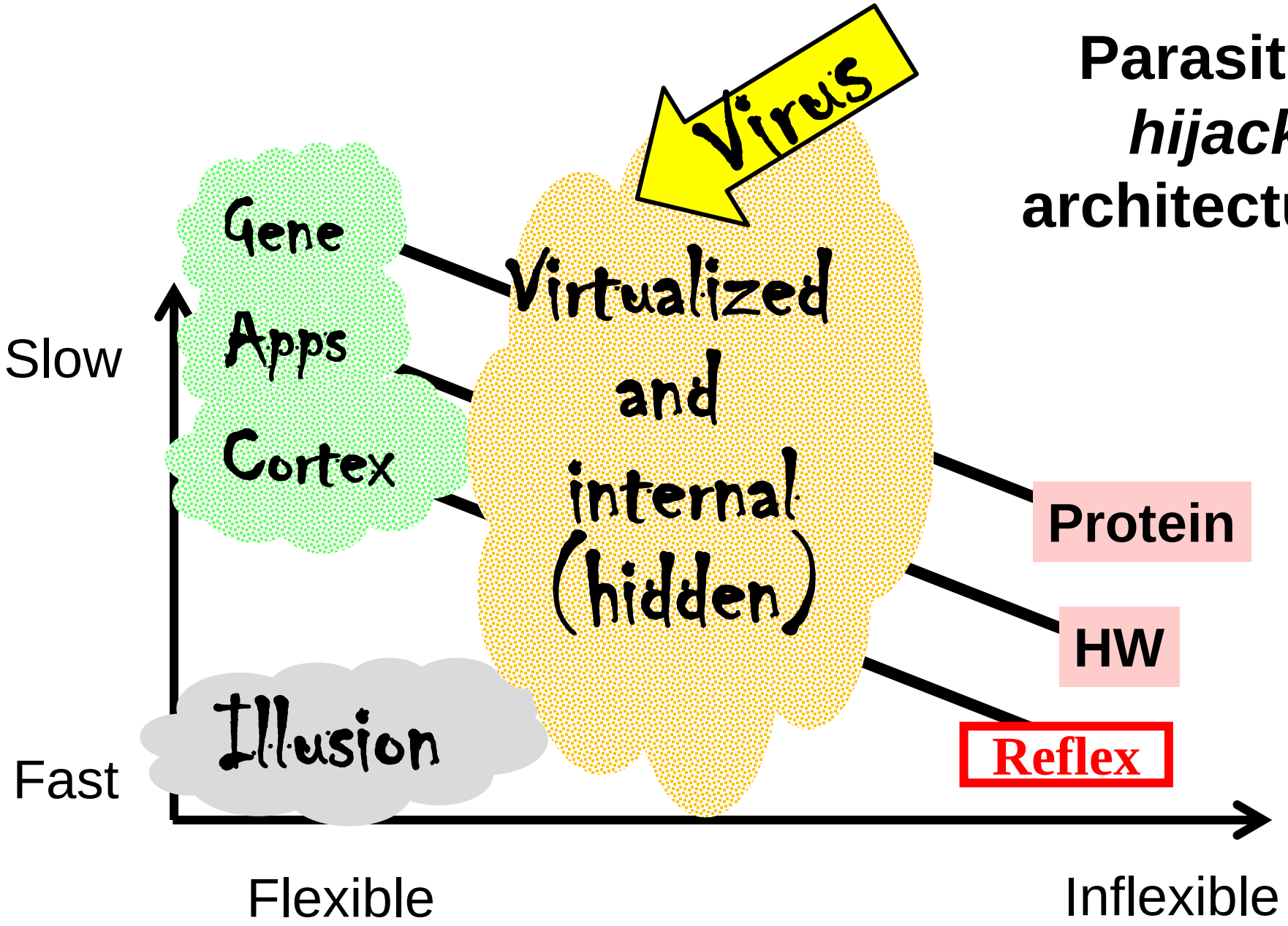
Predators don’t care about architecture

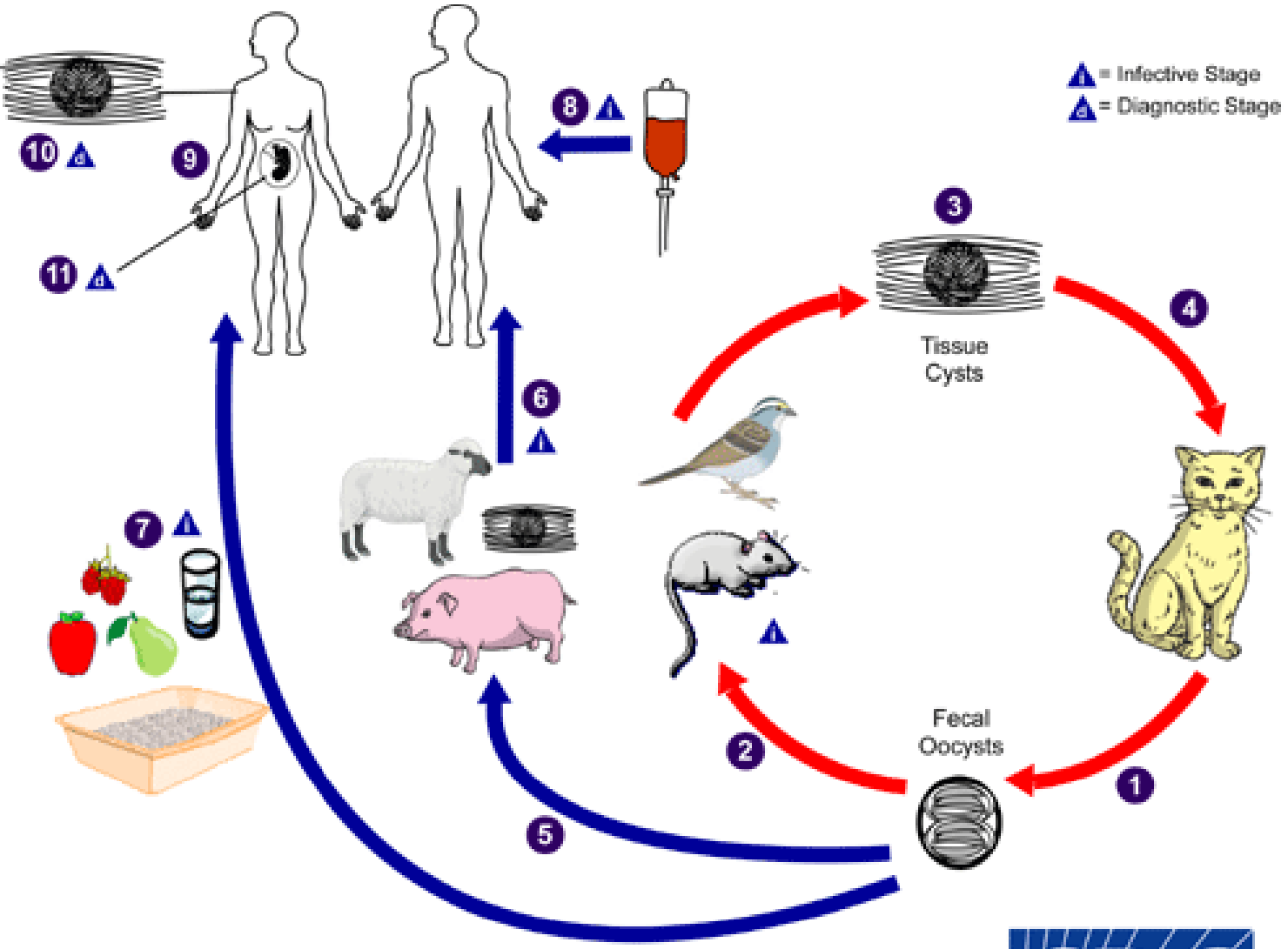
OK to ignore/destroy architecture



Consumes physical building blocks

Parasites *hijack* architectures

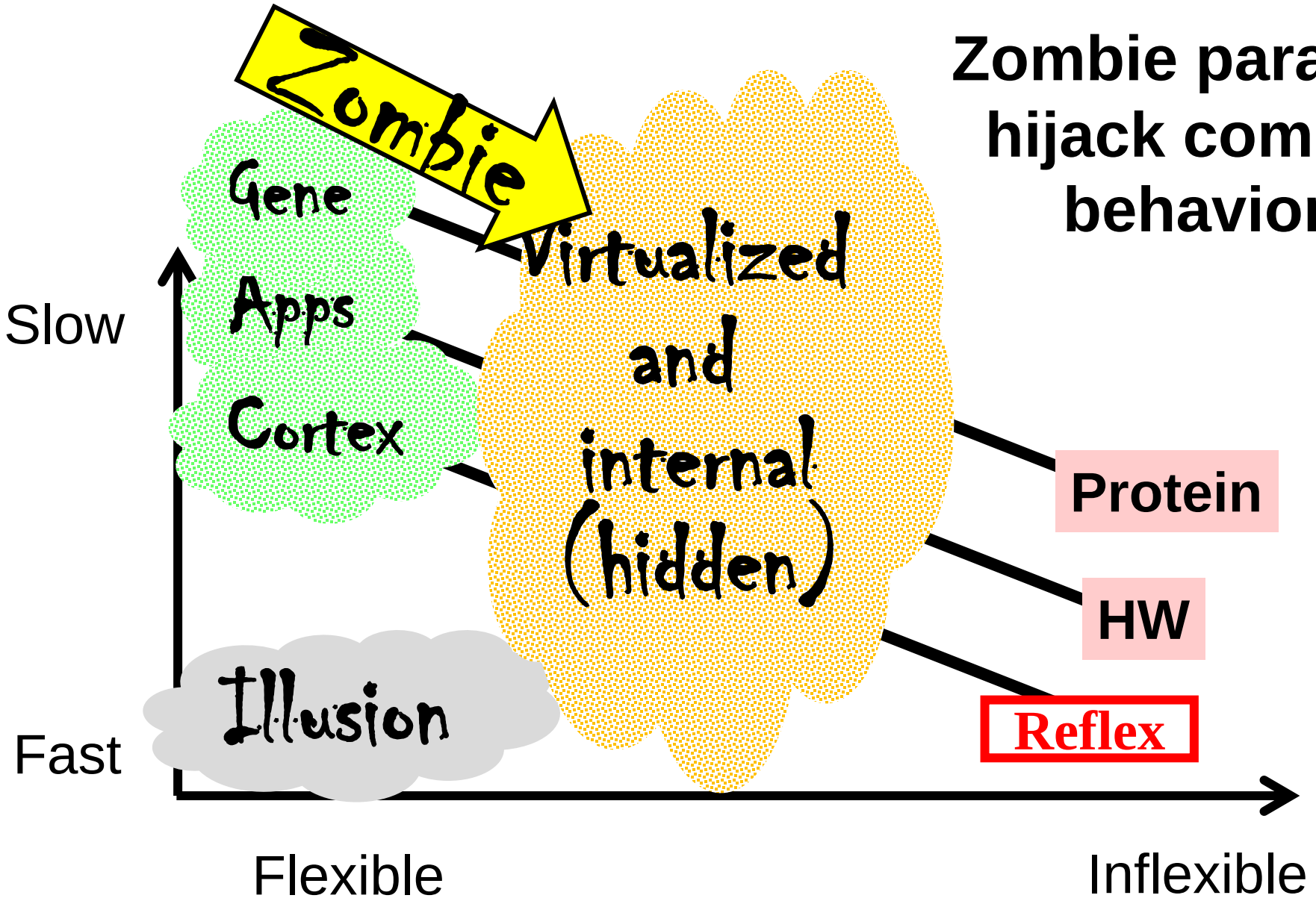


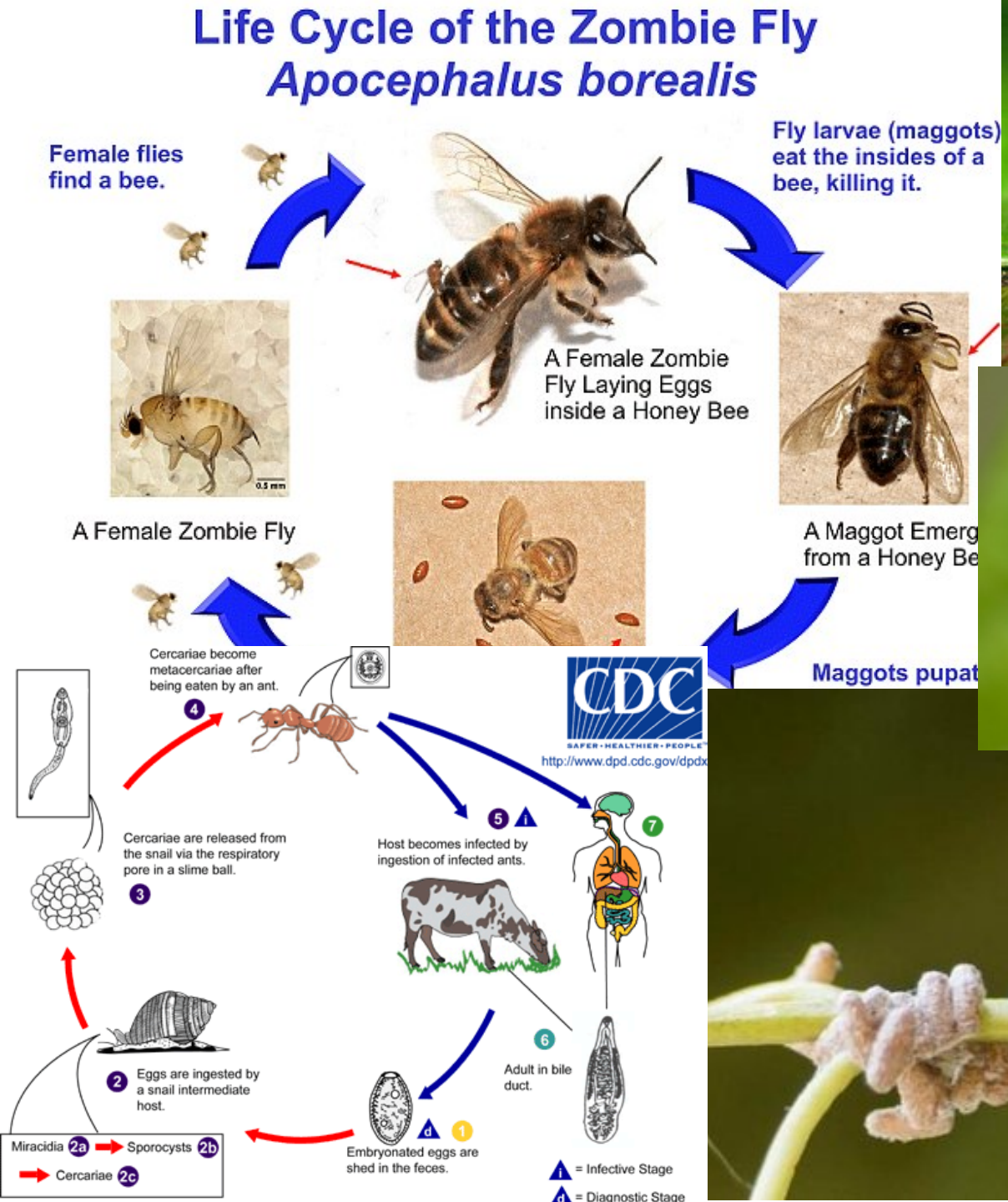


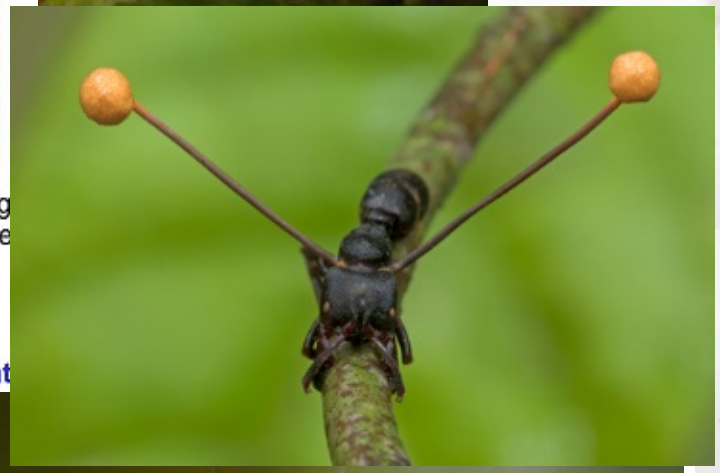
CDC
SAFER · HEALTHIER · PEOPLE™
http://www.dpd.cdc.gov/dpdx

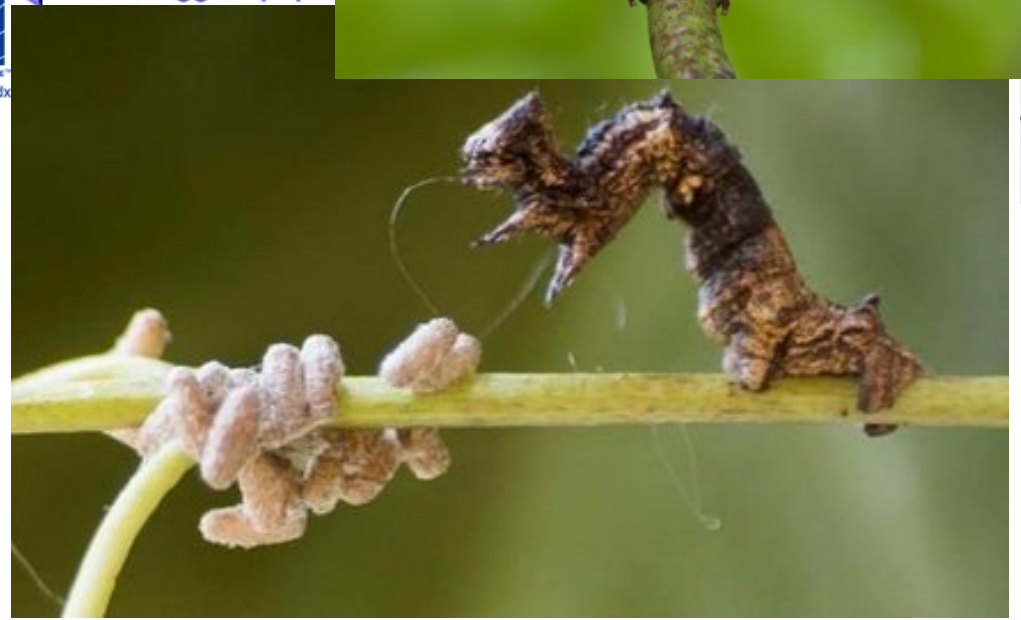
Zombie parasites

Zombie parasites hijack complex behaviors









Zombie parasites

Zombie

Predator?

Ziencefiction

Zombieness

Parasite?

Zombie

Rabies

Zombieness

Predator?

Parasite?

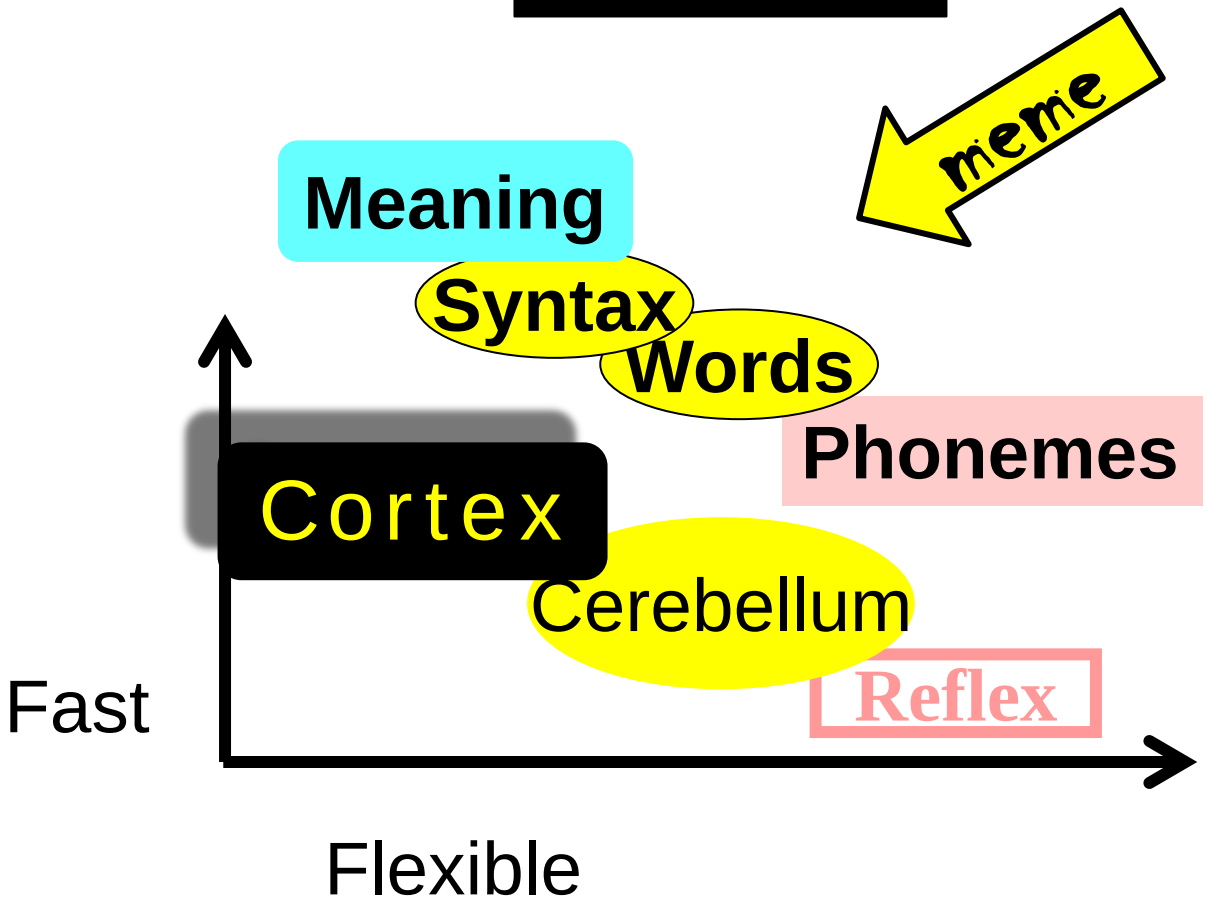
Zombie memes?

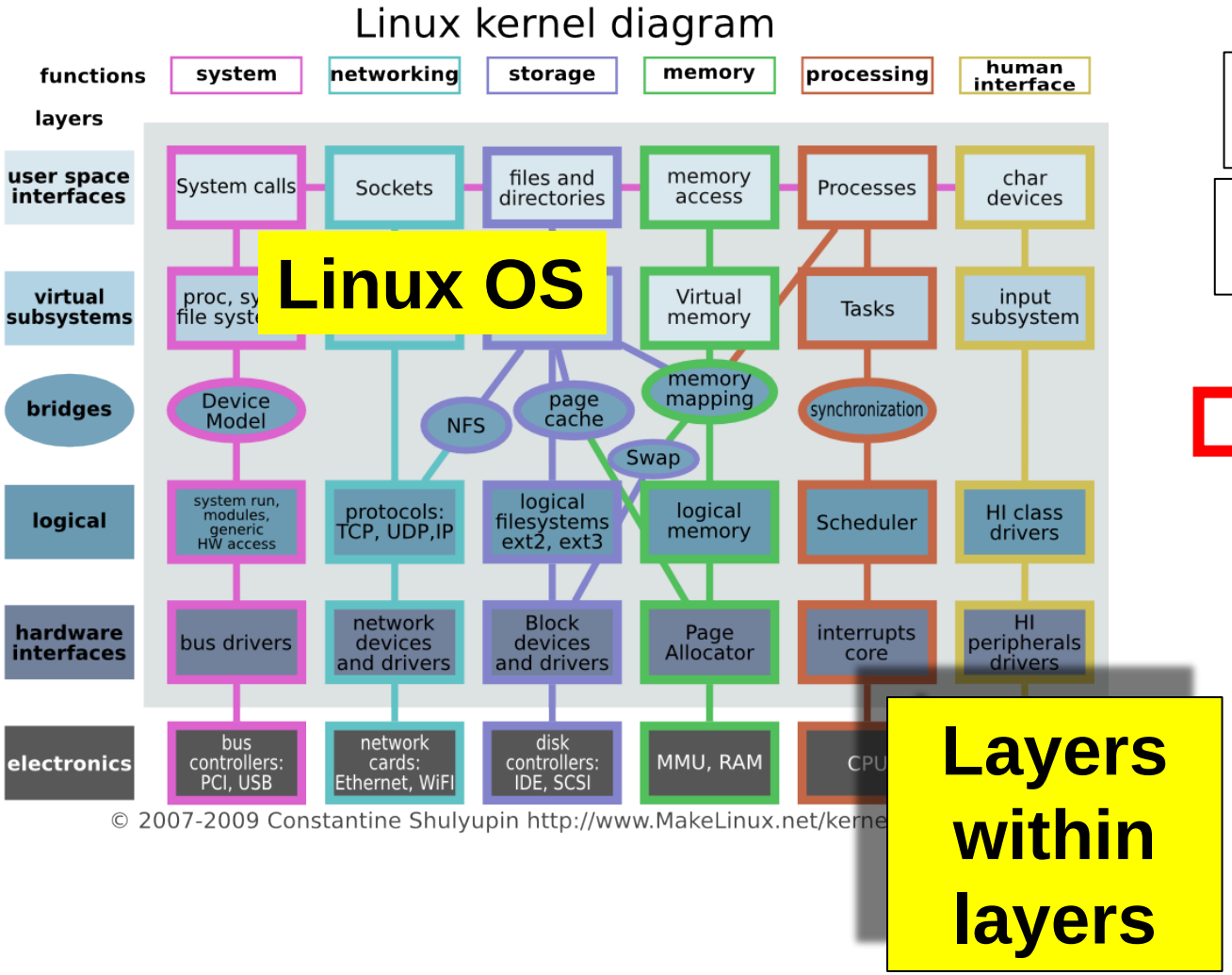
Bad Meme Transfer

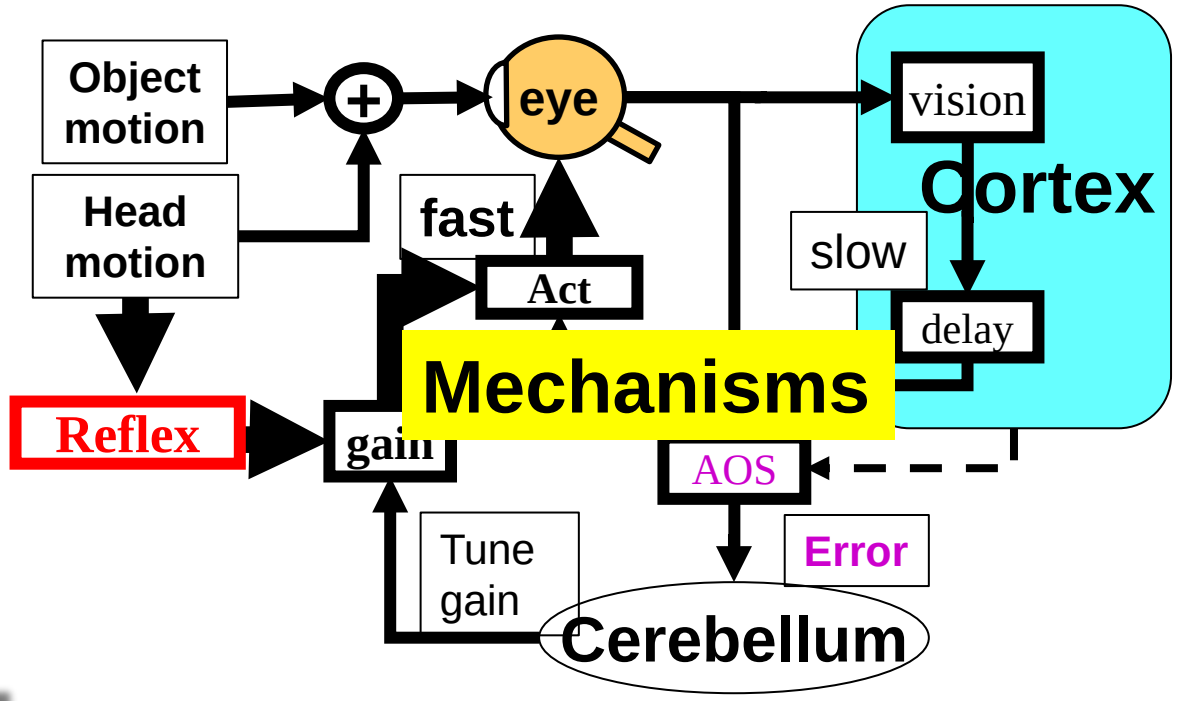
Hijacking?

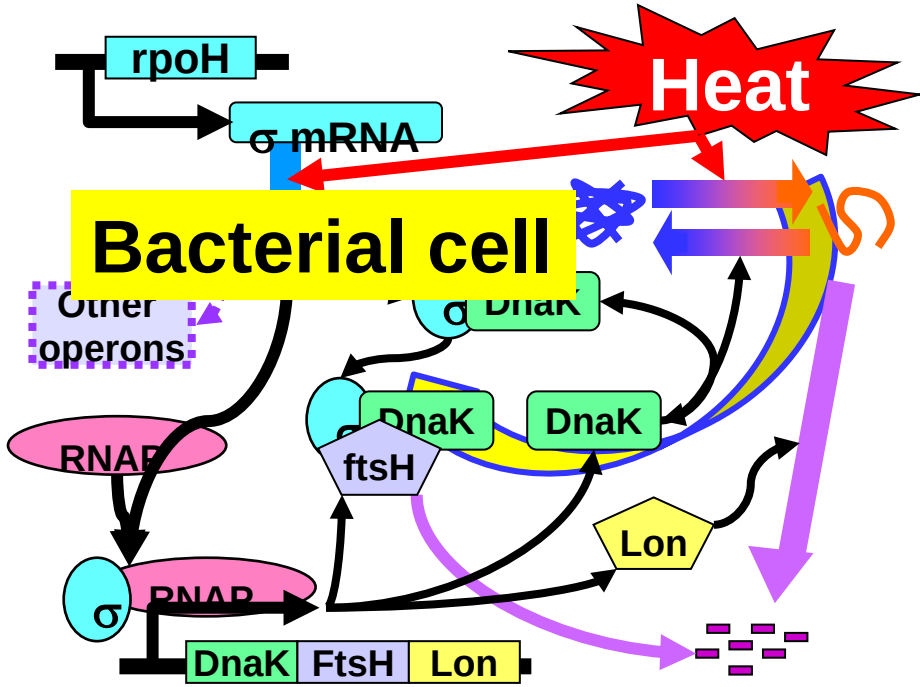
Beliefs

- Viral
- False
- Unhealthy
- Dangerous




Linux kernel diagram
functions
system
networking
storage
memory
processing
human interface
layers
user space interfaces
virtual subsystems
bridges
logical
hardware interfaces
electronics
System calls
Sockets
files and directories
memory access
Processes
char devices
proc, sy file syst
Virtual memory
Tasks
input subsystem
Device Model
NFS
page cache
memory mapping
synchronization
Swap
system run, modules, generic HW access
protocols: TCP, UDP,IP
logical filesystems ext2, ext3
logical memory
Scheduler
HI class drivers
bus drivers
network devices and drivers
Block devices and drivers
Page Allocator
interrupts core
HI peripherals drivers
bus controllers: PCI, USB
network cards: Ethernet, WiFi
disk controllers: IDE, SCSI
MMU, RAM
CPU
© 2007-2009 Constantine Shulyupin http://www.MakeLinux.net/kerne
Linux OS
Layers within layers


Object motion
Head motion
eye
vision
Cortex
fast
slow
Act
delay
Mechanisms
Reflex
gain
AOS
Tune gain
Error
Cerebellum


rpoH
σ mRNA
Heat
Bacterial cell
Other operons
σ DnaK
DnaK
DnaK
ftsH
RNAP
Lon
σ RNAP
DnaK FtsH Lon

Layered architectures

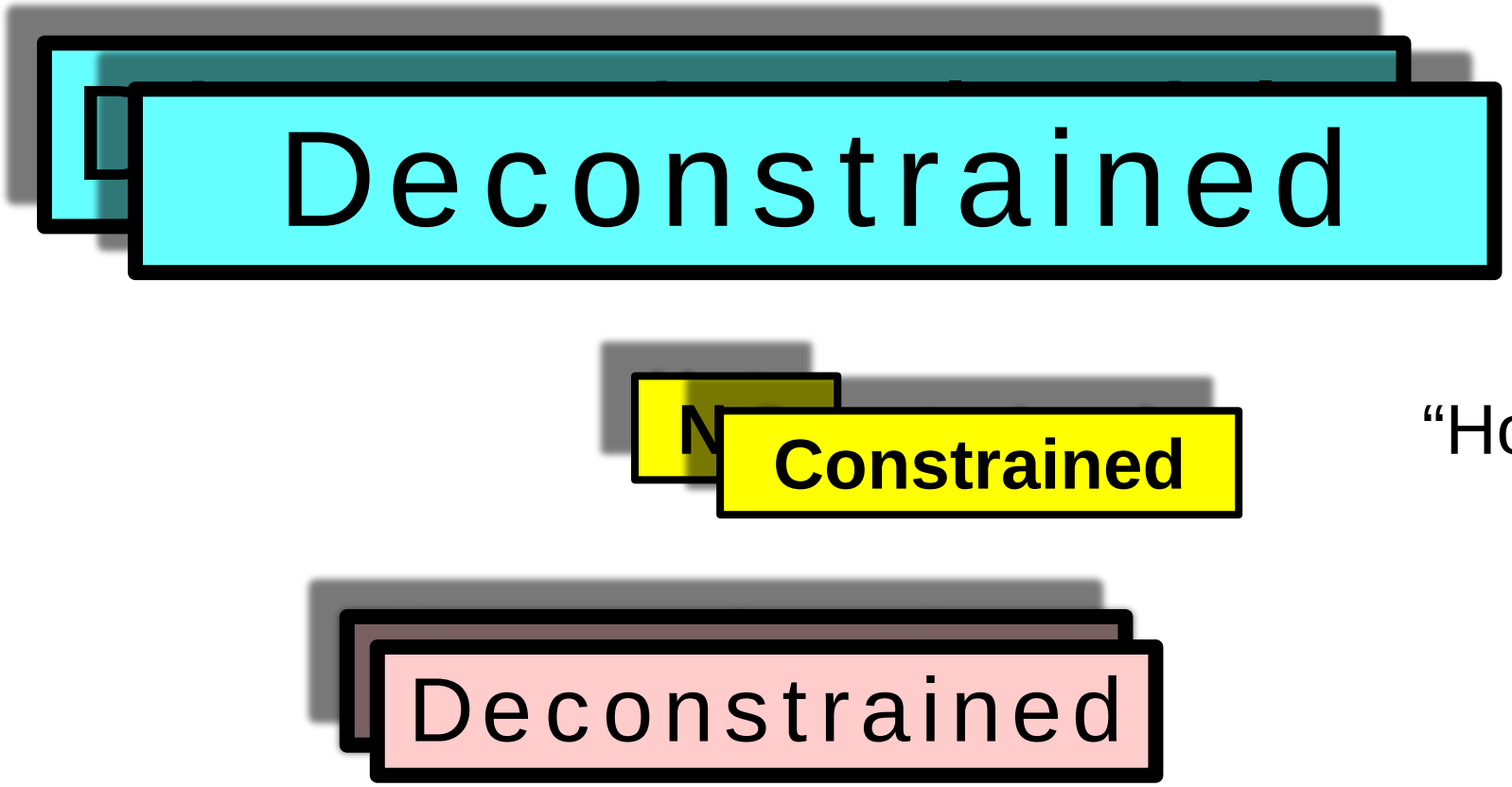
Diverse/evolvable

Not

Diverse/evolve

“Hourglass”

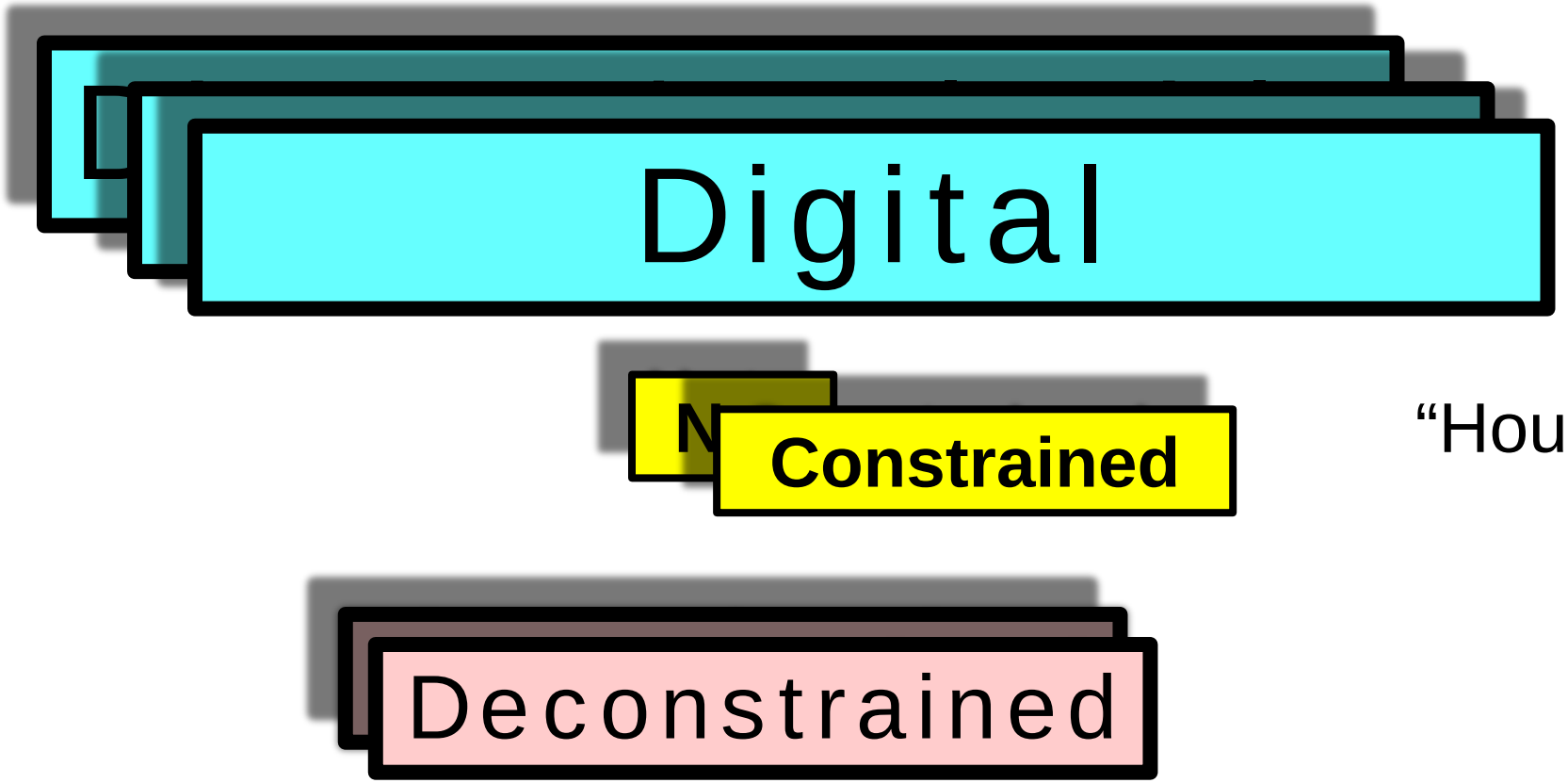
Layered architectures
“Constraints that deconstrain”



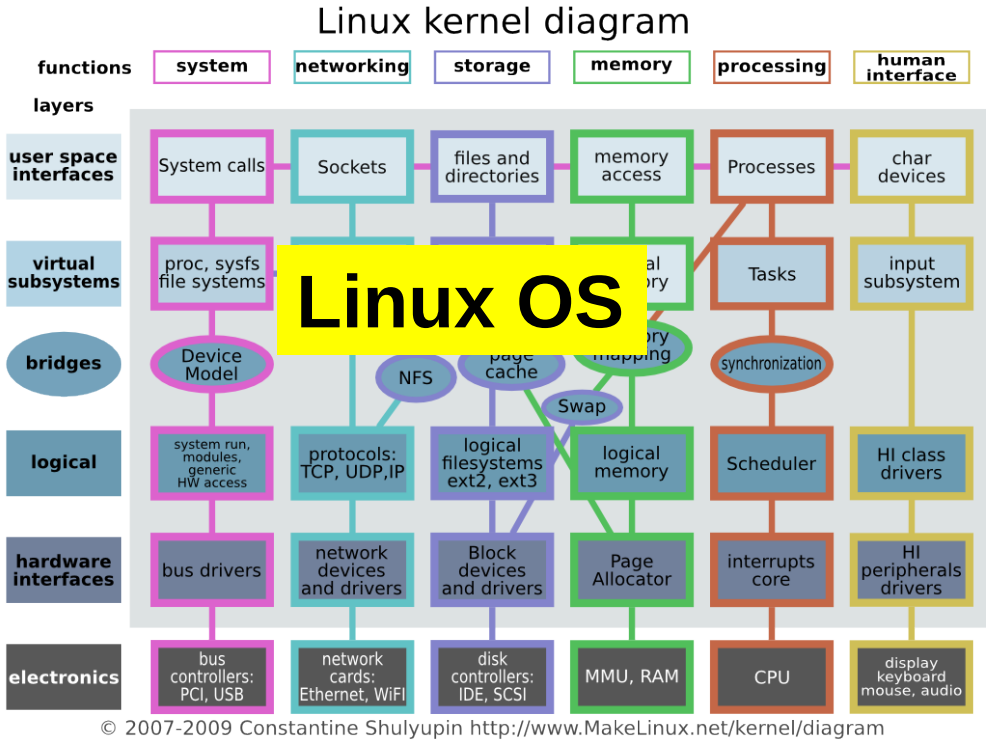
“Hourglass”

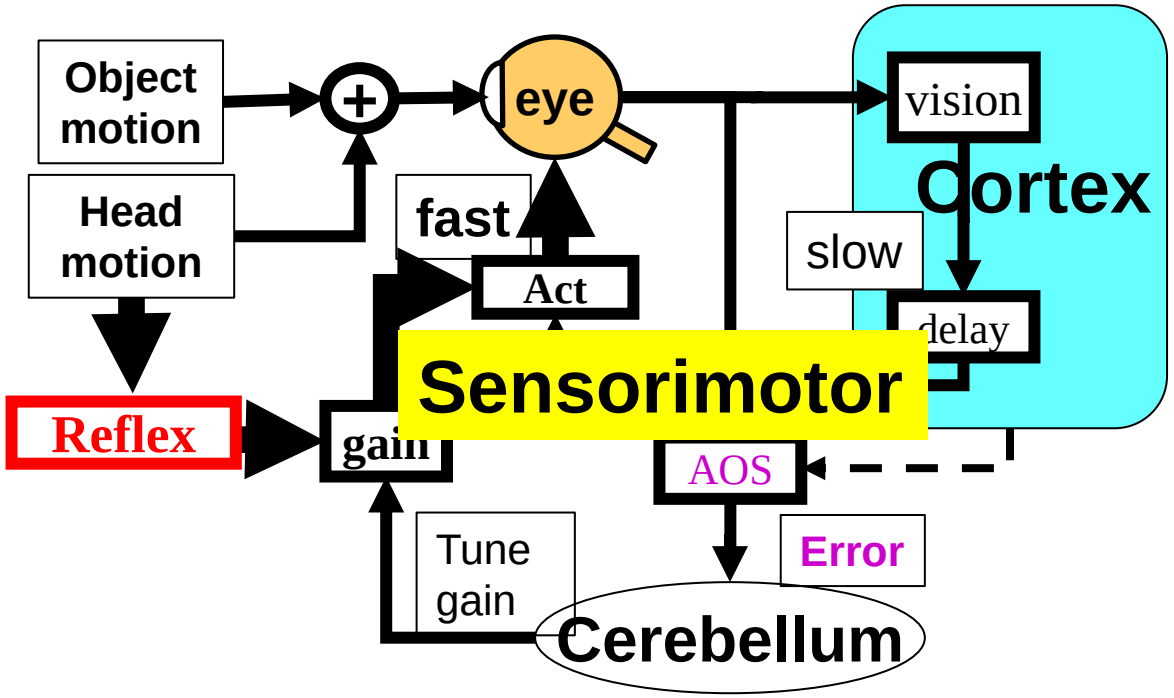
Caporale, Gerhart and Kirschner

Layered architectures
“Constraints that deconstrain”

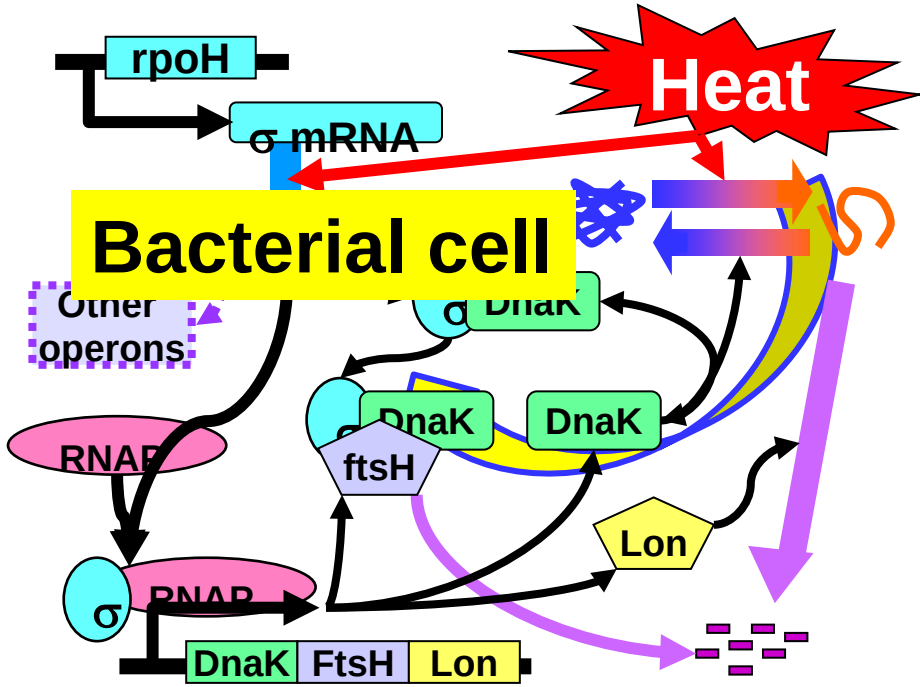


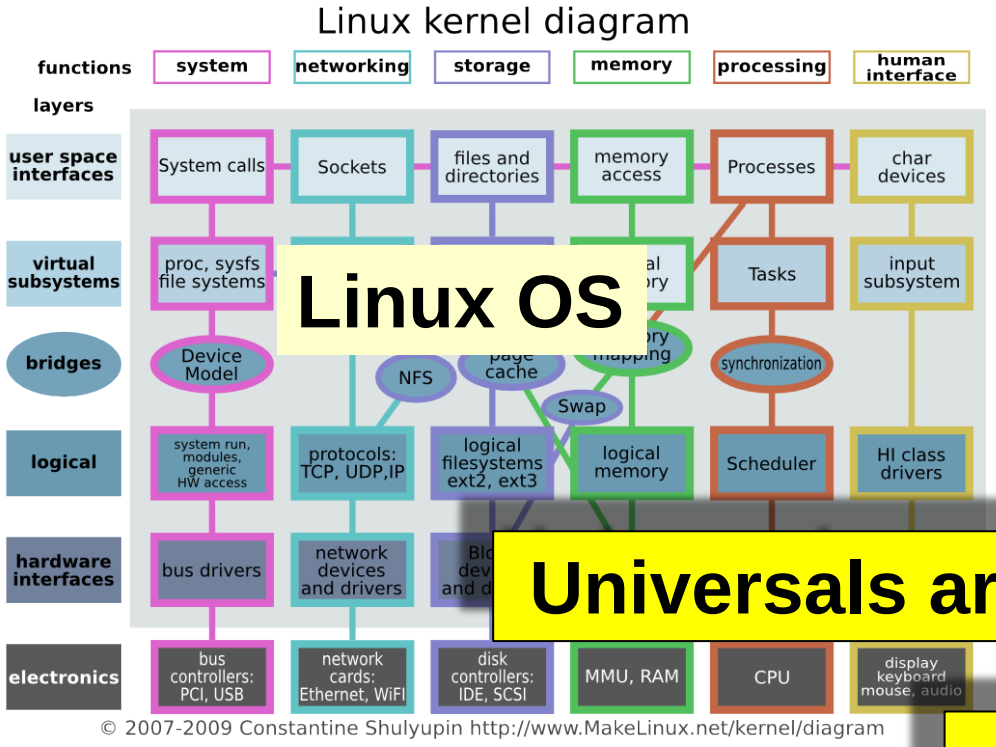
“Hourglass”



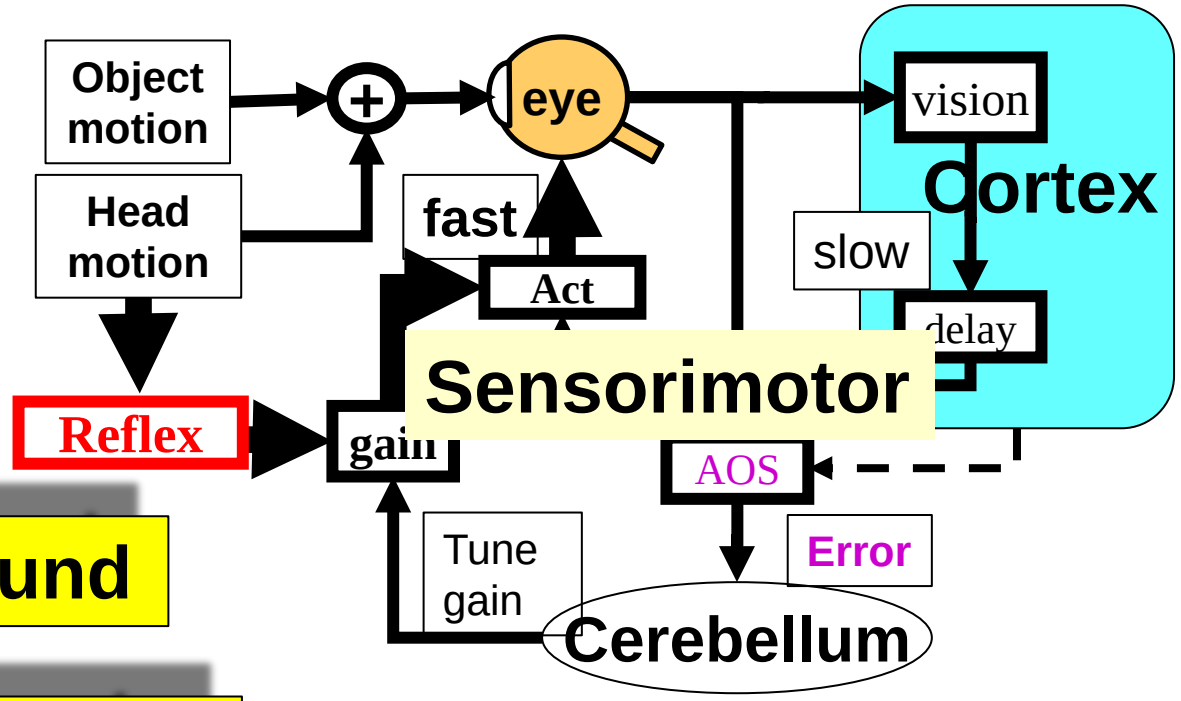


Vastly more different than similar



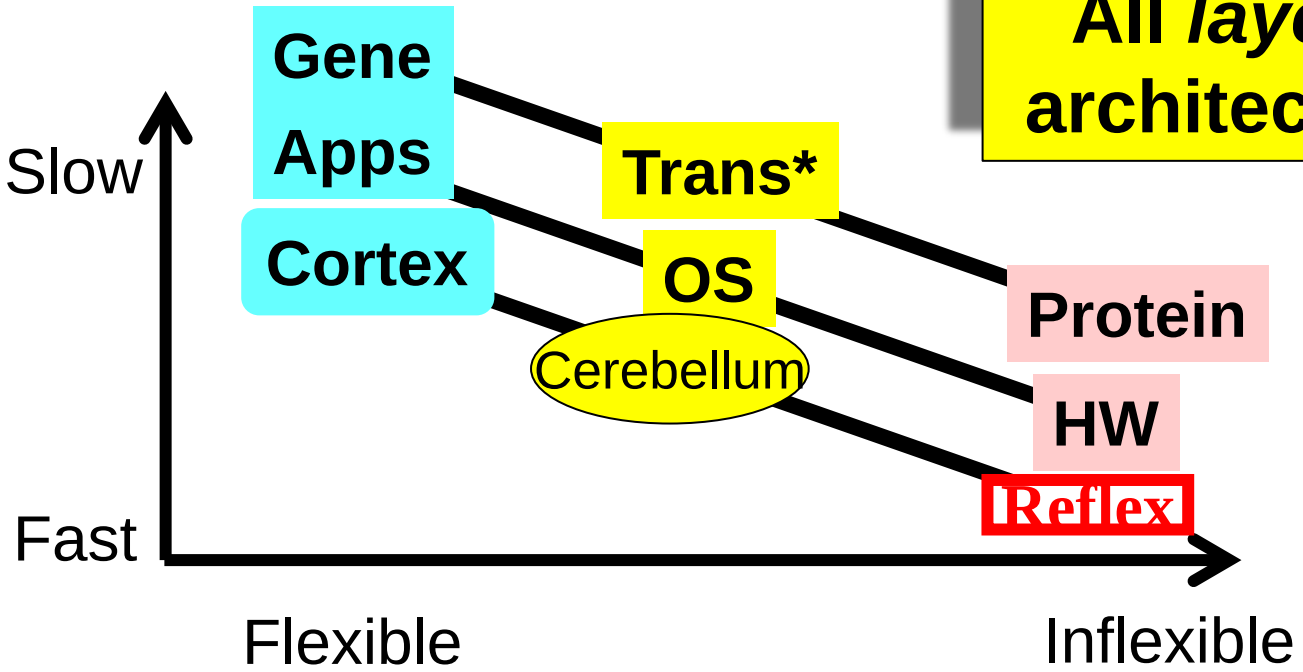


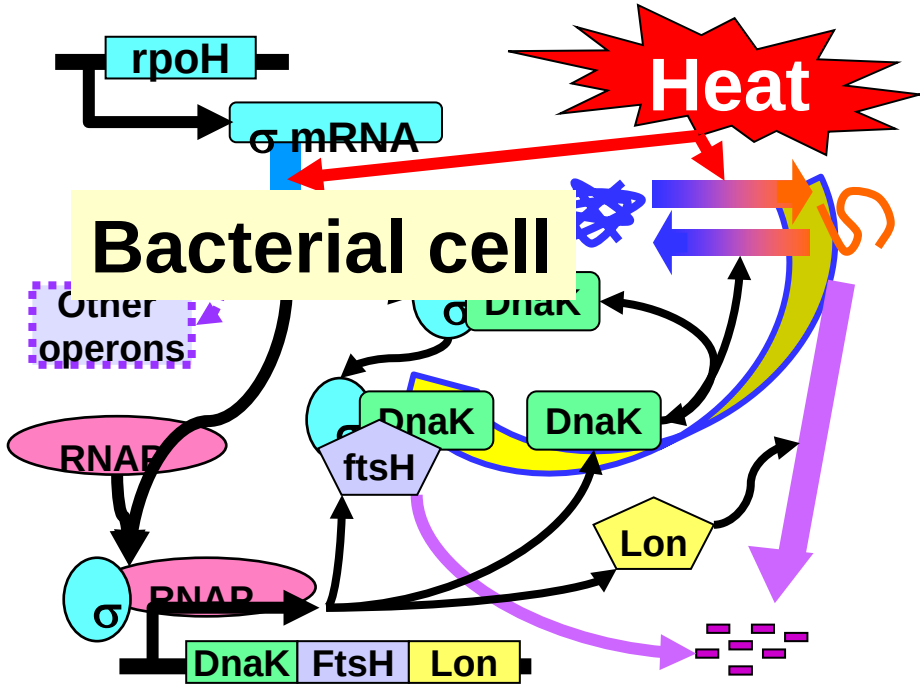
Linux OS



Universals are profound

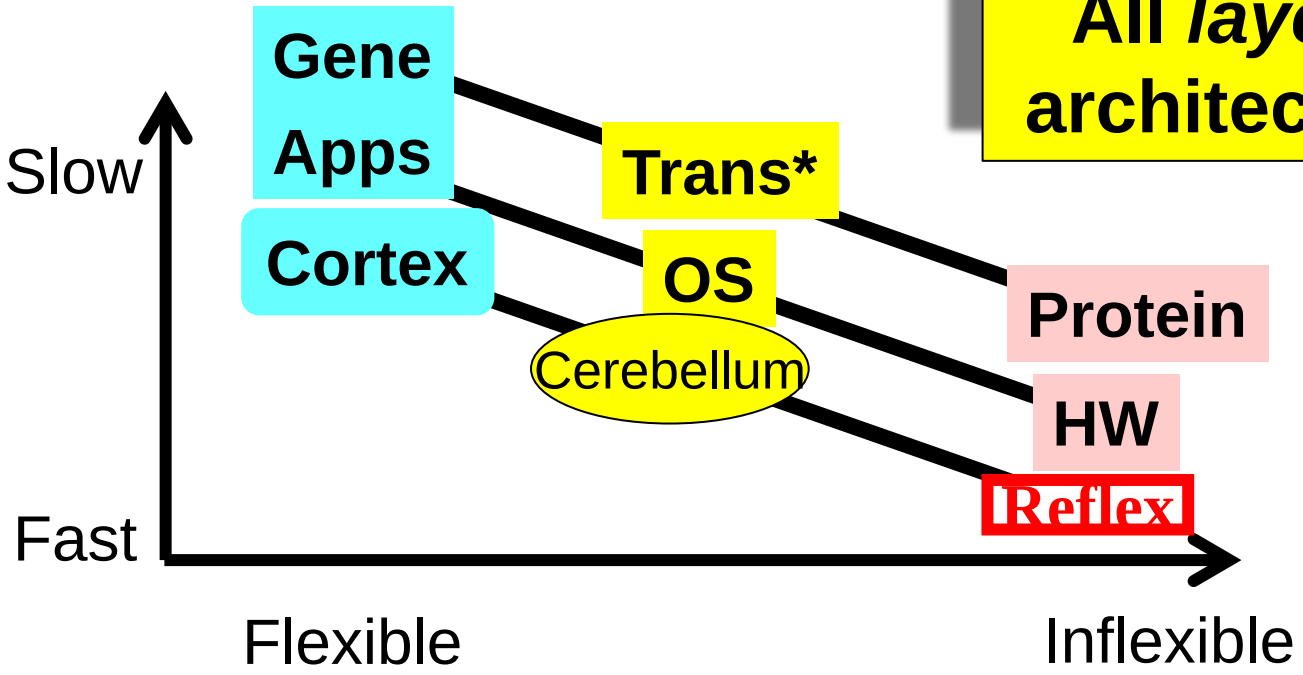
All *layered* architectures





Universals are profound

All *layered* architectures



Diverse

Not

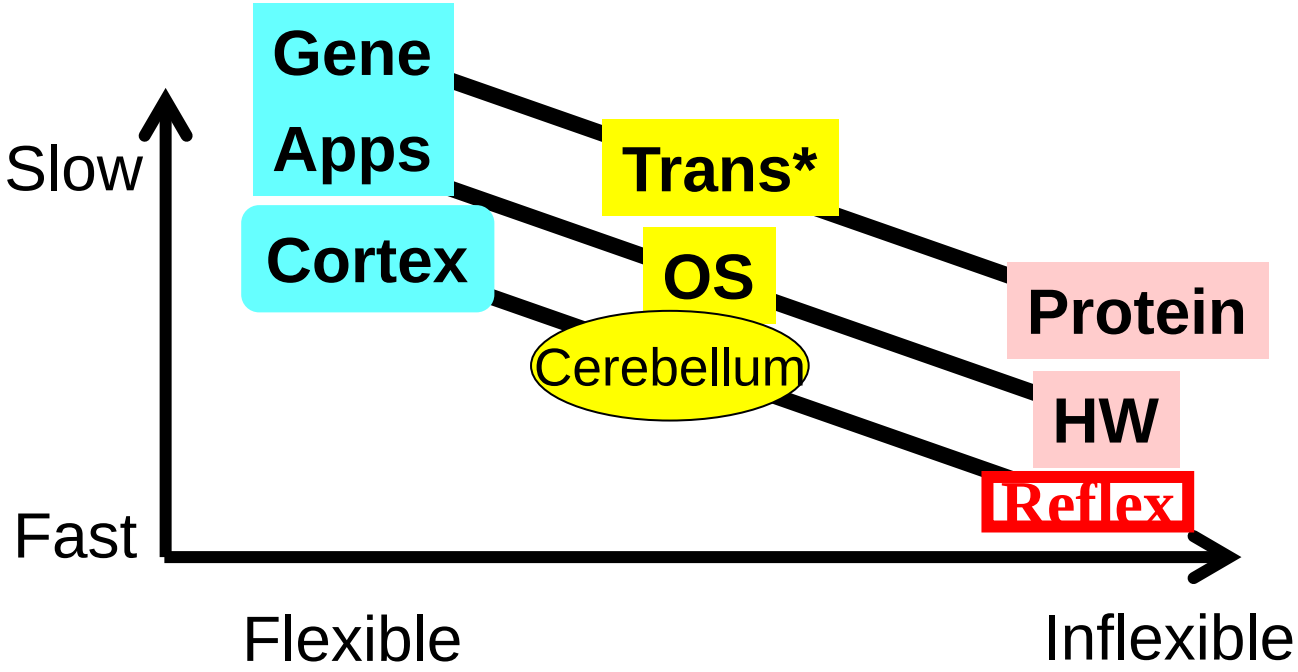
Diverse

Universals are profound

- Tradeoffs
- Evolvability
- Hourglass diversity

"Constraints that deconstrain"

Tradeoffs



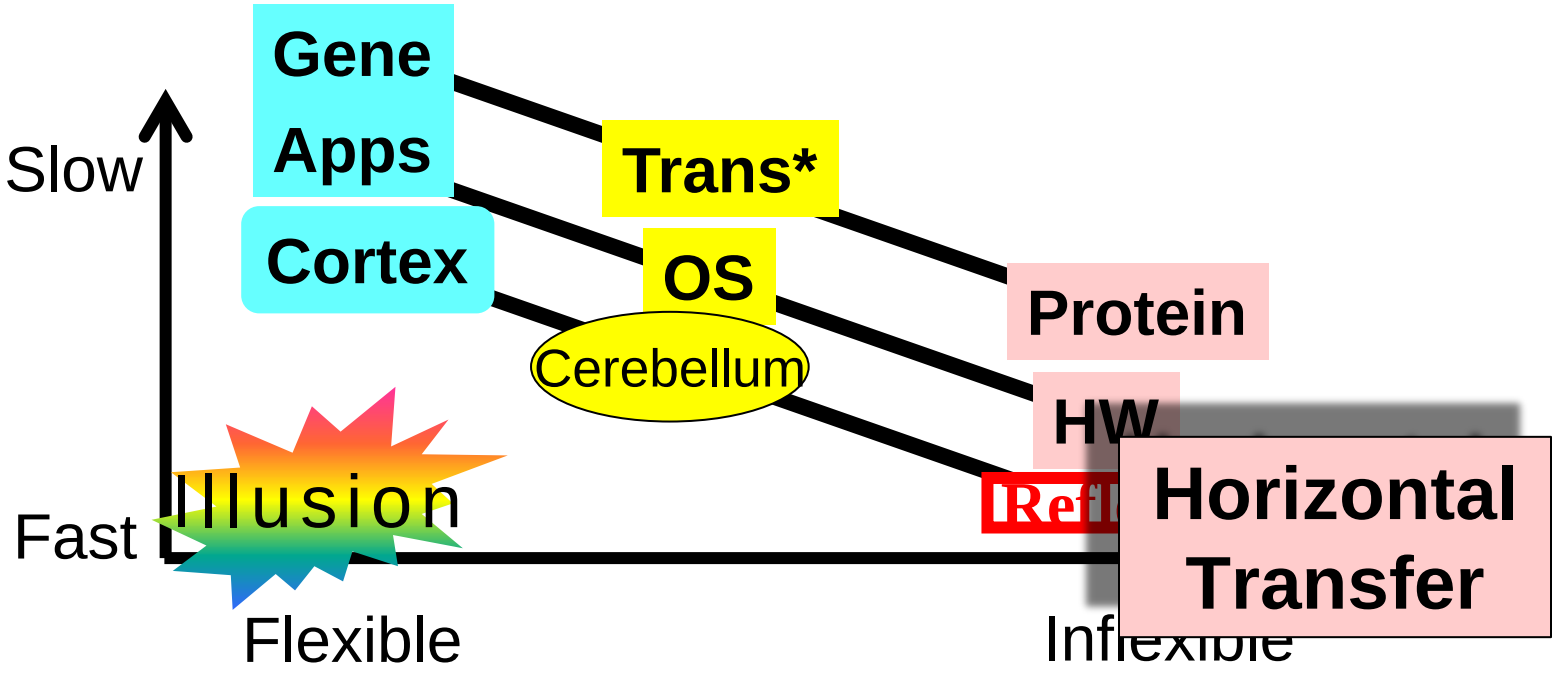
Diverse

Not

Diverse

Horizontal Transfer

Tradeoffs



Universals are profound

- Tradeoffs
- Evolvability
- Hourglass diversity
- Horizontal transfer
- Tunable (illusion)

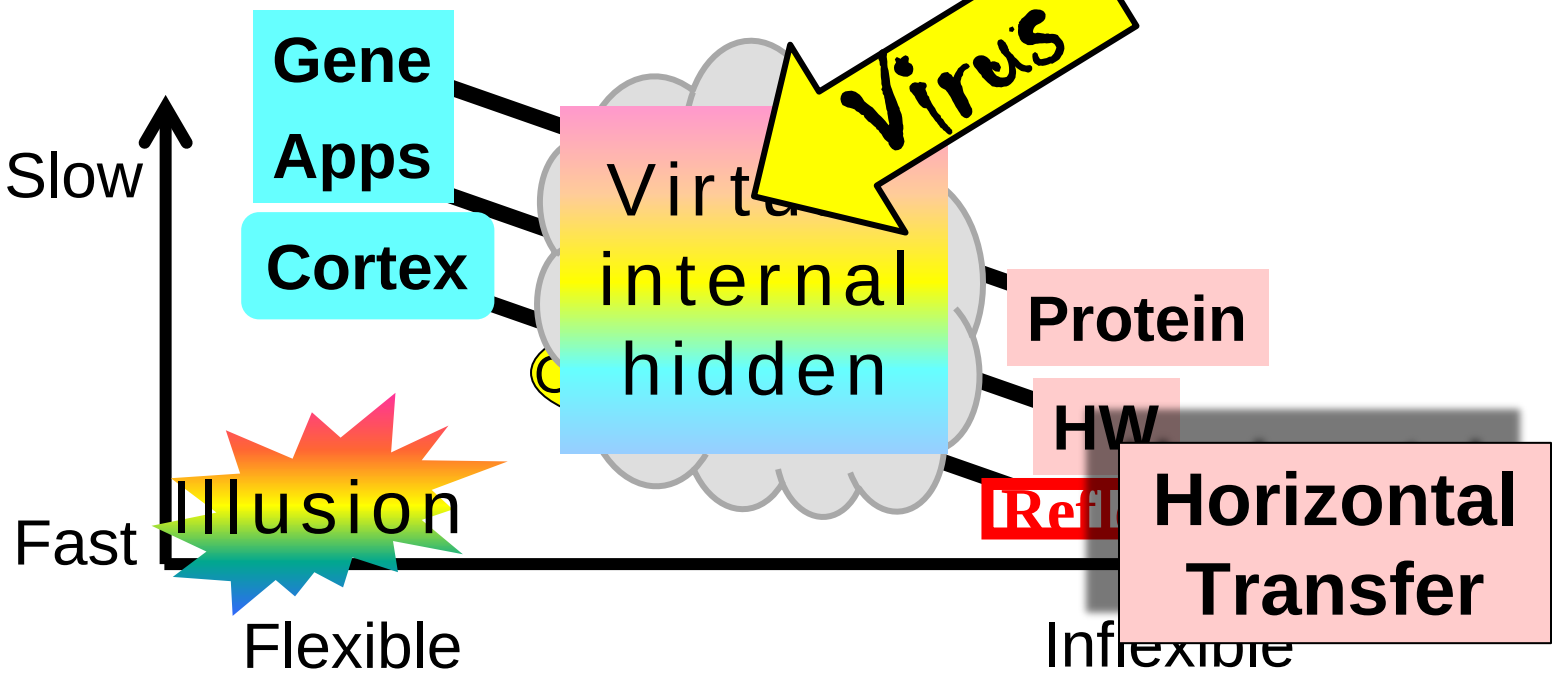
Diverse

Not

Diverse

Horizontal Transfer

Tradeoffs



Horizontal Transfer

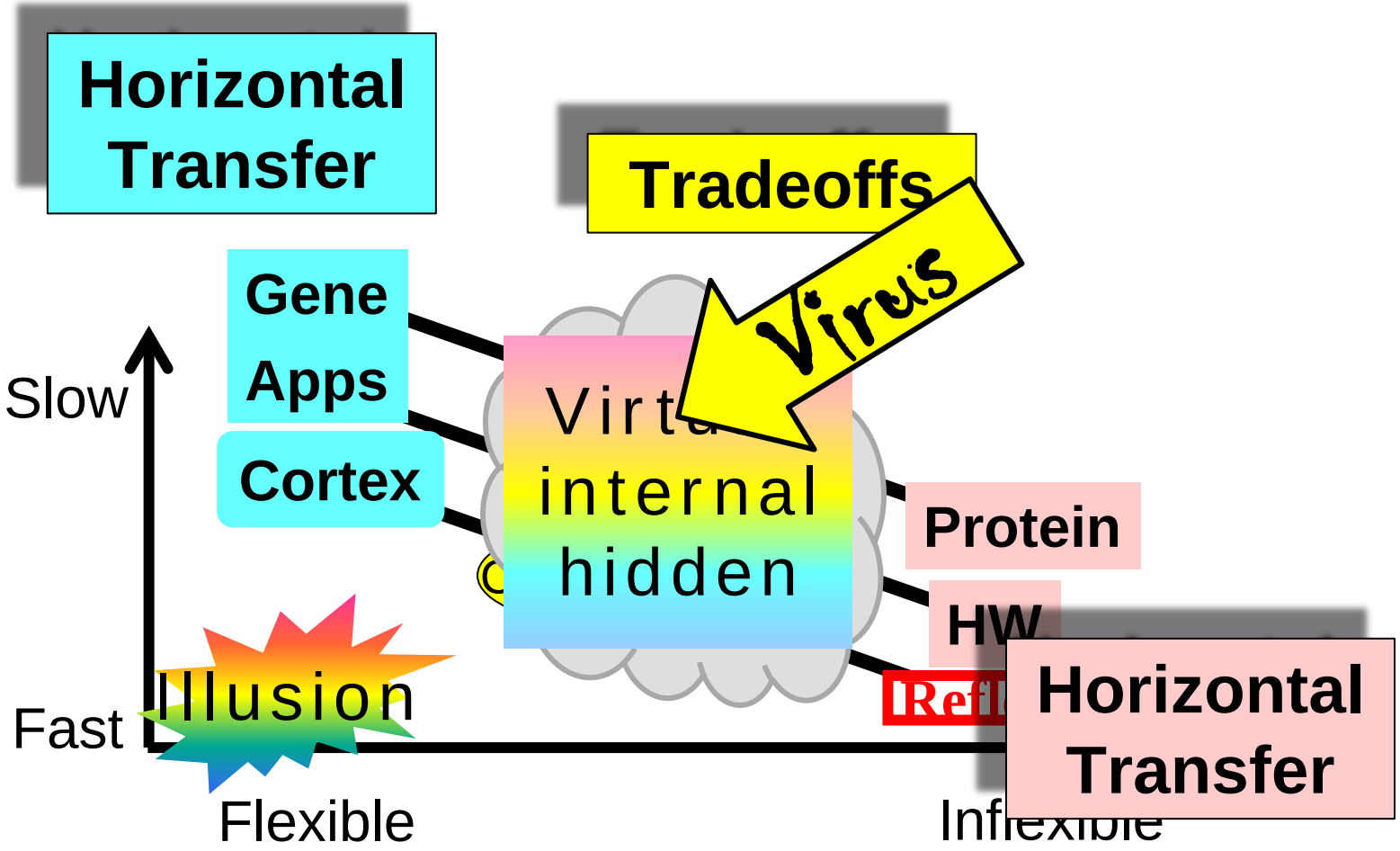
Universals are profound

- Tradeoffs
- Evolvability
- Hourglass diversity
- Horizontal transfer
- Tunable (illusion)
- Virtual/Internal
- Hijacking

Diverse/Digital

Not

Diverse



Universals are profound

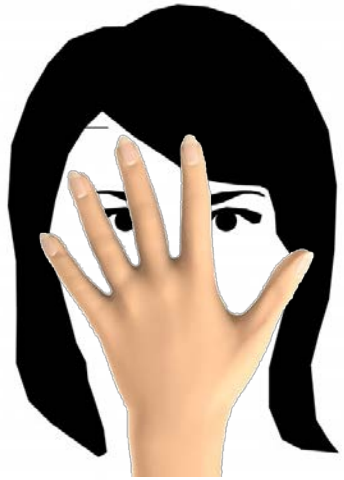
- Tradeoffs
- Evolvability
- Hourglass diversity
- Horizontal transfer
- Tunable (illusion)
- Virtual/Internal
- Hijacking
- Digital
- **Robustness**

- SDN, NFV, IOT, IOE, CPS
- Compute/control technology
- **Industrialization**
- **Money/finance/lobbyists/etc**
- **Society/agriculture/states**
- **Weapons**
- Bipedalism (balance, cardiovascular physiology)
- **Maternal care**
- **Warm blood**
- Flight (turbulence)
- **Mitochondria**
- **Oxygen**
- **Translation (ribosomes)**
- Glycolysis
- Cells

Universals are profound

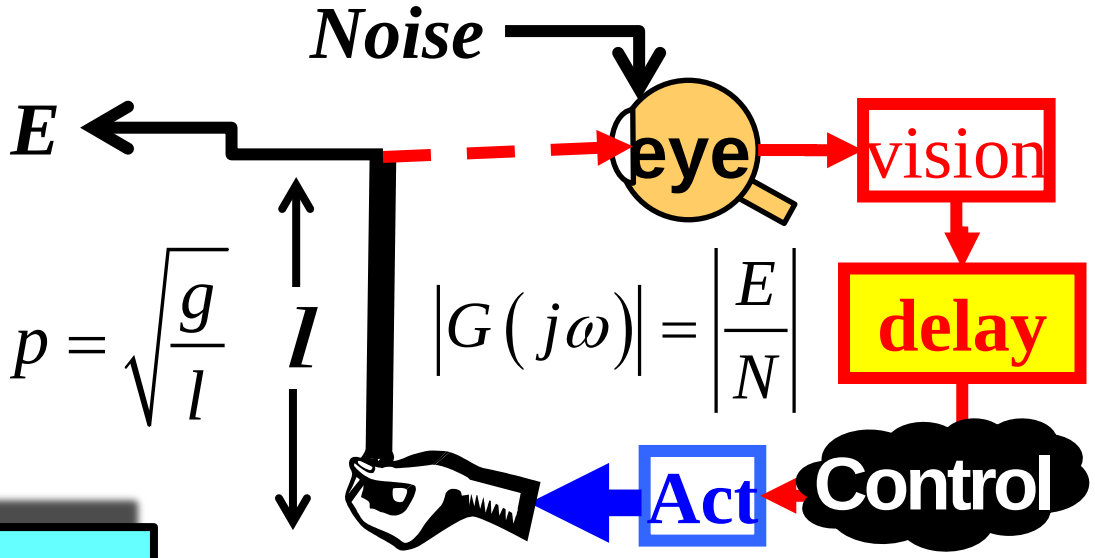
- Tradeoffs
- Evolvability
- Hourglass diversity
- Horizontal transfer
- Tunable (illusion)
- Virtual/Internal
- Hijacking
- Digital
- **Robustness**

Slow

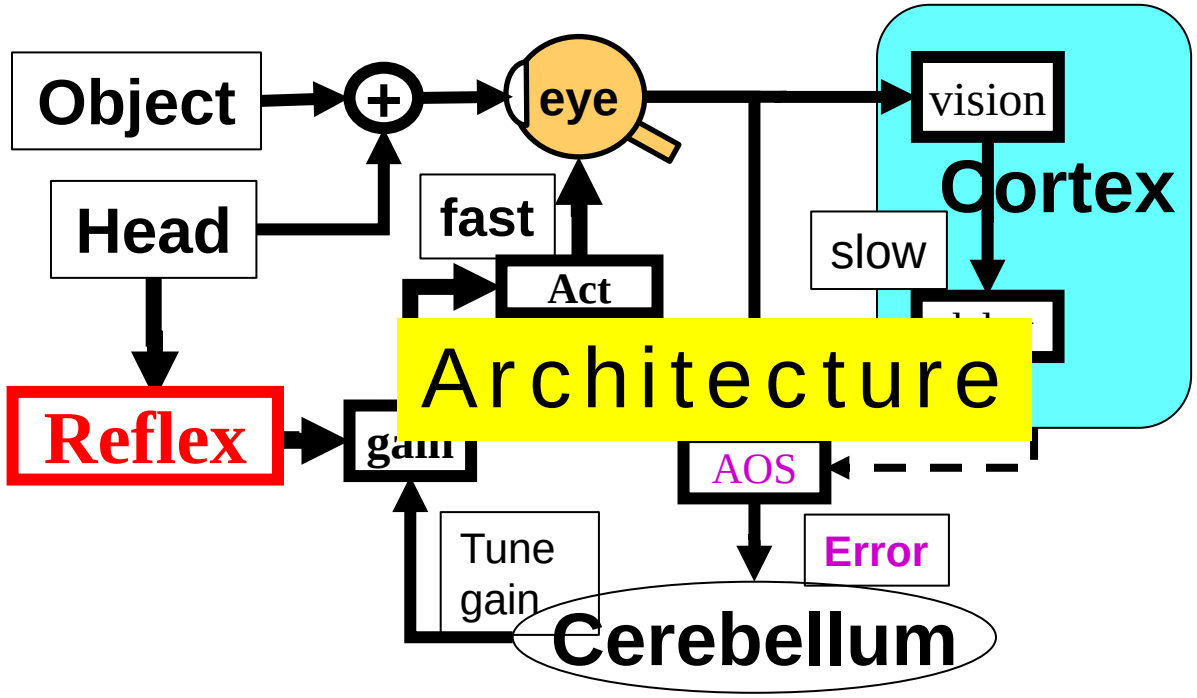


Fast

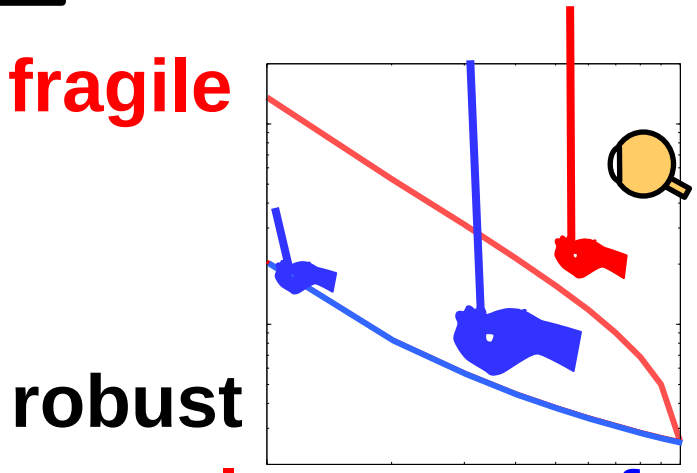
Behavior



Theory?



Architecture



Laws

slow
unstable
efficient

fast
stable
costly

General

Low layers

past
reflex
delayed
unstable (real)

small ||w|| *but*
large ||CL|| >1
need speed
distributed
local

General

layered/recursive
dynamic/feedback
saturation
digital/quantized
worst case ||●||
scale and sparsity
(state, sense, act,
compute, comms)
co-design (control, plant,
comms, sense, act)
other nonlinearities

High layers

future
planning
advance warning
stable (virtual)

large ||r|| *but*
small ||CL|| <<1
need accuracy
centralized
global

---

unconscious
>>10Mb/sec

Brains

conscious
<100 bits/sec

General

layered/recursive
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||$
scale and sparsity
(state, sense, act,
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future
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High layers
future
planning
advance warning
stable (virtual)

large ||r|| *but*
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need accuracy
centralized
global

conscious
<100 bits/sec

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past
reflex
delayed
unstable (real)

small ||w|| *but*
large ||CL|| >1
need speed
distributed
local

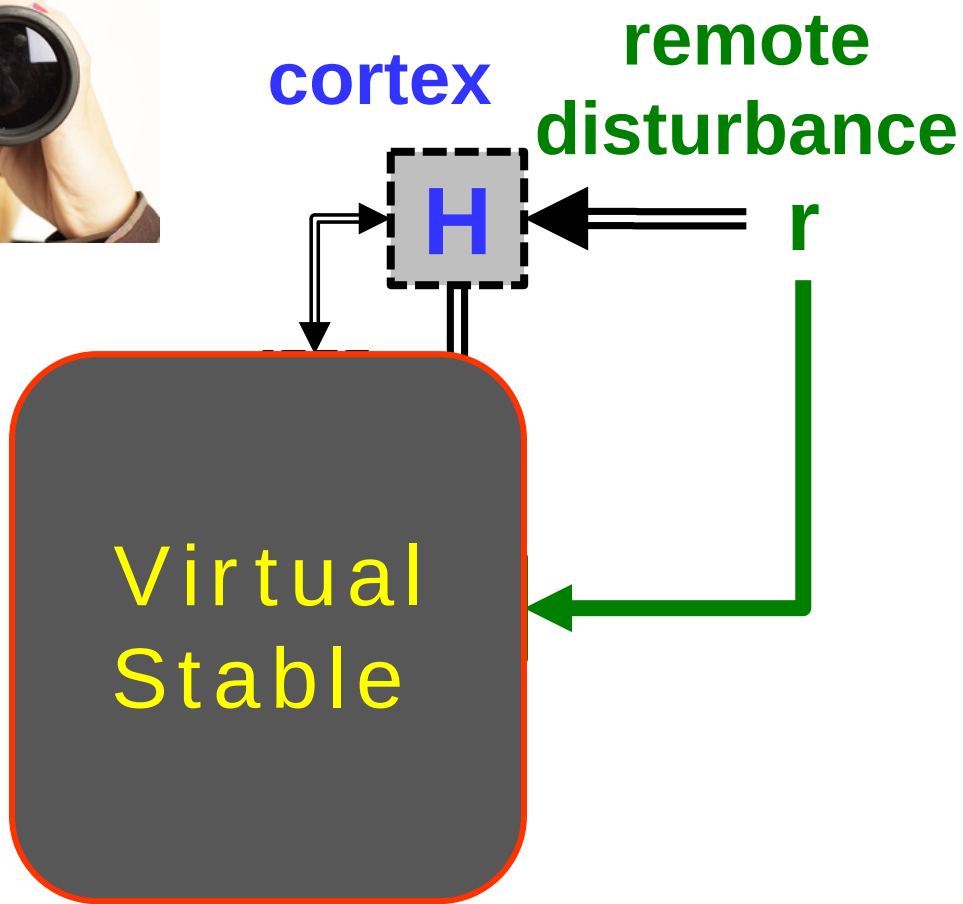
unconscious
>>10Mb/sec

High layers
future
planning
advance warning
stable (virtual)

large ||r|| *but*
small ||CL|| <<1
need accuracy
centralized
global

conscious
<100 bits/sec

layered/recursive
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$



High layers
future
planning (MPC)
advance warning
stable (virtual)
large ||r|| *but*
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized

Low layers

past

reflex

delayed

unstable (real)

small ||w|| *but*

large ||CL|| >1

need speed

unconscious

>>10Mb/sec

distributed

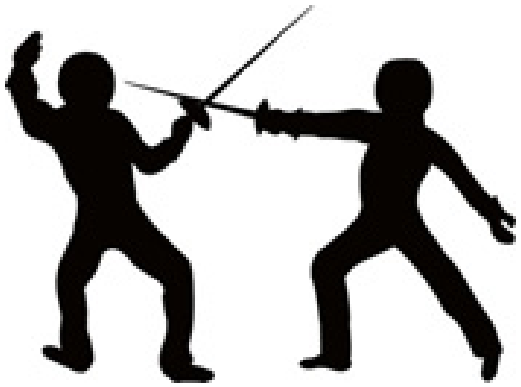
layered/recursive

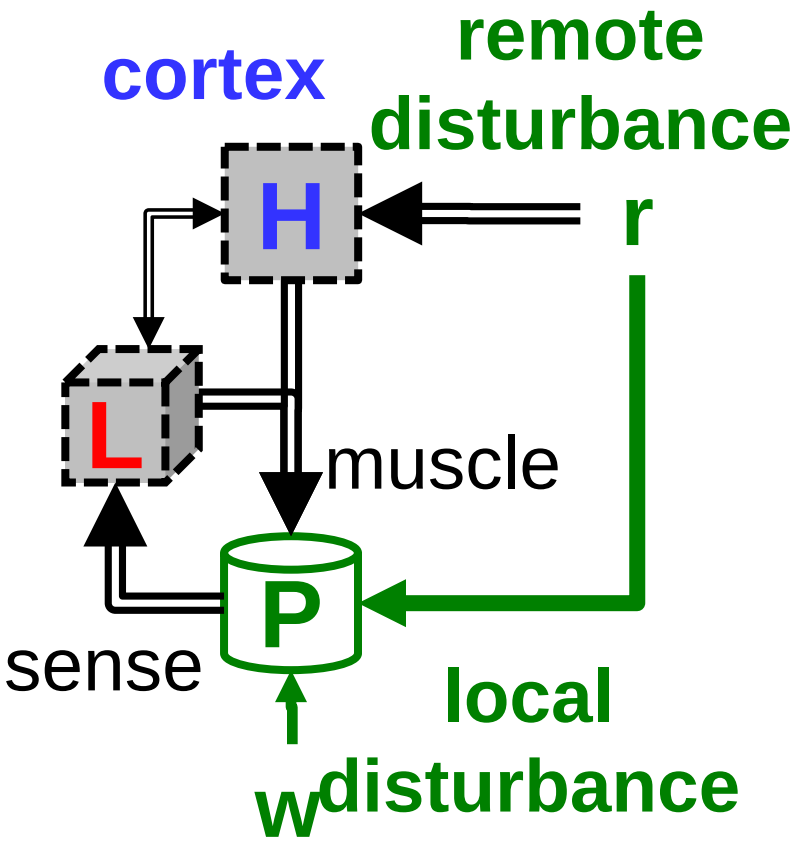
dynamic/feedback

saturation

digital/quantized

worst case $||\bullet||_\infty$





Low layers
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
unconscious
>>10Mb/sec
distributed

layered/recursive
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$

e
nce

P

local
disturbance

w

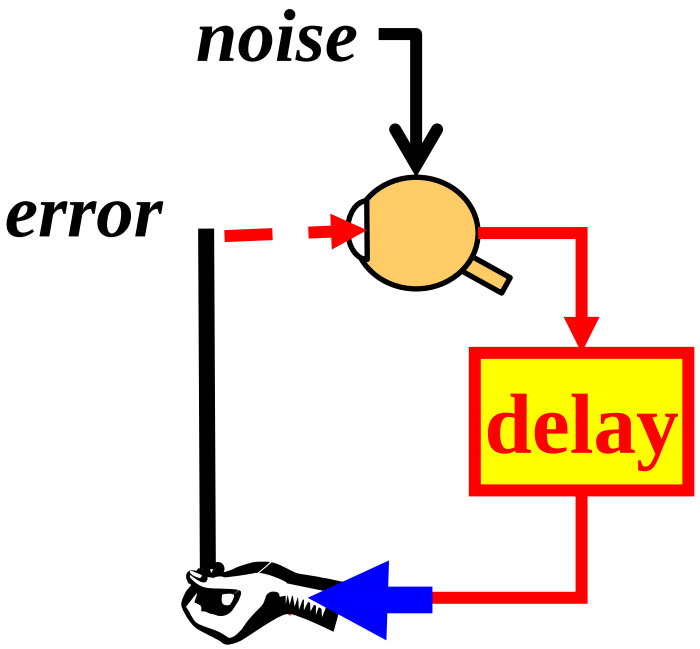
Low layers
past
reflex
delayed
unstable (real)
small ||w|| but
large ||CL|| >1
need speed
u us
> ec
distributed
local

$$p = \sqrt{\frac{g}{l}}$$

Instability

$$\left.\begin{matrix} \exp\left(\int \ln|T|\right) \\ \|T\|_\infty \end{matrix}\right\} \geq \exp(p\tau)$$

High layers
~~future~~
planning
~~advance warning~~
~~stable (virtual)~~
large ||r|| but
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized
global



Low layers
past
reflex
delayed
unstable (real)

small ||w|| *but*
large ||CL|| >1
need speed
distributed
local

unconscious
>>10Mb/sec

High layers
future
planning
advance warning
stable (virtual)

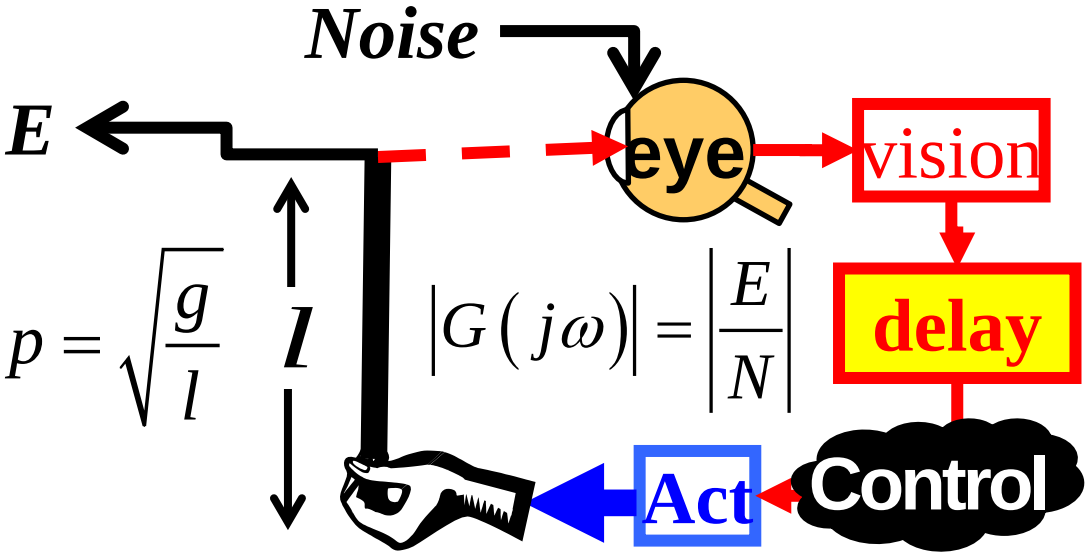
large ||r|| *but*
small ||CL|| <<1
need accuracy
centralized
global

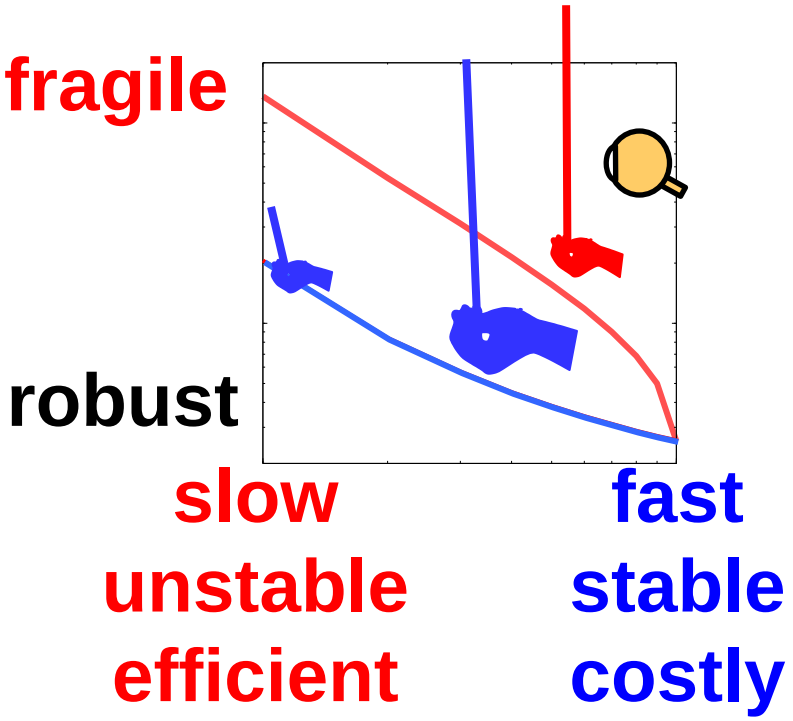
conscious
<100 bits/sec

Tradeoffs

Low layers
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1

balance
posture
SaO2
BP
pH
infections
...





Low layers
past
reflex
delayed
unstable (real)
small ||w|| but
large ||CL|| >1
need speed
unconscious
>>10Mb/sec
distributed

layered/recursive
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$

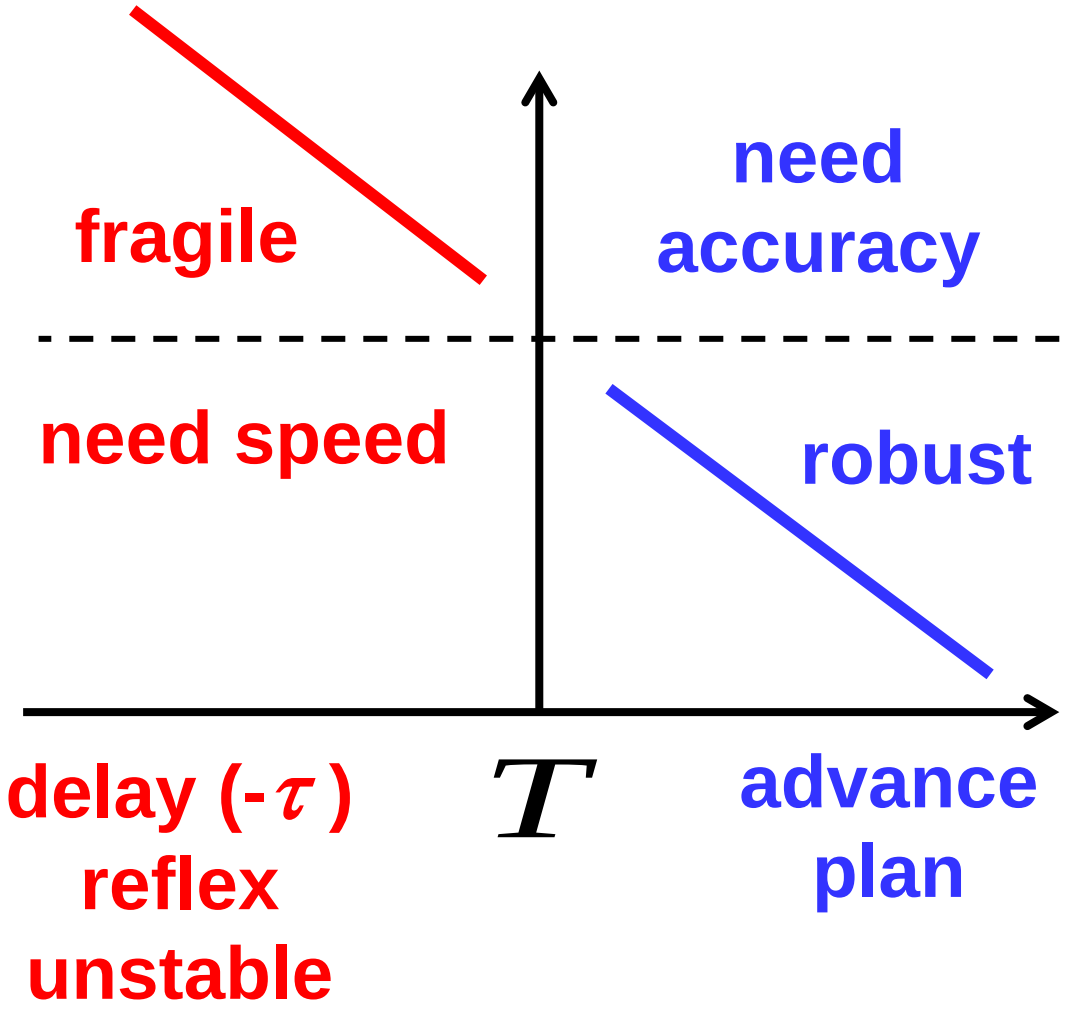
local
disturbance

High layers
future
planning
advance warning
stable (virtual)
large ||r|| *but*
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized

Low layers
past
reflex
delayed
unstable (real)

small ||w|| *but*
large ||CL|| >1
need speed
distributed
local

unconscious
>>10Mb/sec



T = external delays
- internal delays

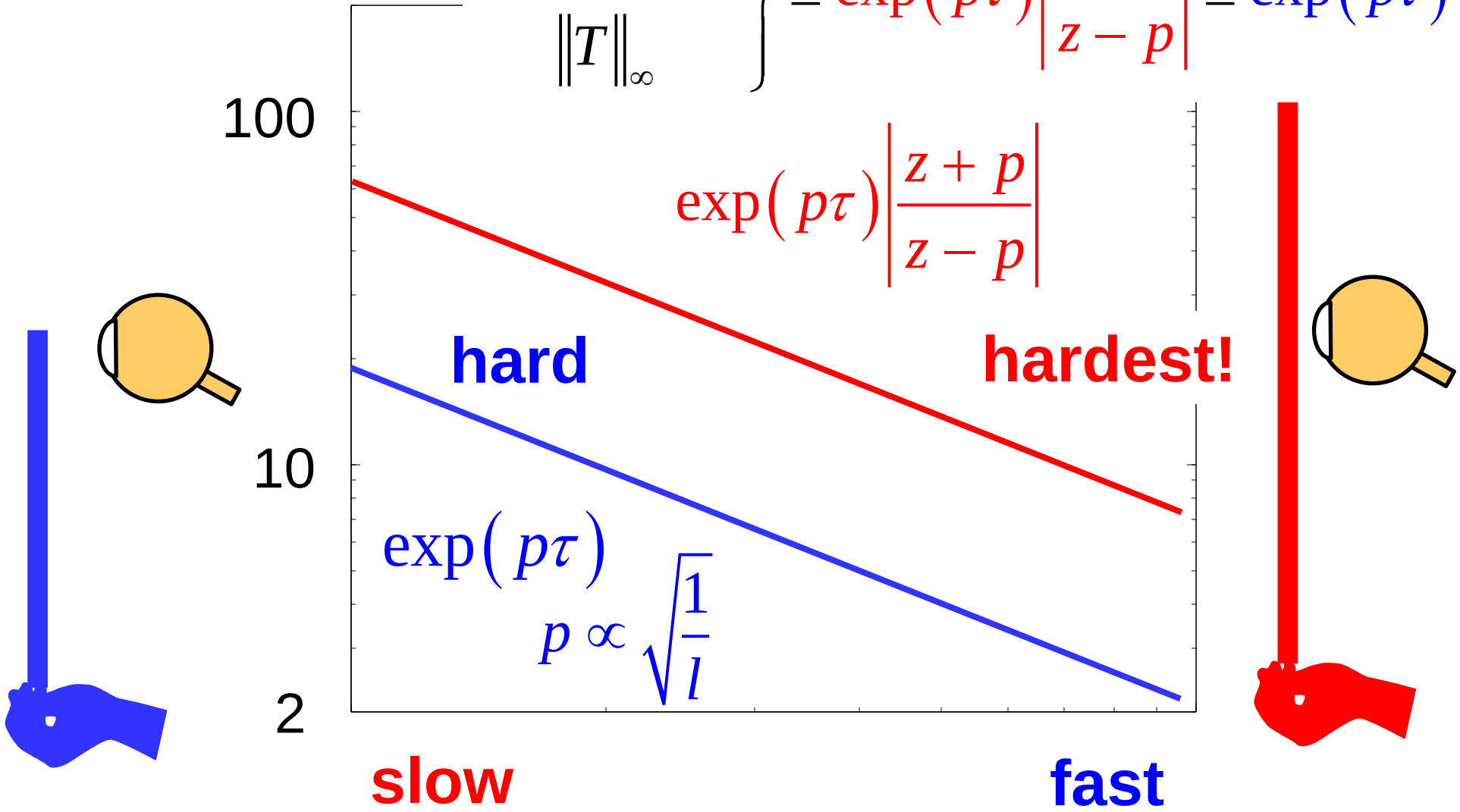
High layers
future
planning
advance warning
stable (virtual)

large ||r|| *but*
small ||CL|| <<1
need accuracy
centralized
global

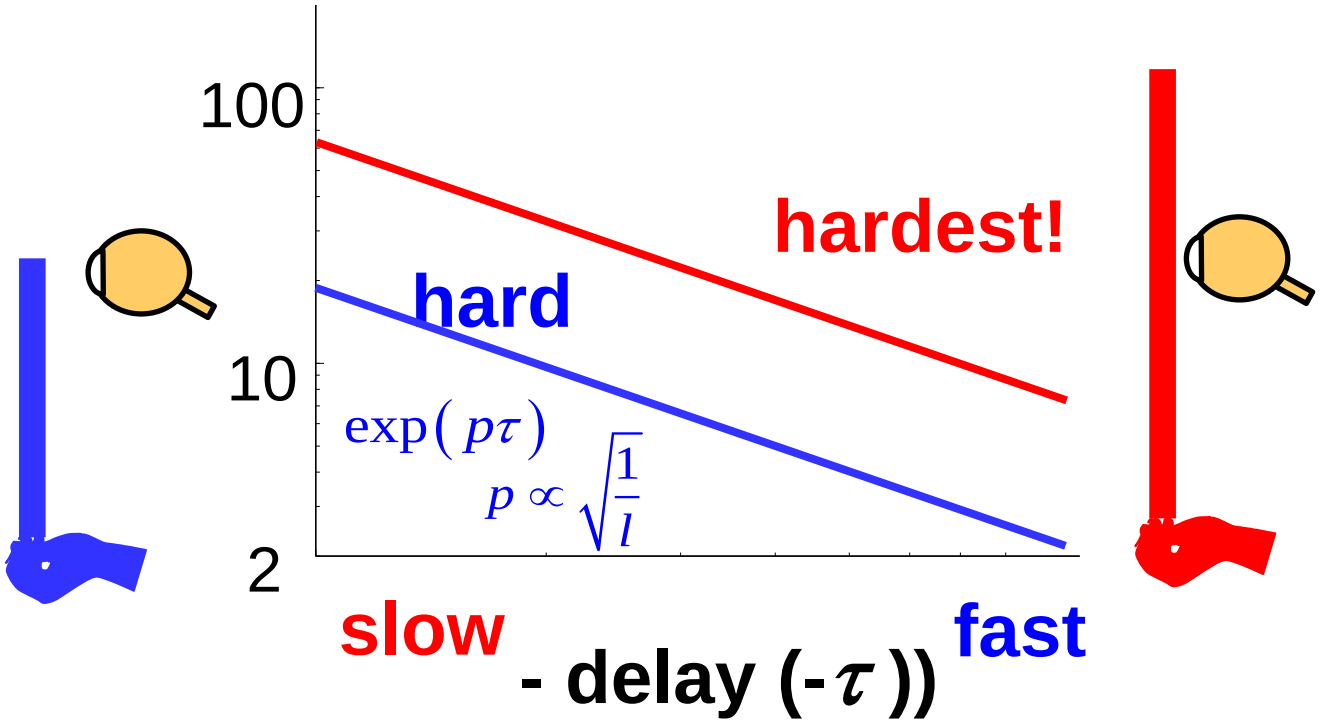
conscious
<100 bits/sec

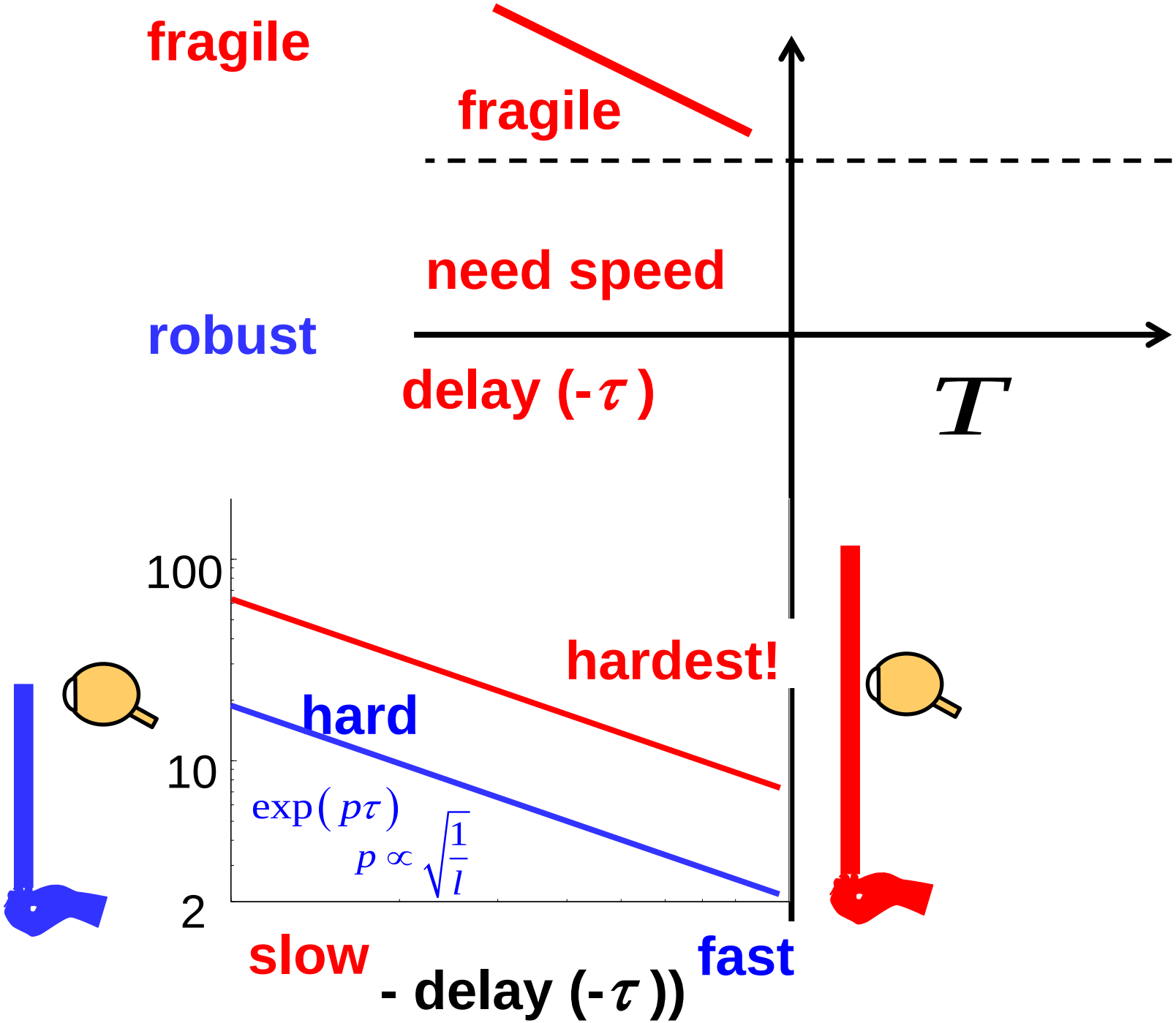
$$\tau = .3s$$

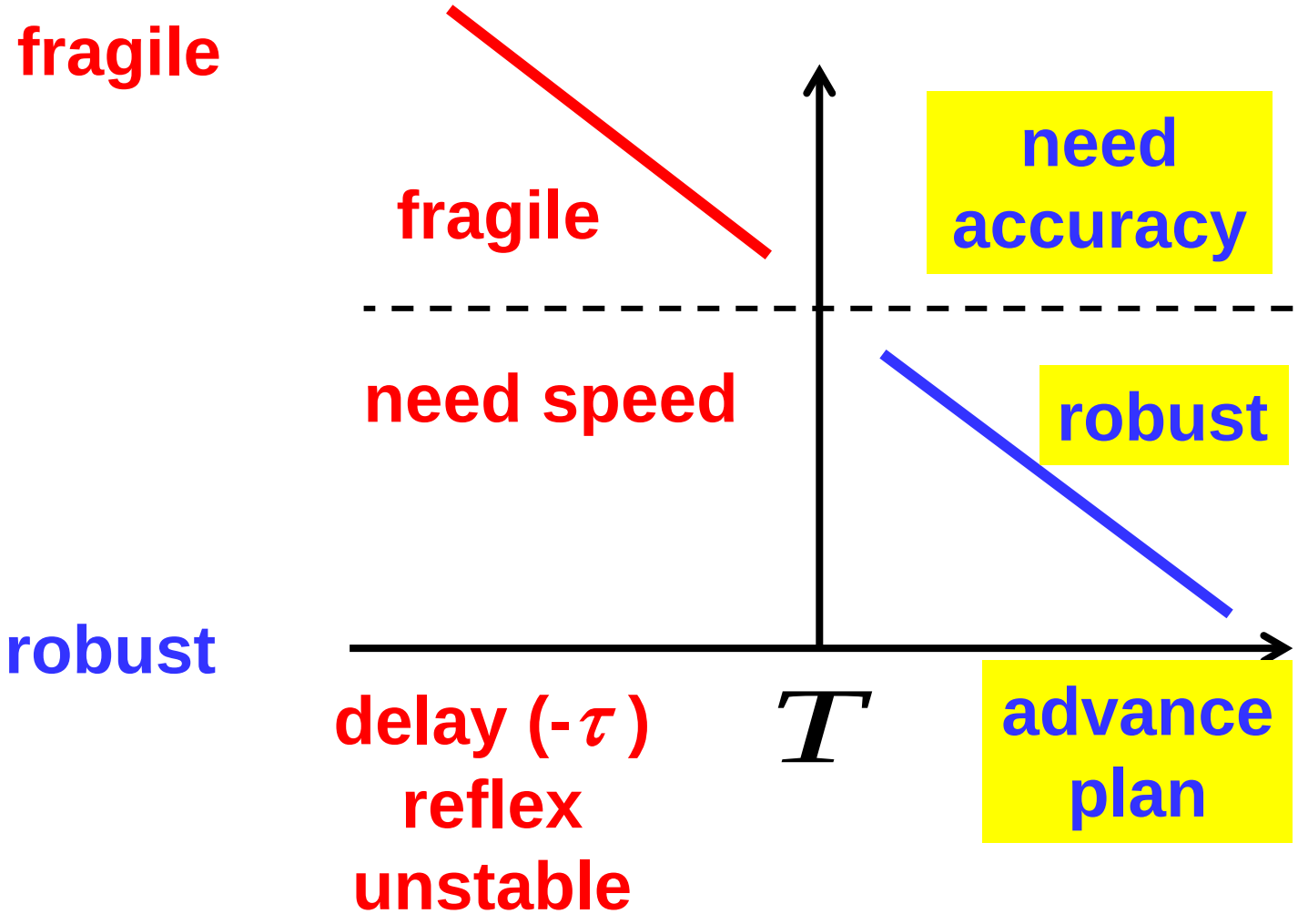
$$\left.\begin{matrix}\exp\left(\int \ln|T|\right) \\ \|T\|_\infty\end{matrix}\right\} \geq \exp(p\tau)\left|\frac{z+p}{z-p}\right| \geq \exp(p\tau)$$



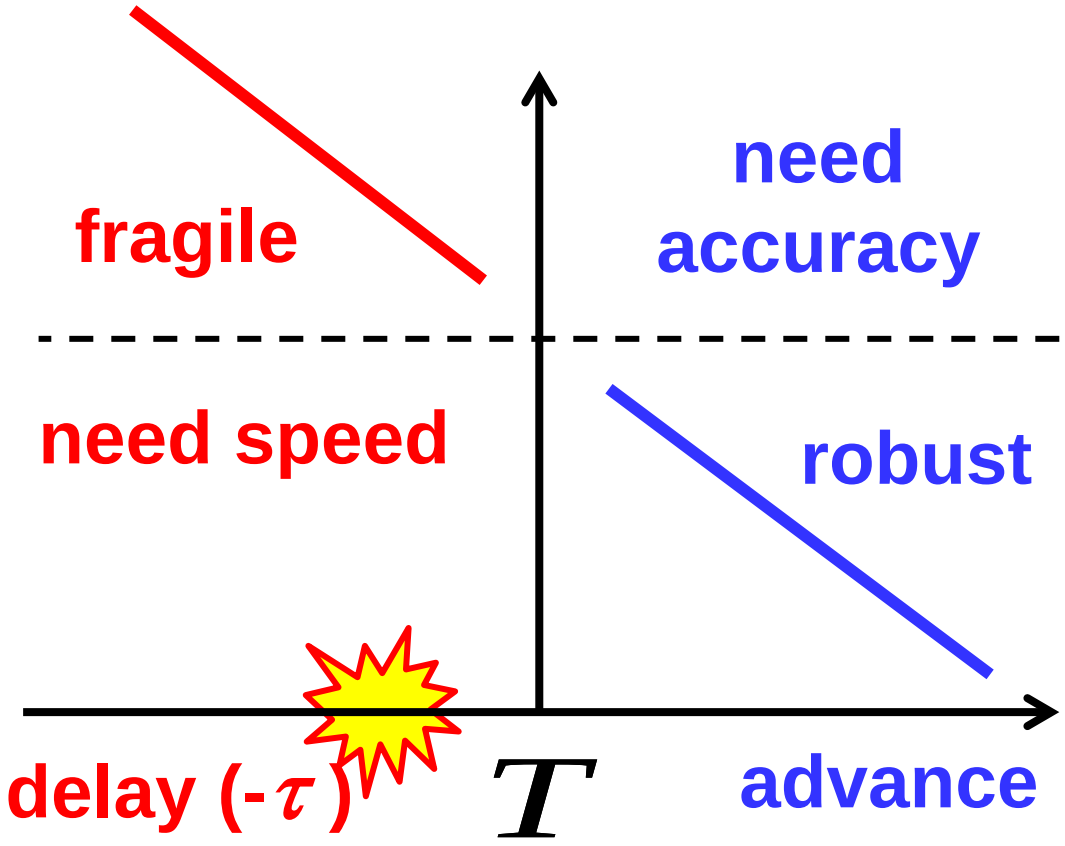
- delay (-τ))


100
10
2
hardest!
hard
$\exp(p\tau)$
$p \propto \sqrt{\frac{1}{l}}$
slow
fast
- delay (-τ))


fragile
fragile
need speed
robust
delay (-τ)
T
100
10
2
hardest!
hard
exp(pτ)
p ∝ √(1/l)
slow
fast
- delay (-τ))

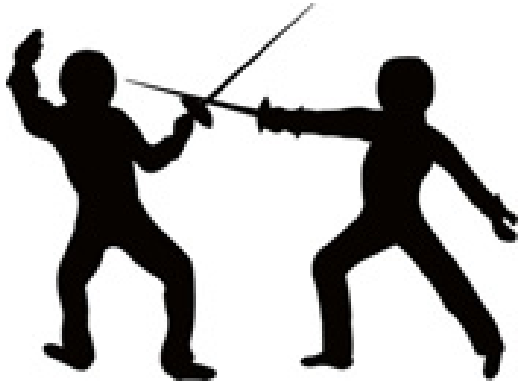


T = external delays
- internal delays



My delays = your advance

Your delays = my advance

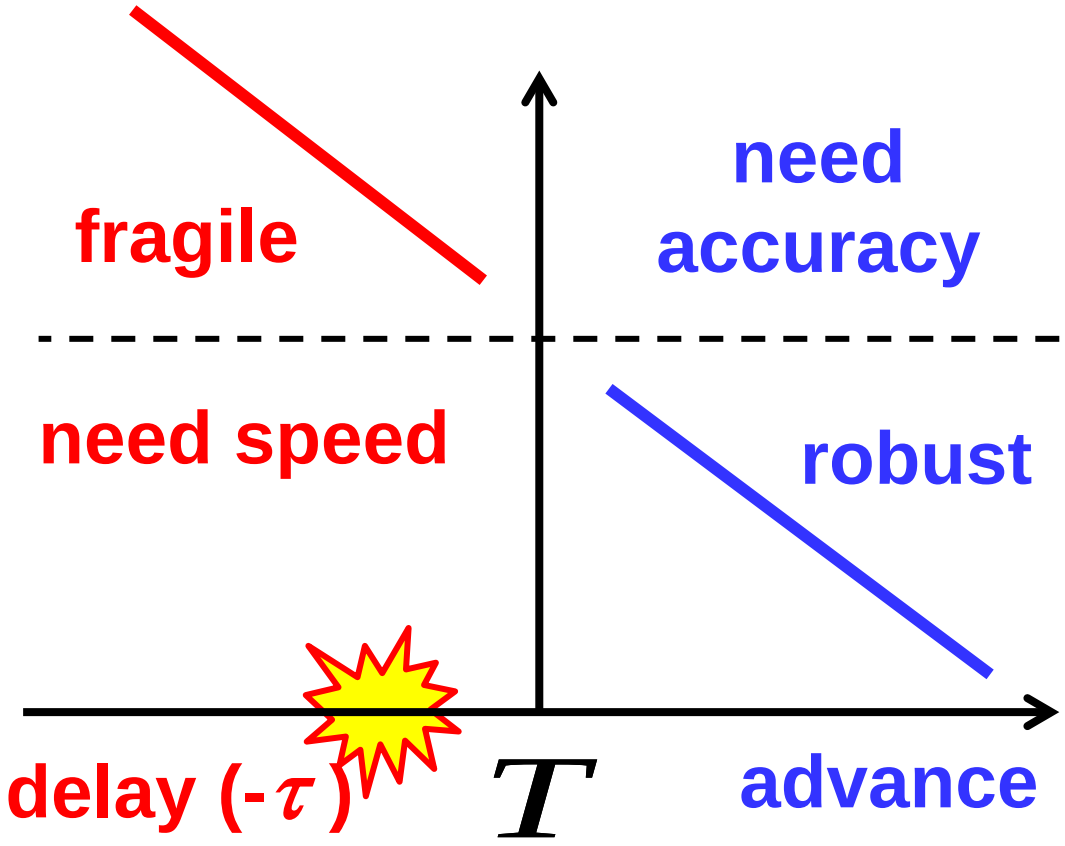


you me

You are faster

→ $T < 0$

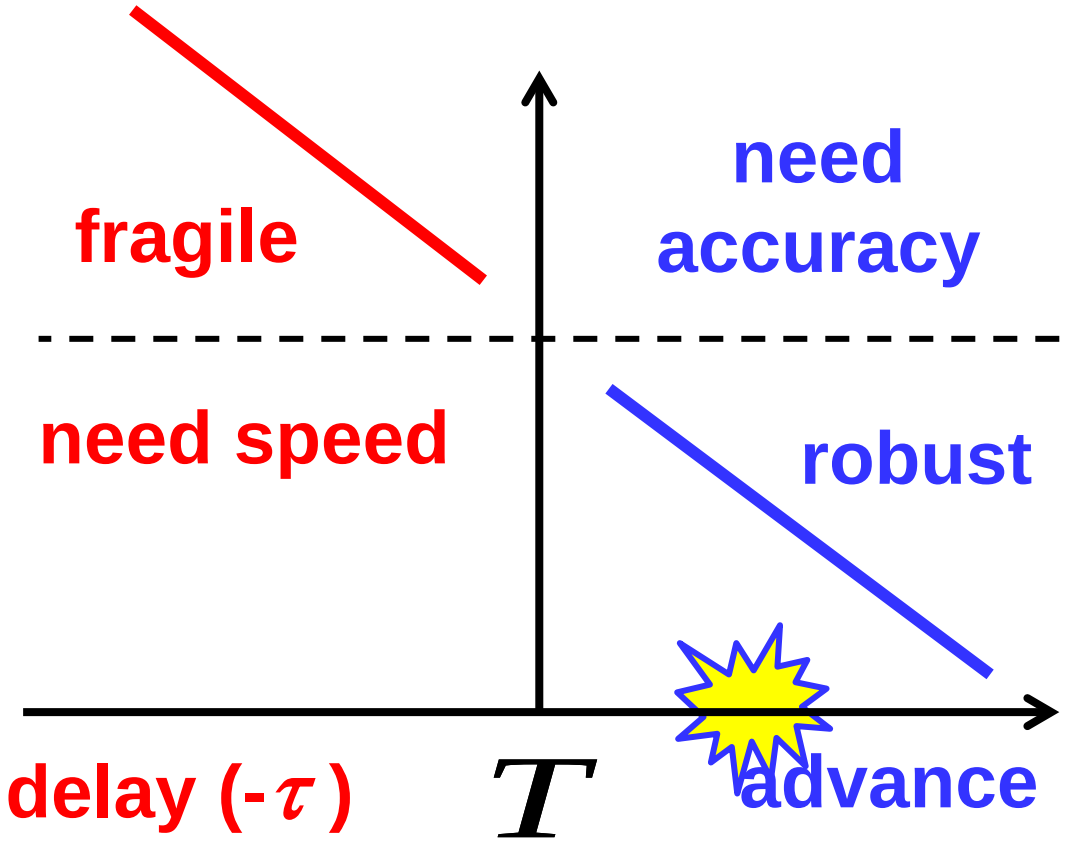
T = your delays - my delays



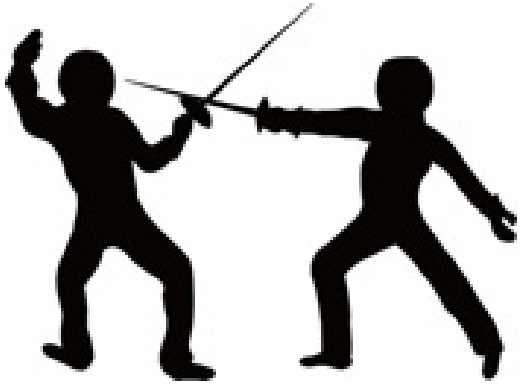
You are faster

→ $T < 0$

You win



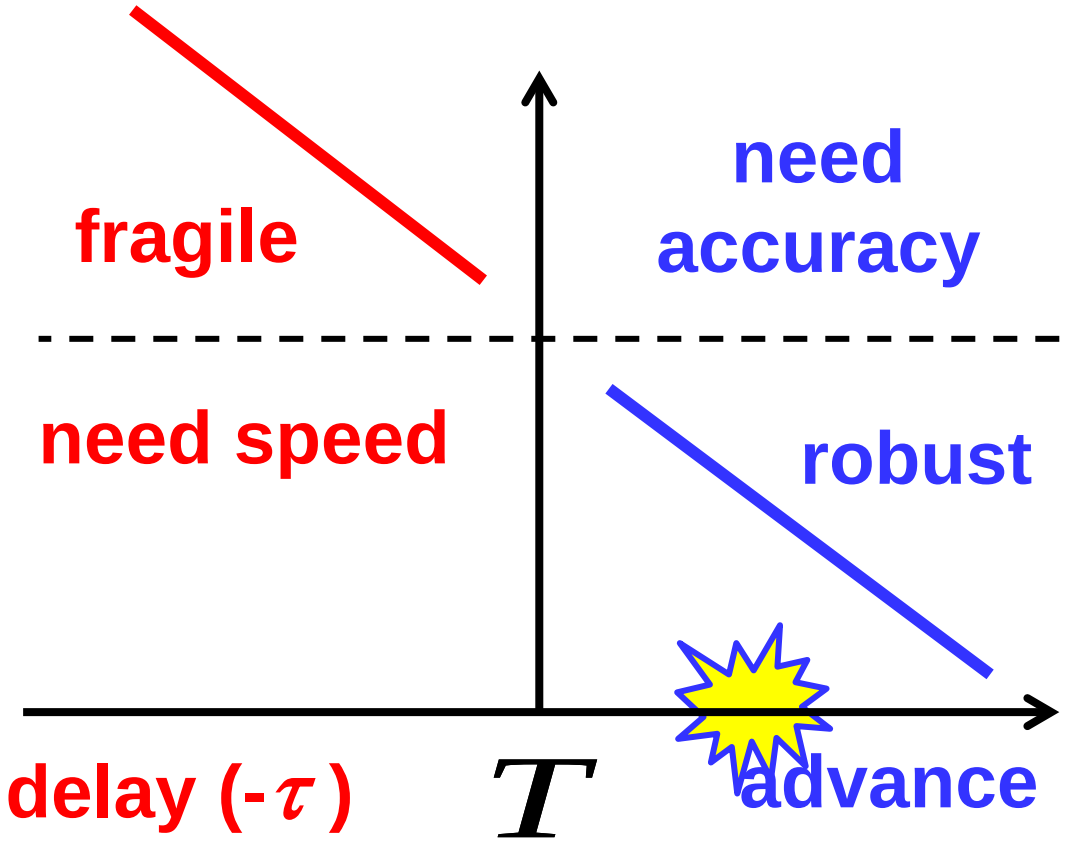
T = your delays
- my delays



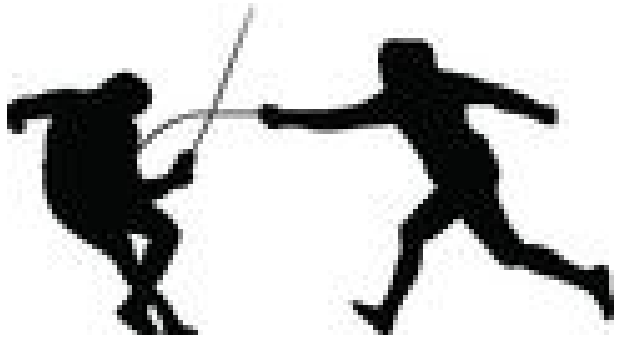
you me

I am faster

$\rightarrow T > 0$

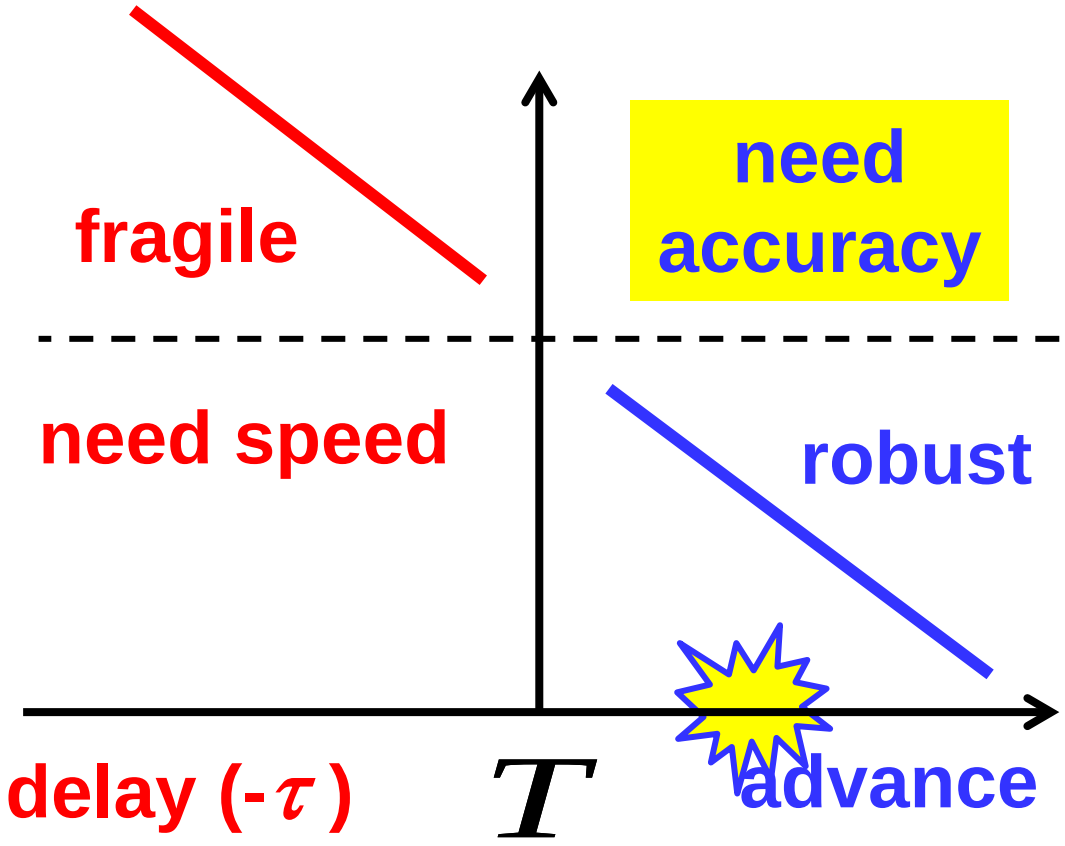


T = your delays
- my delays

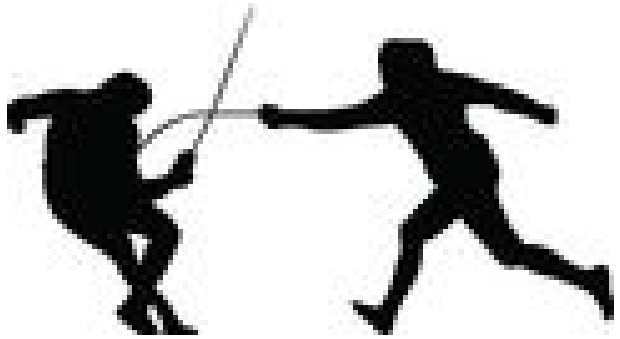


I am faster
$\rightarrow T > 0$

I win

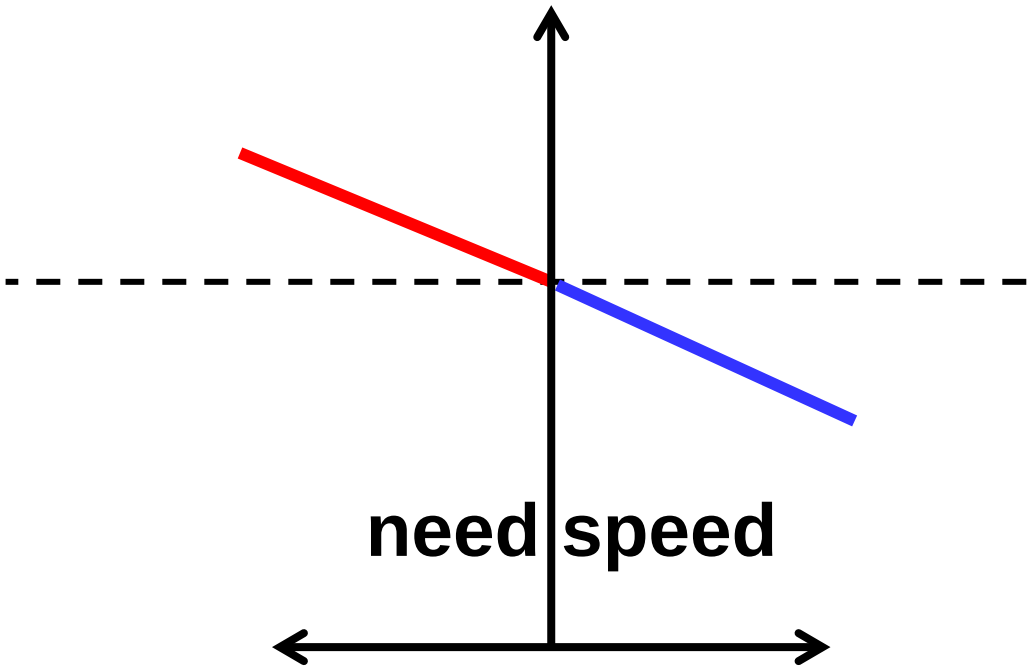


T = your delays
- my delays



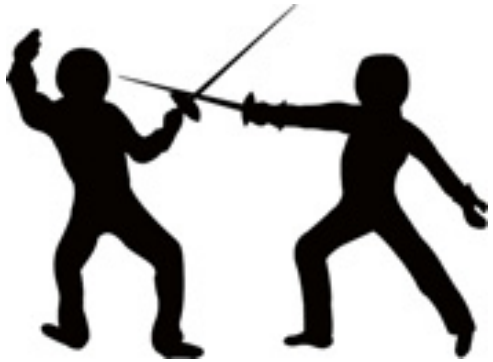
I am faster
$\rightarrow T > 0$

I win

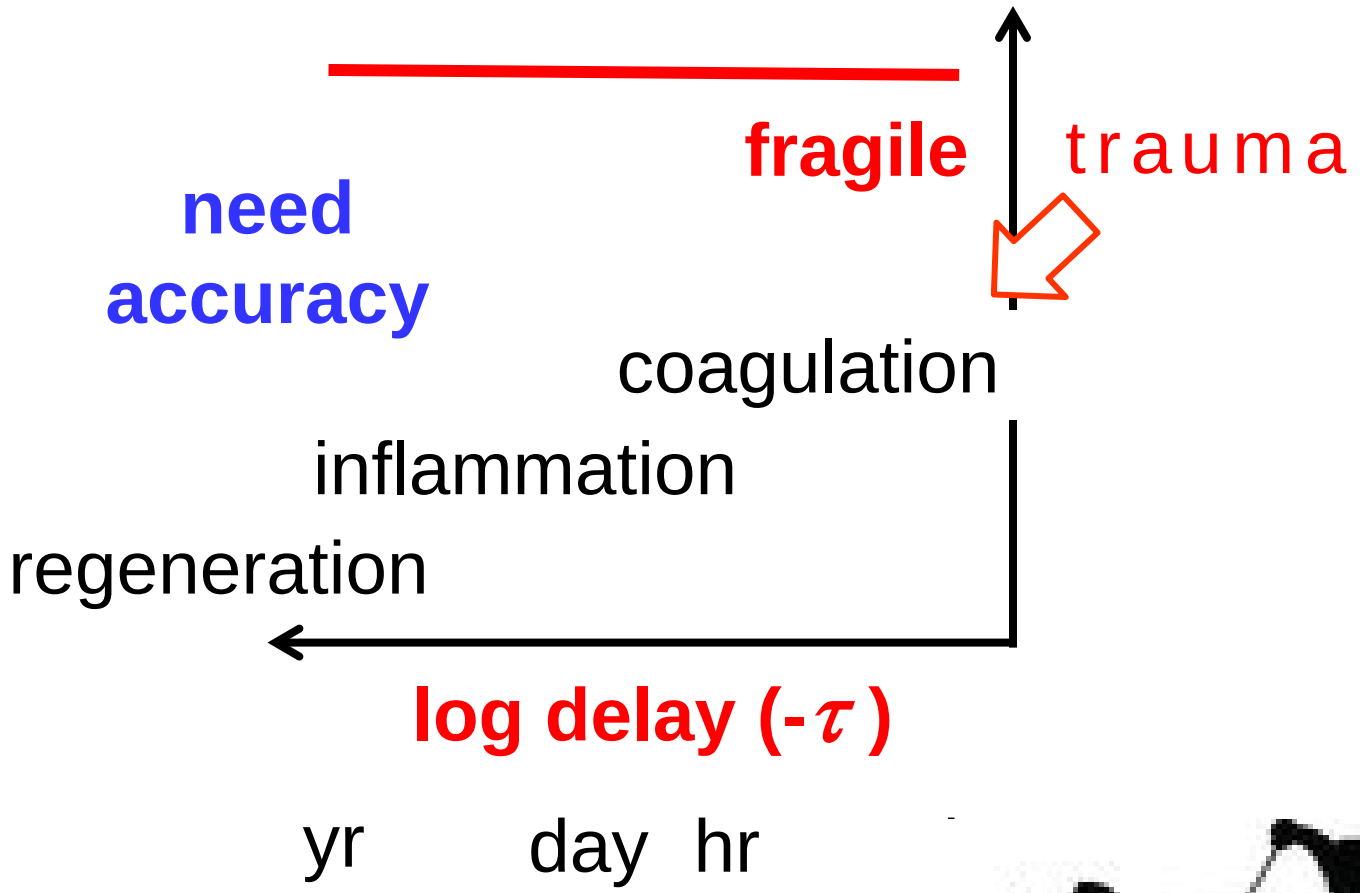


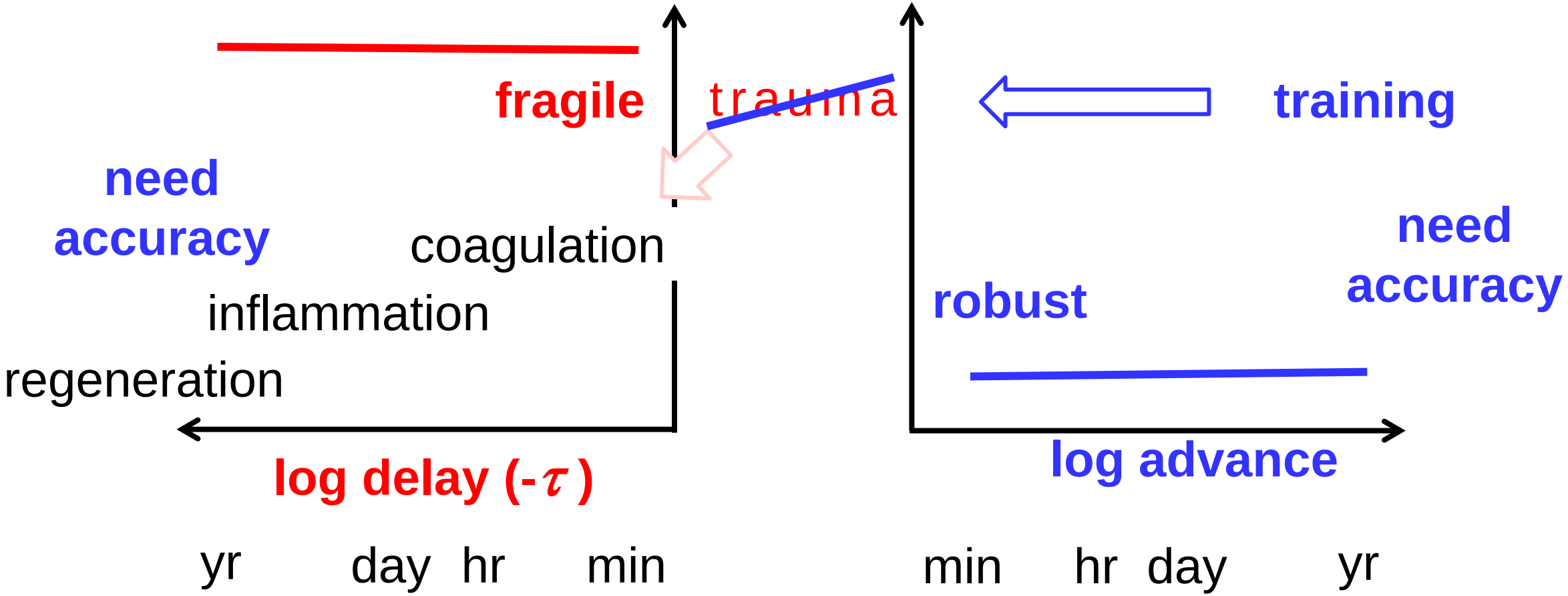
sec T sec

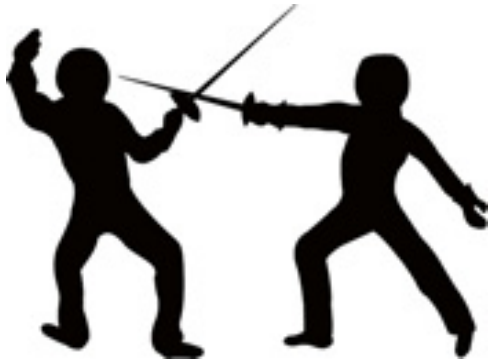
reacting to past disturbances

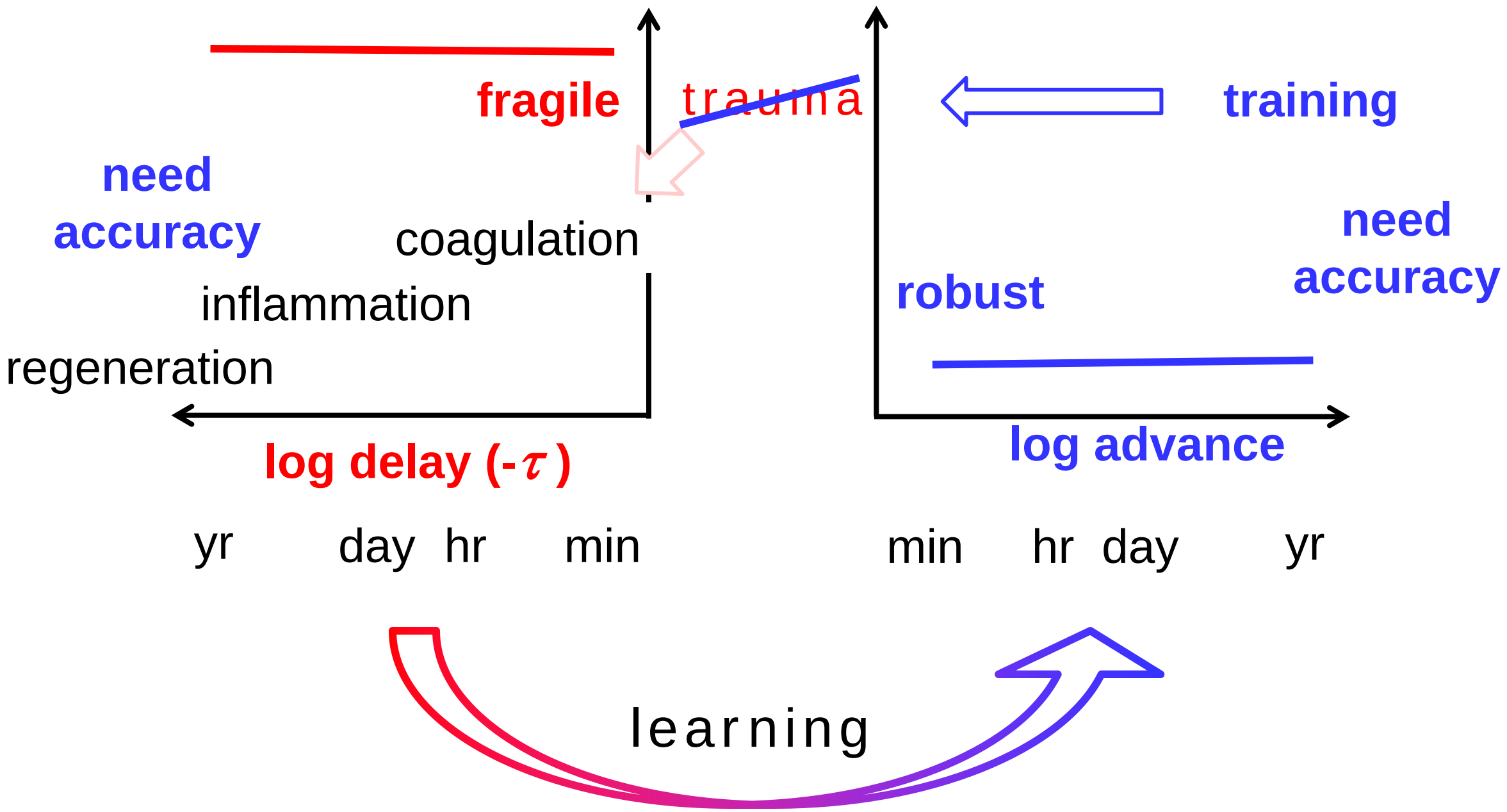


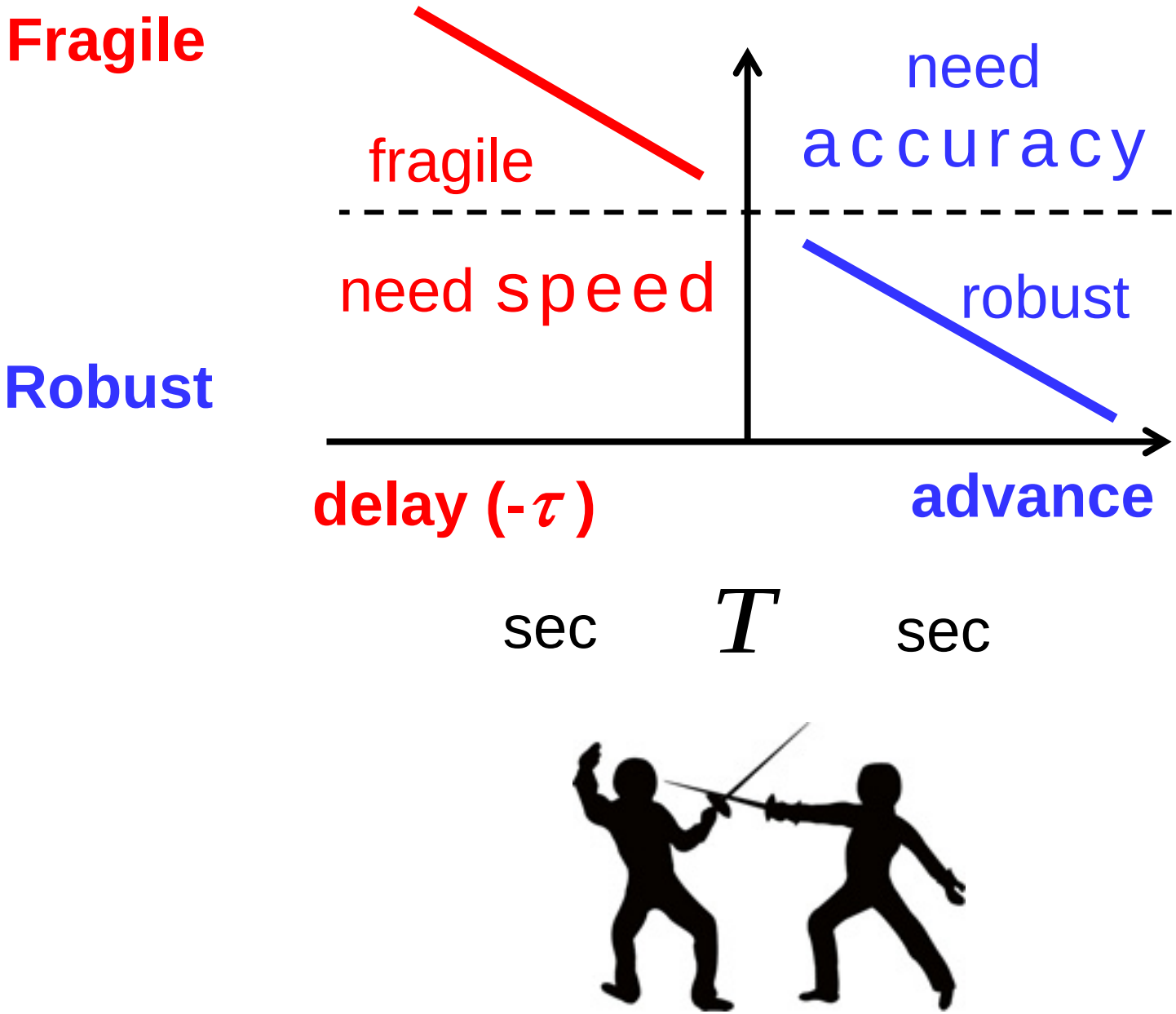
planning for future disturbances

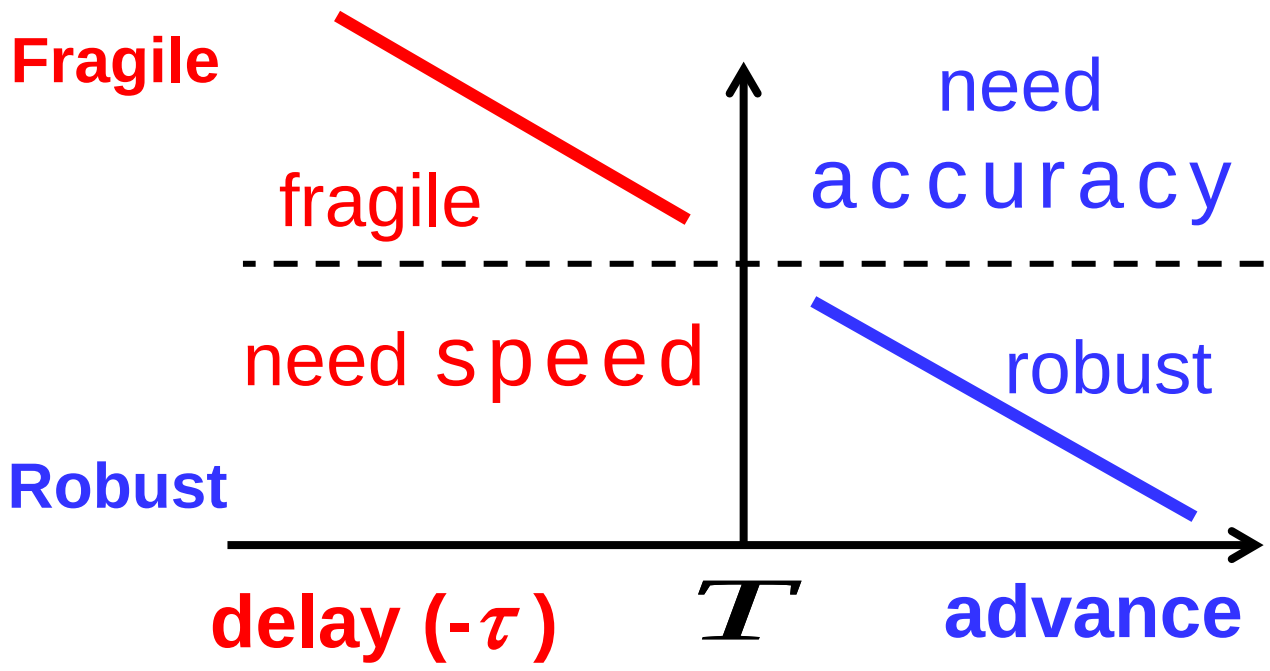

fragile
trauma
need
accuracy
coagulation
inflammation
regeneration
log delay (-τ)
yr
day
hr

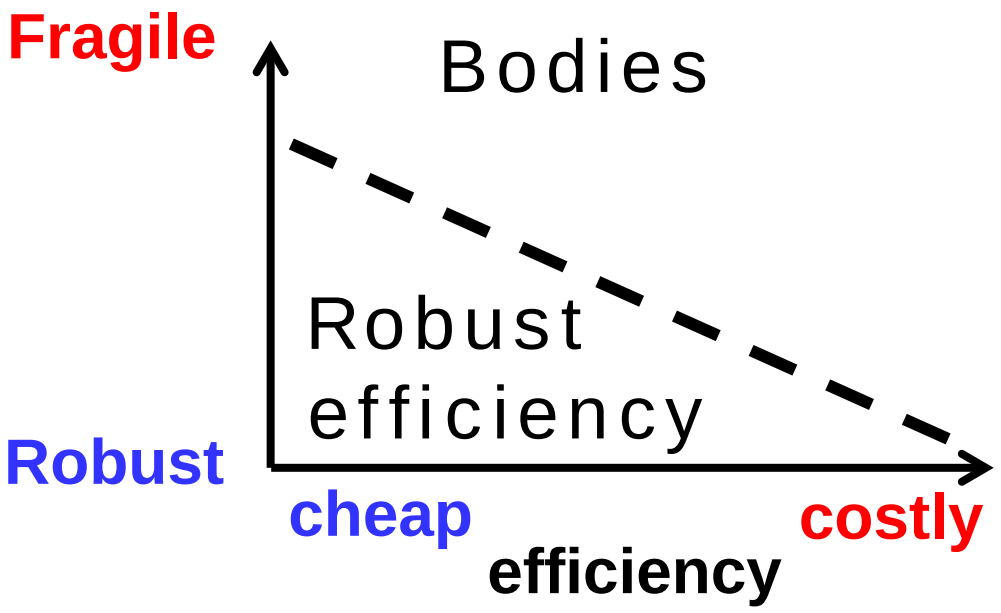

fragile
trauma
training
need
accuracy
coagulation
need
accuracy
inflammation
robust
regeneration
log delay (-τ)
log advance
yr
day
hr
min
min
hr
day
yr

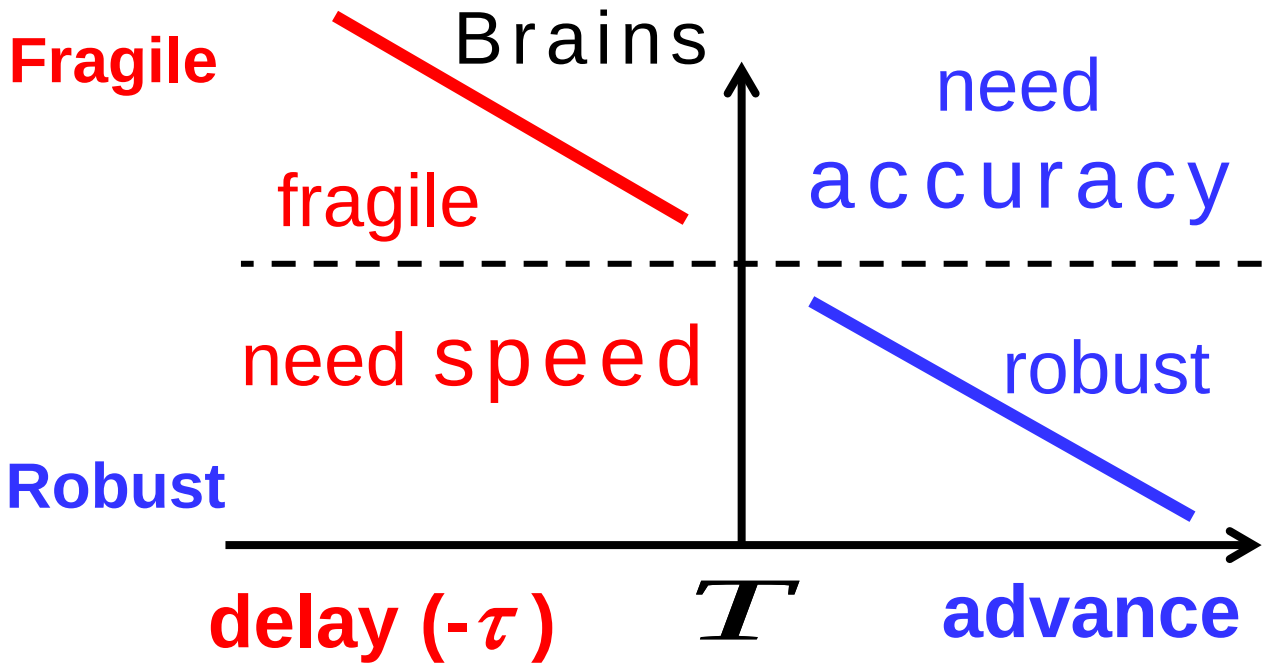


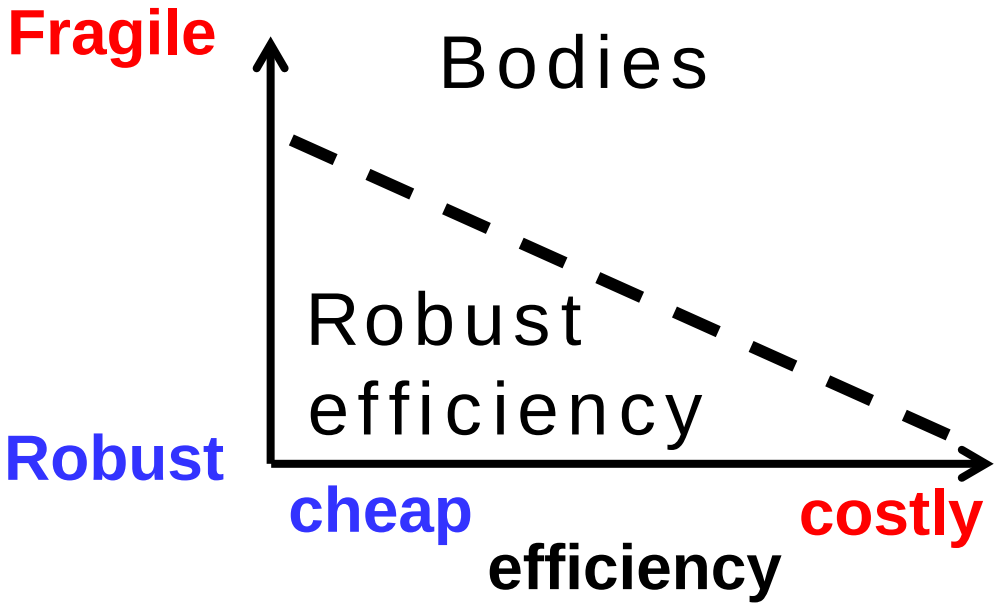

fragile
trauma
training
need
accuracy
coagulation
need
accuracy
inflammation
robust
regeneration
log delay (-τ)
log advance
yr
day
hr
min
min
hr
day
yr
learning

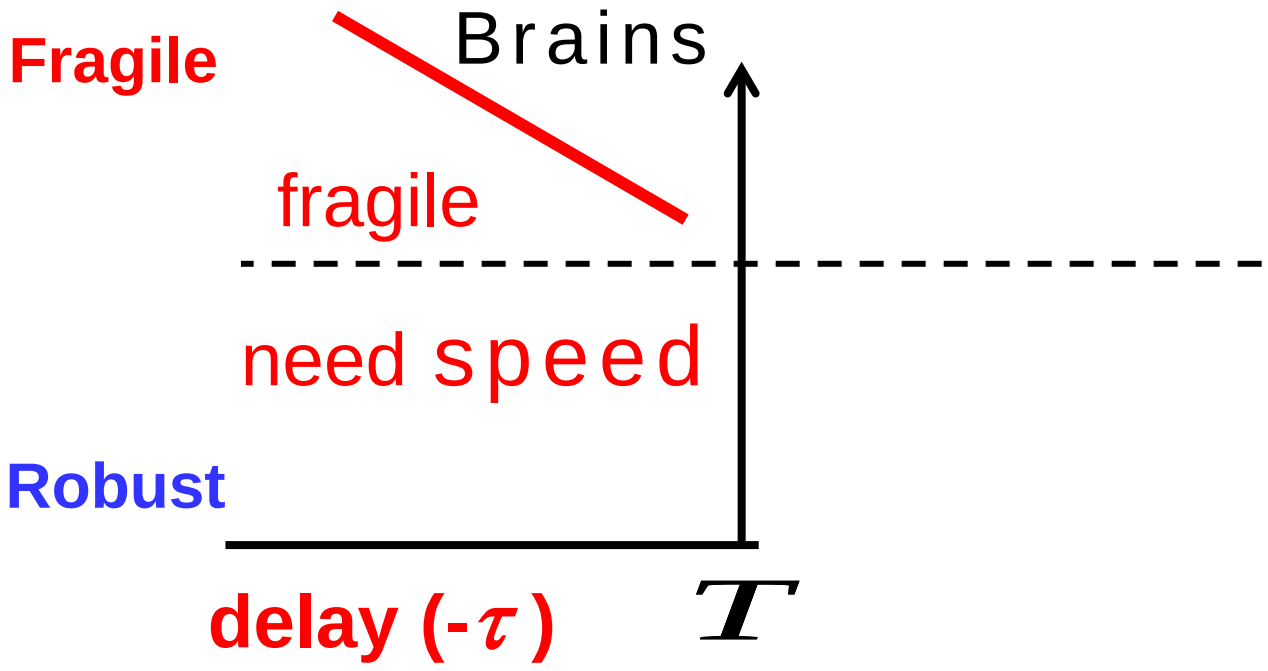

Fragile
fragile
need
accuracy
need speed
robust
Robust
delay (-τ)
advance
sec
T
sec

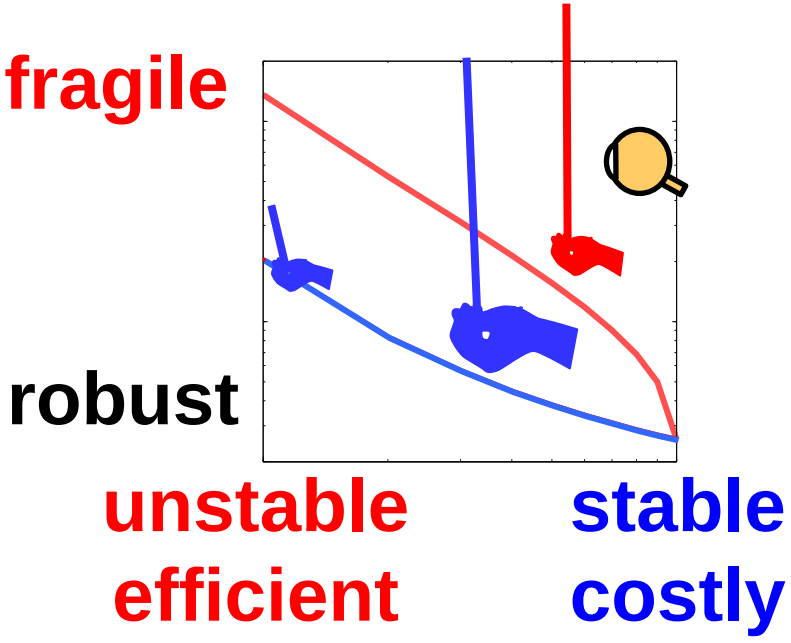

Fragile
fragile
need
accuracy
need speed
robust
Robust
delay (-τ)
T
advance

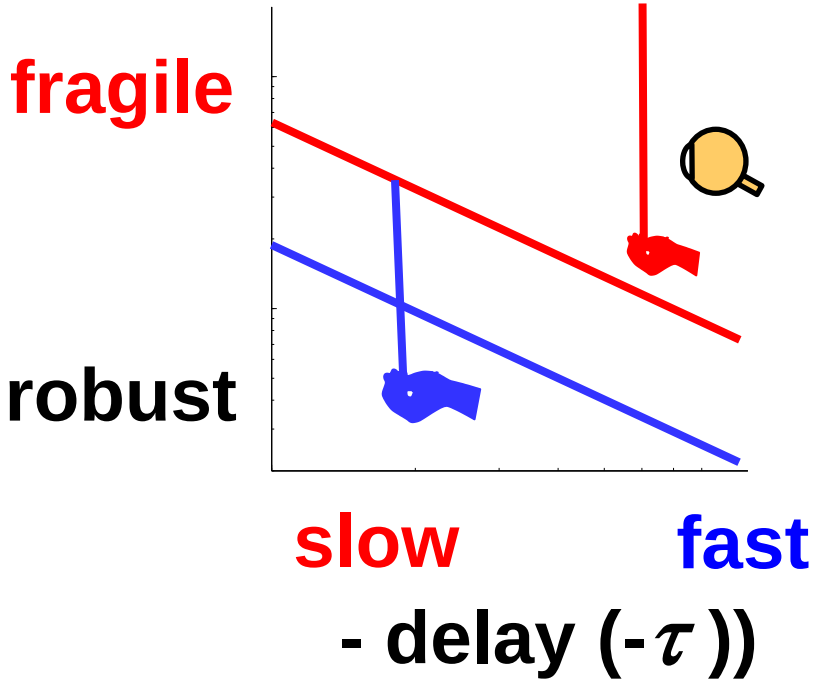

Fragile
Bodies
Robust
efficiency
Robust
cheap
costly
efficiency

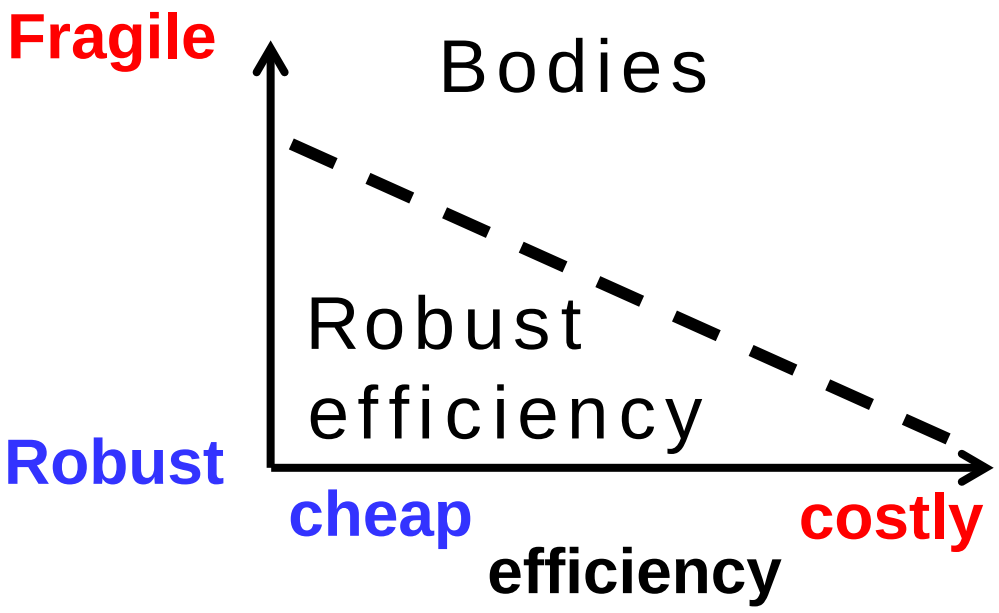

Fragile
Brains
need
accuracy
fragile
need speed
robust
Robust
delay (-τ)
T
advance

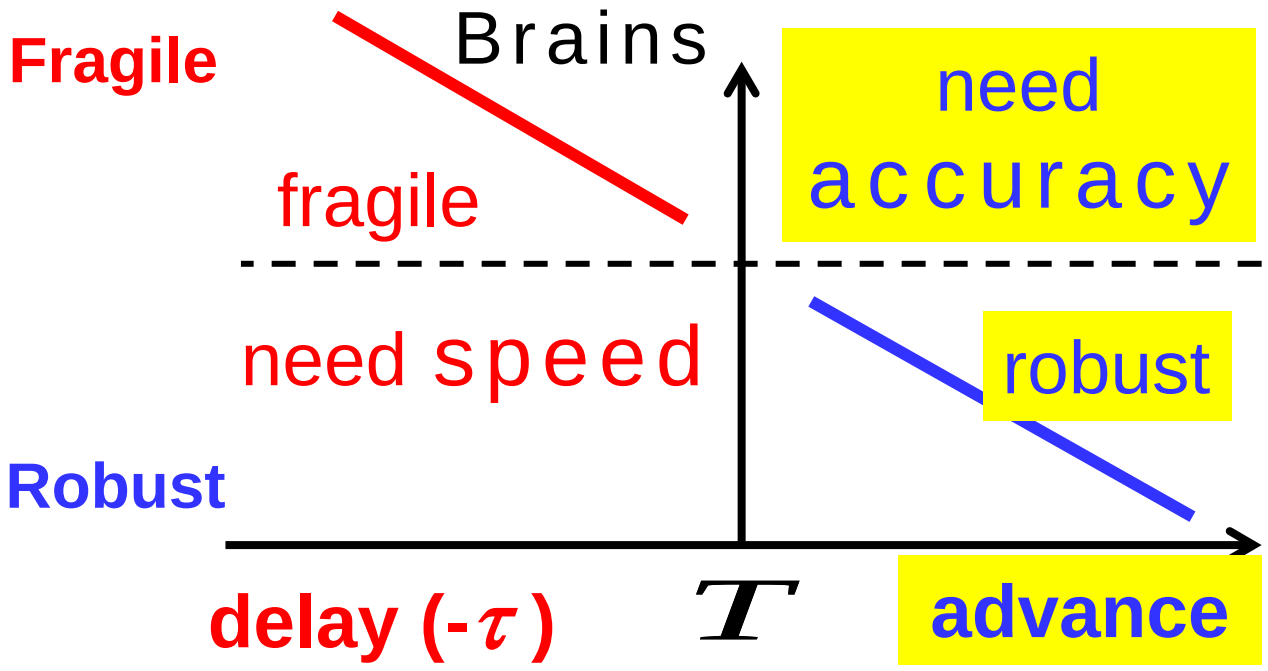

Fragile
Bodies
Robust
efficiency
Robust
cheap
costly
efficiency

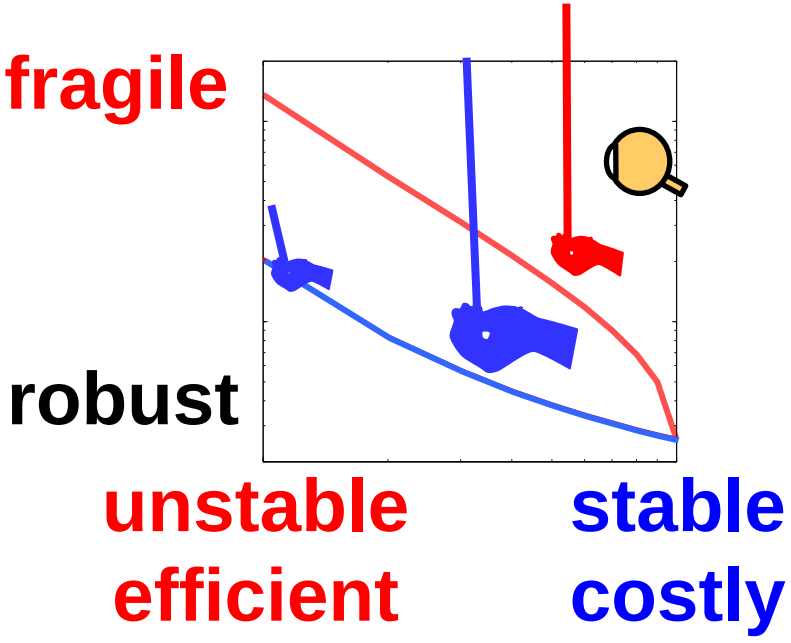

Fragile
Brains
fragile
need speed
Robust
delay (-τ)
T

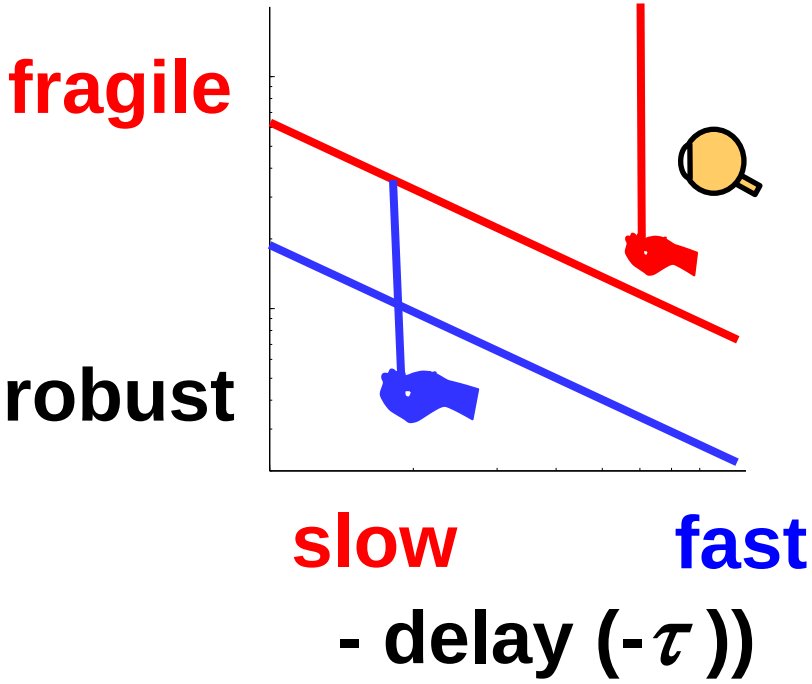

fragile
robust
unstable
efficient
stable
costly

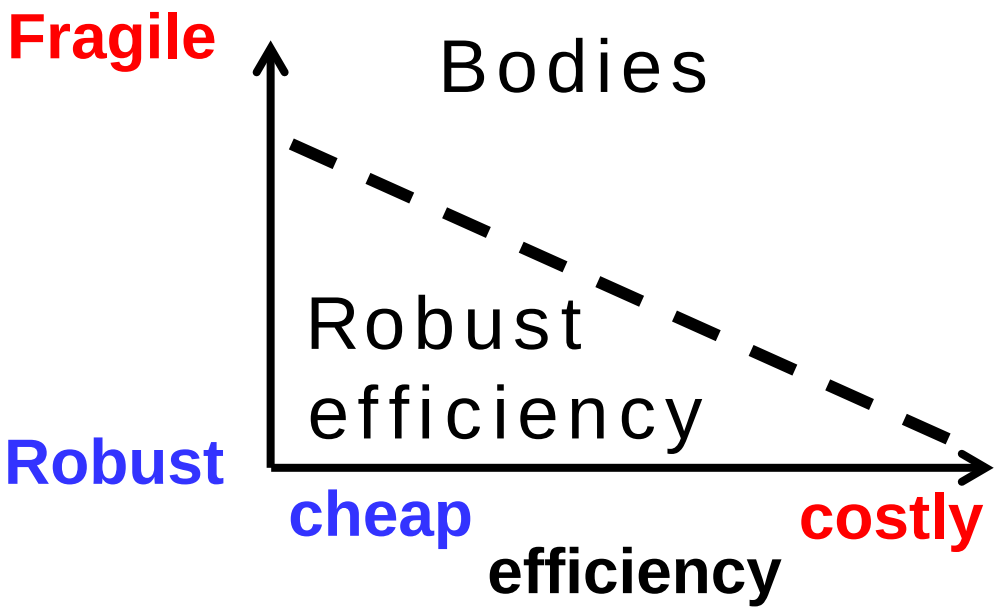

fragile
robust
slow
fast
- delay (-τ))

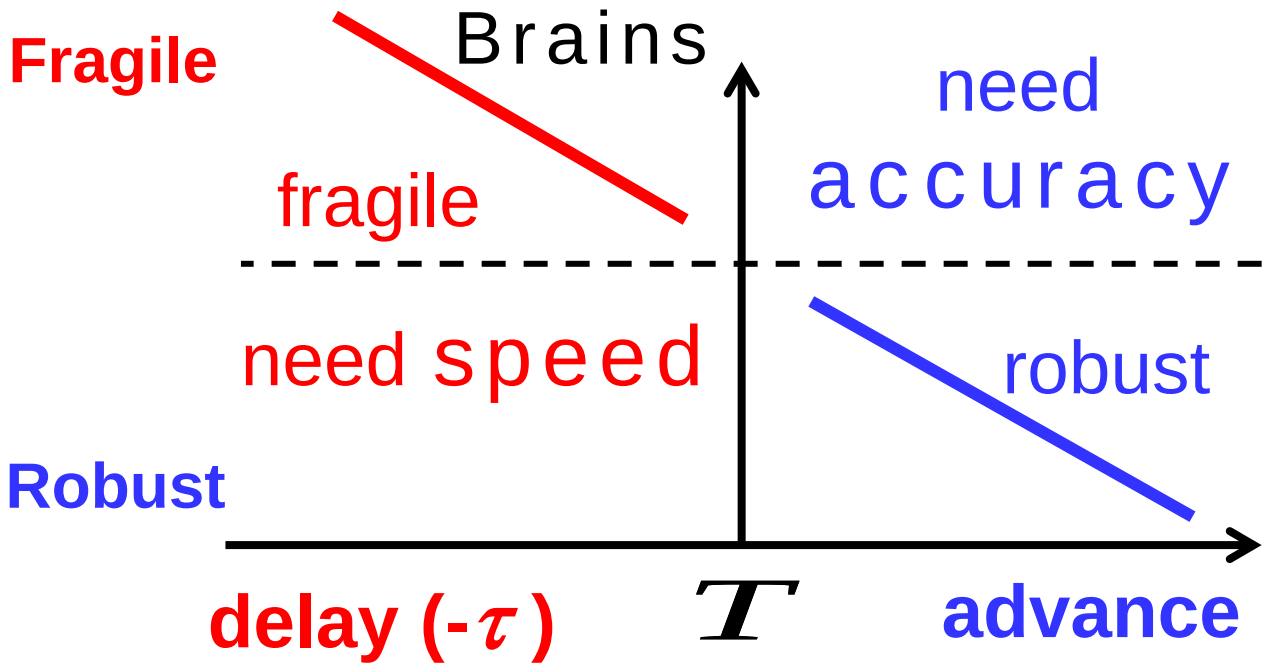

Fragile
Bodies
Robust
efficiency
Robust
cheap
costly
efficiency

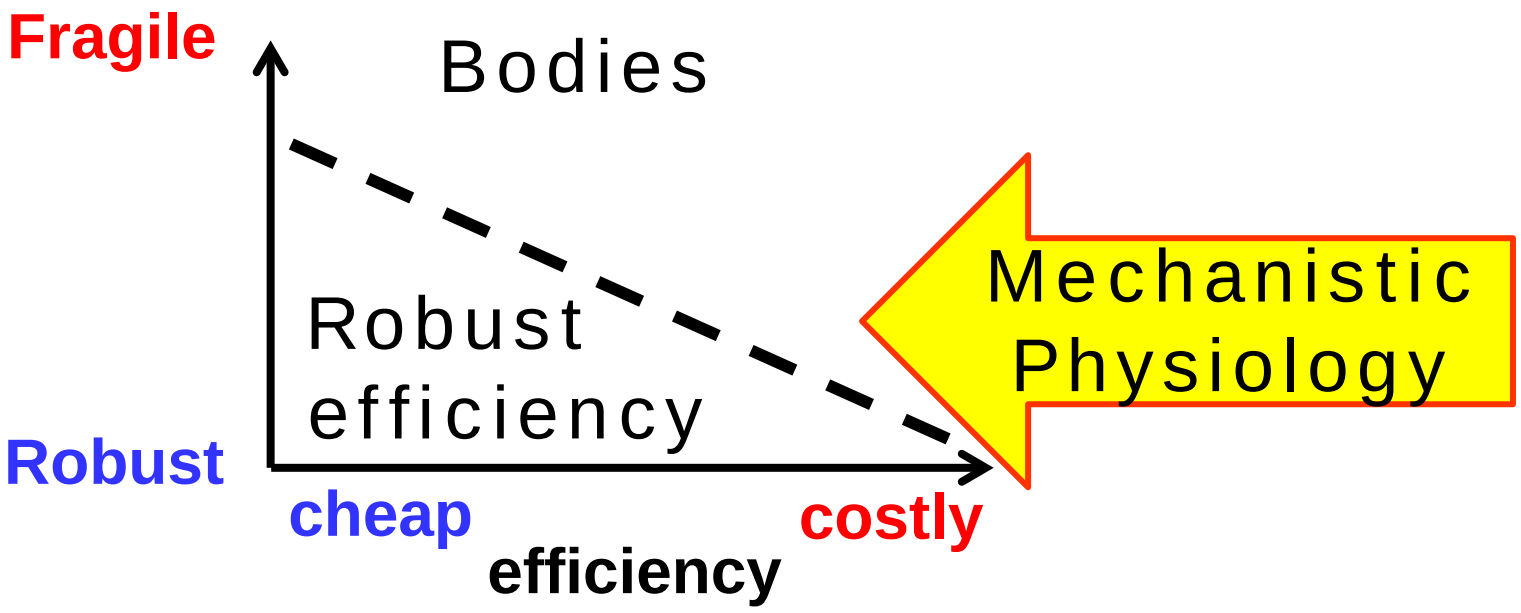

Fragile
Brains
need
accuracy
fragile
need speed
robust
Robust
delay (-τ)
T
advance

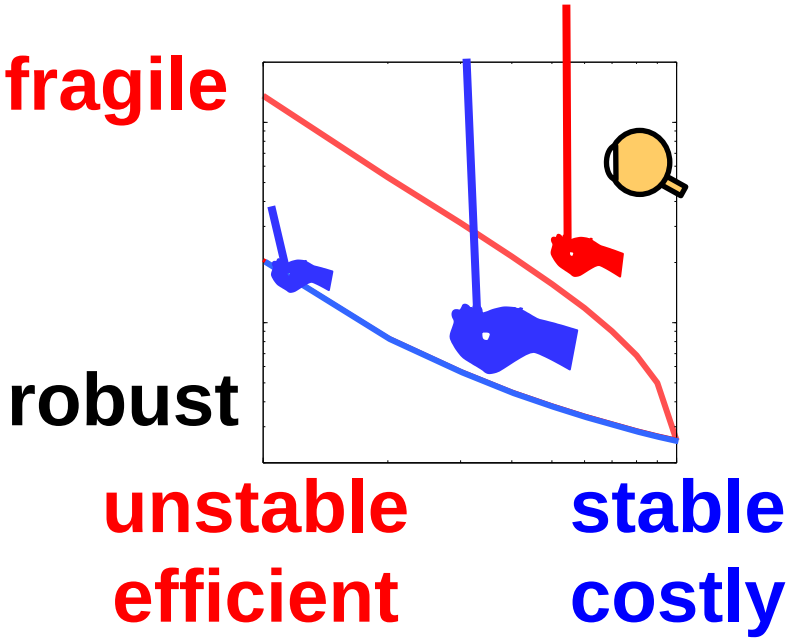

fragile
robust
unstable
efficient
stable
costly

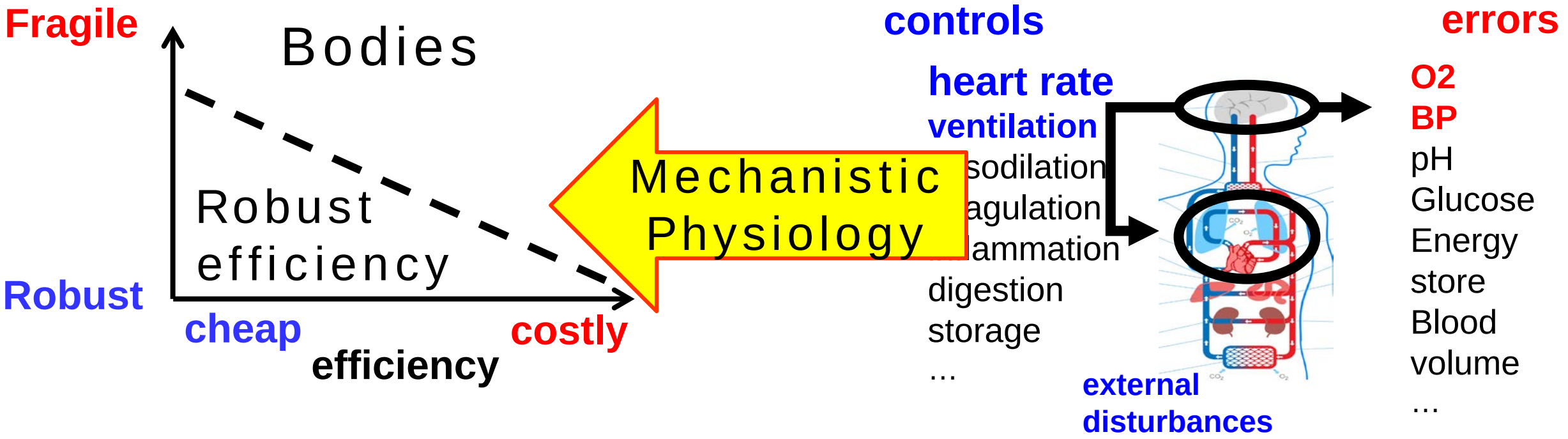

fragile
robust
slow
fast
- delay (-τ))


Fragile
Bodies
Robust
efficiency
Robust
cheap
costly
efficiency


Fragile
Brains
need
accuracy
fragile
need speed
robust
Robust
delay (-τ)
T
advance


Fragile
Bodies
Robust
efficiency
Robust
cheap
costly
efficiency
Mechanistic
Physiology

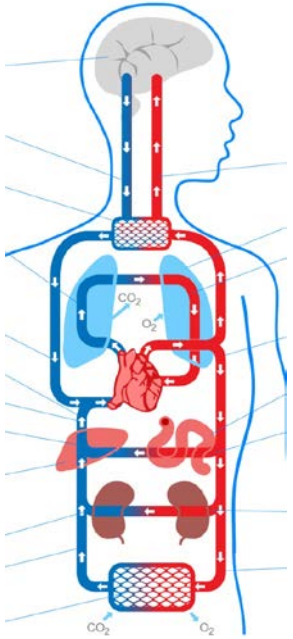

fragile
robust
unstable
efficient
stable
costly

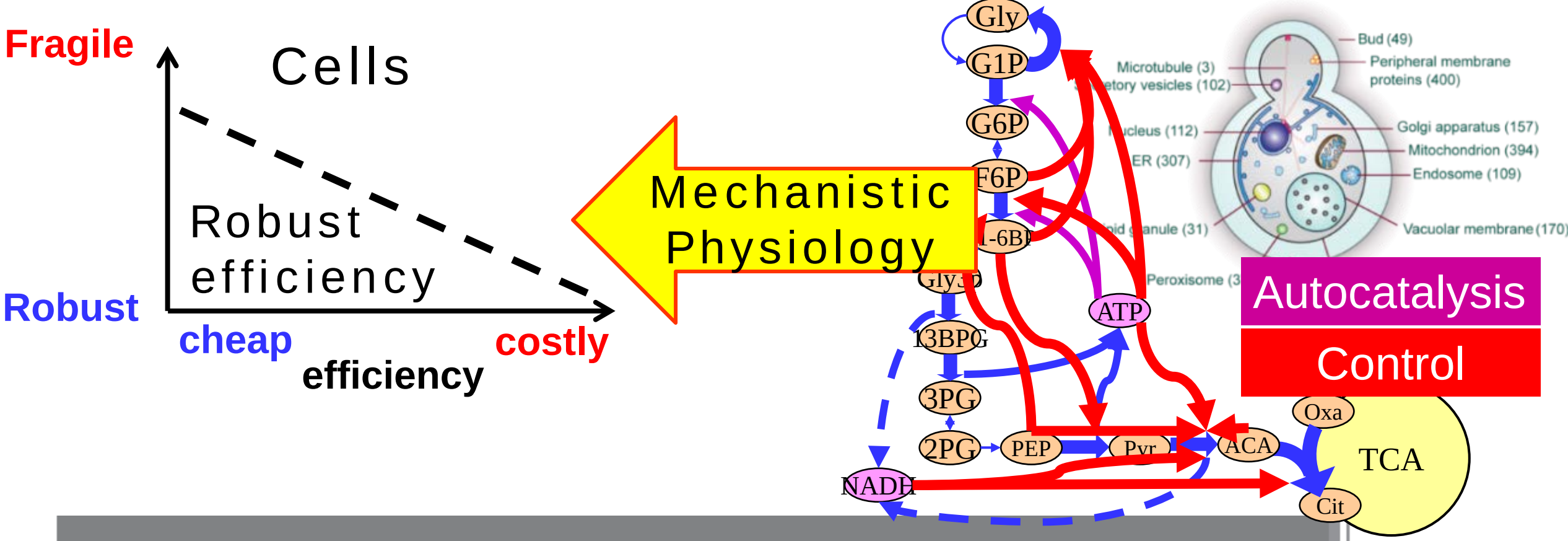


Li, Cruz, Chien, Sojoudi, Recht, Stone, Csete, Bahmiller, Doyle

Robust efficiency and actuator saturation explain healthy heart rate control and variability

***Proc. Nat. Acad. Sci. PLUS* 2014**





RESEARCH ARTICLES

Chandra, Buzi, and Doyle

Glycolytic Oscillations and Limits on Robust Efficiency

Fiona A. Chandra,[1]* Gentian Buzi,[2] John C. Doyle[2]

uvate kinase (PK) produces $q + 1$ molecules of y for a net (normalized) production of one unit, which is consumed in a final reaction modeling the cell's use of ATP. In glycoly molecules are consumed upstrear produced downstream, which norm (each y molecule produces two dov kinetic exponent $a = 1$. To high trade-offs with the simplest possibl

Science
AAAS

Major transitions

- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- Weapons
- Bipedali
- Maternal care
- Warm blood
- Flight
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis

Cardio-physiology

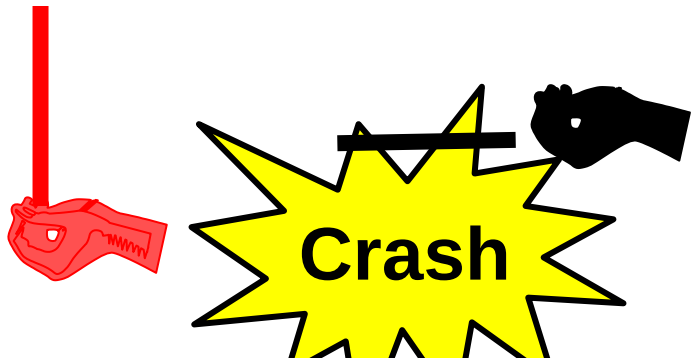
Balance

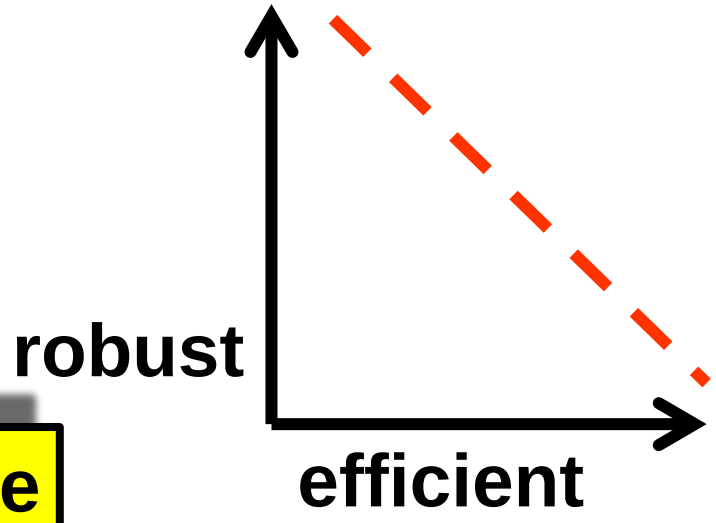
Efficient but unstable

Turbulence

Needs complex control

Glycolysis





Similar
- Tradeoffs
- Laws
- Mechanisms

But ***simpler***
- Models
- Experiments

Bad

- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- Weapons
- Balance (biped)
- Maternal care
- Warm blood
- Flight (streamlining)
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis

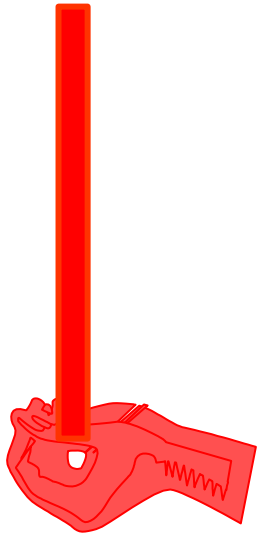
Mass extinction

Warfare (“up”)

~~Weapons~~

Prey (“back”)

Crash



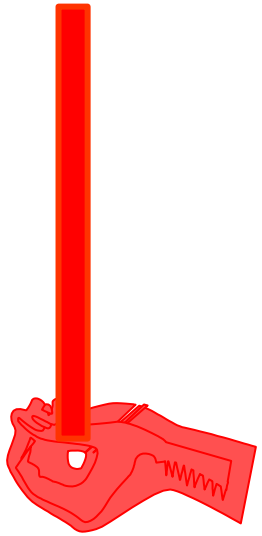
- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- **Weapons**
- Balance (biped)
- Maternal care
- Warm blood
- Flight (streamlining)
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis

Bad

Slavery

Famine

Crash



- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- **Weapons**
- Balance (biped)
- Maternal care
- Warm blood
- Flight (streamlining)
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis

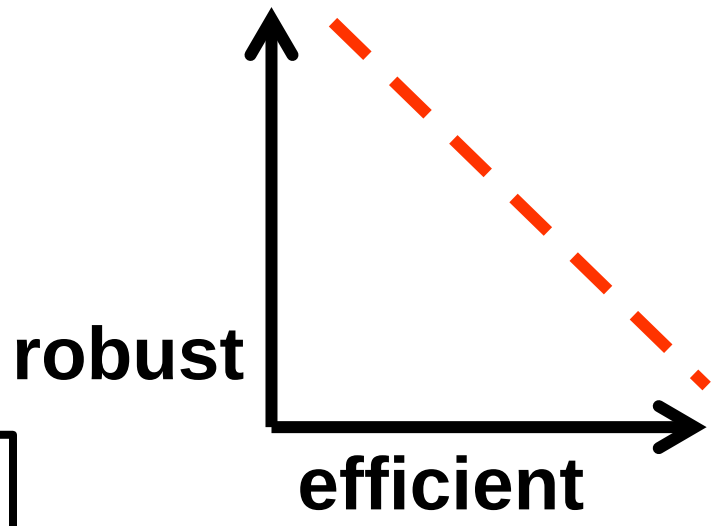
Slavery → **forward**

Famine → **back**

- Crashes “back” can hurt everyone
- Crashes “forward” benefit those (few) that cause and maintain them
- Systematic study is missing

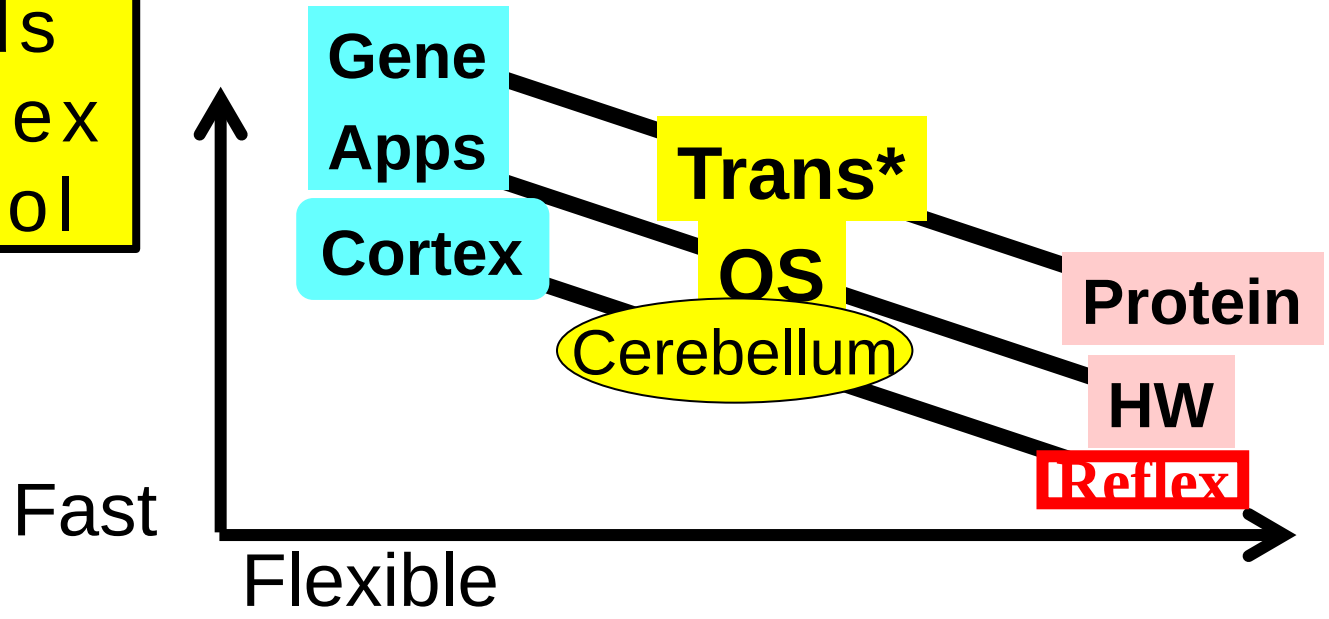
Major transitions

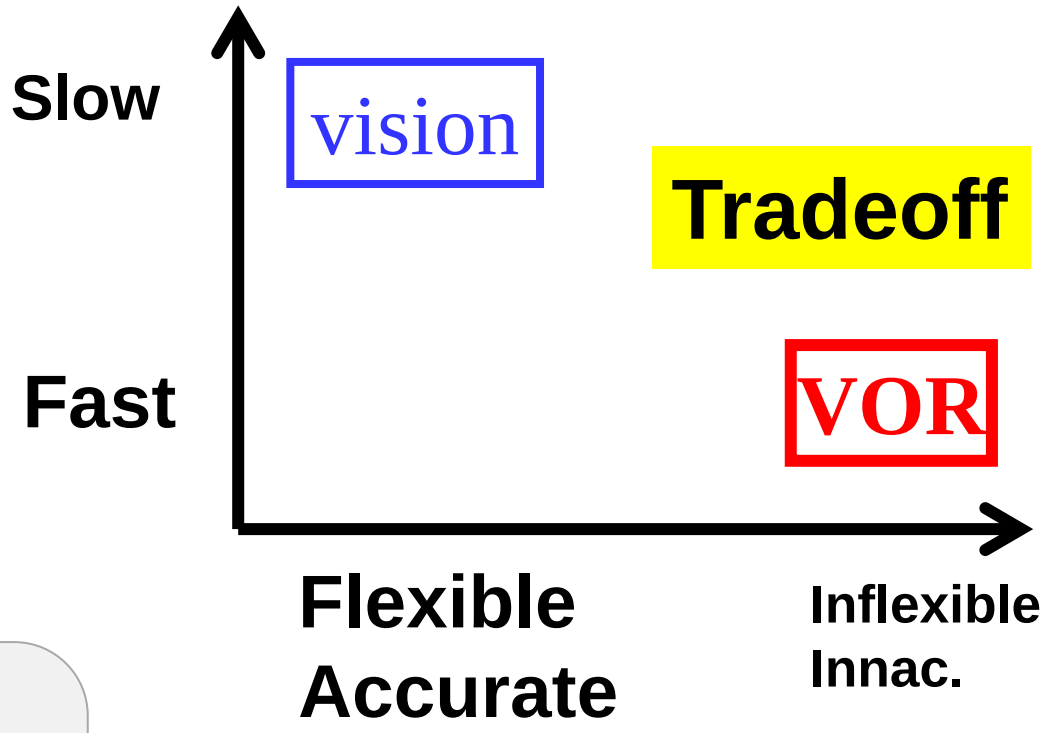
- Compute/control technology
- Industrialization
- Money/finance/lobbyists/etc
- Society/agriculture/states
- Weapons
- Bipedalism
- Maternal care
- Warm blood
- Flight
- Mitochondria
- Oxygen
- Translation (ribosomes)
- Glycolysis



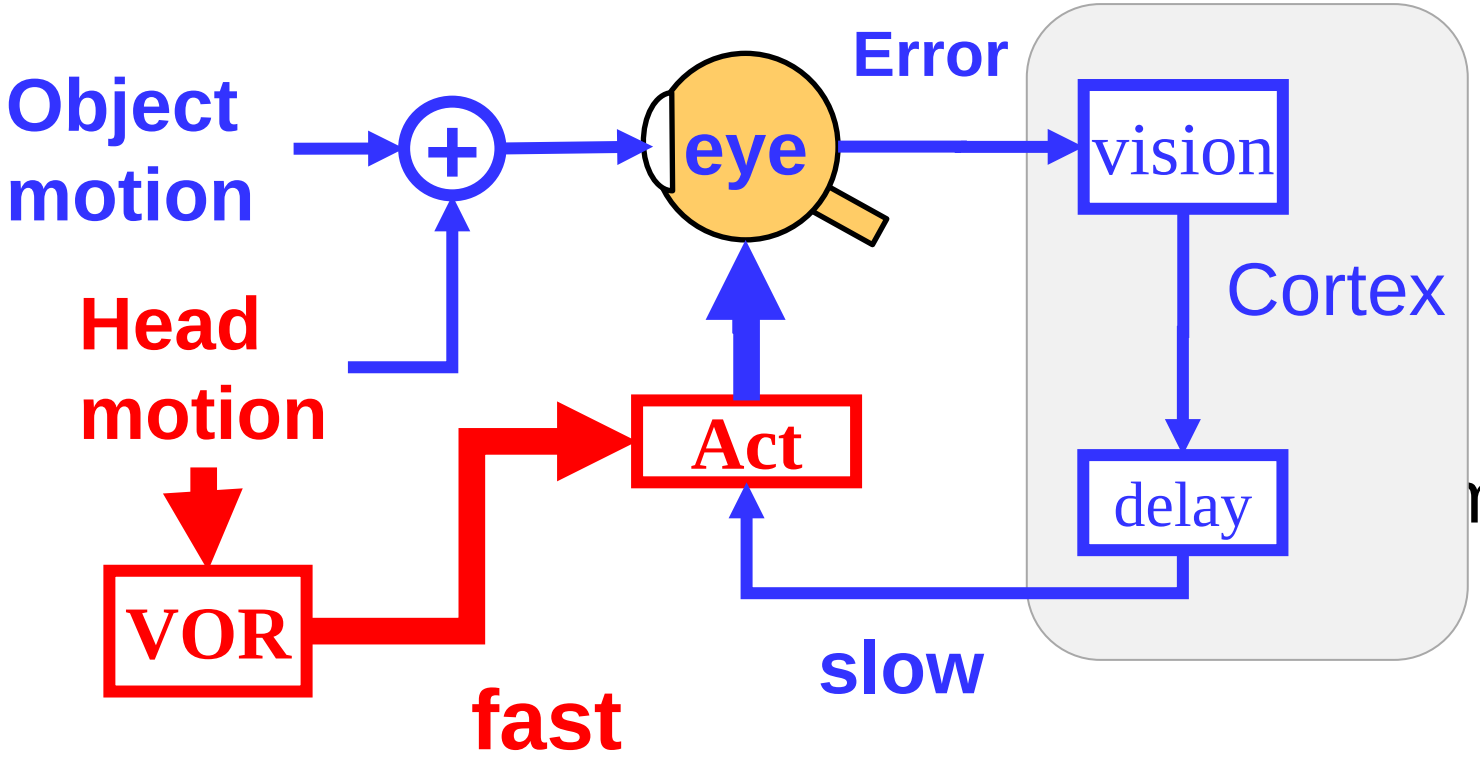
Efficient but unstable

Needs complex control

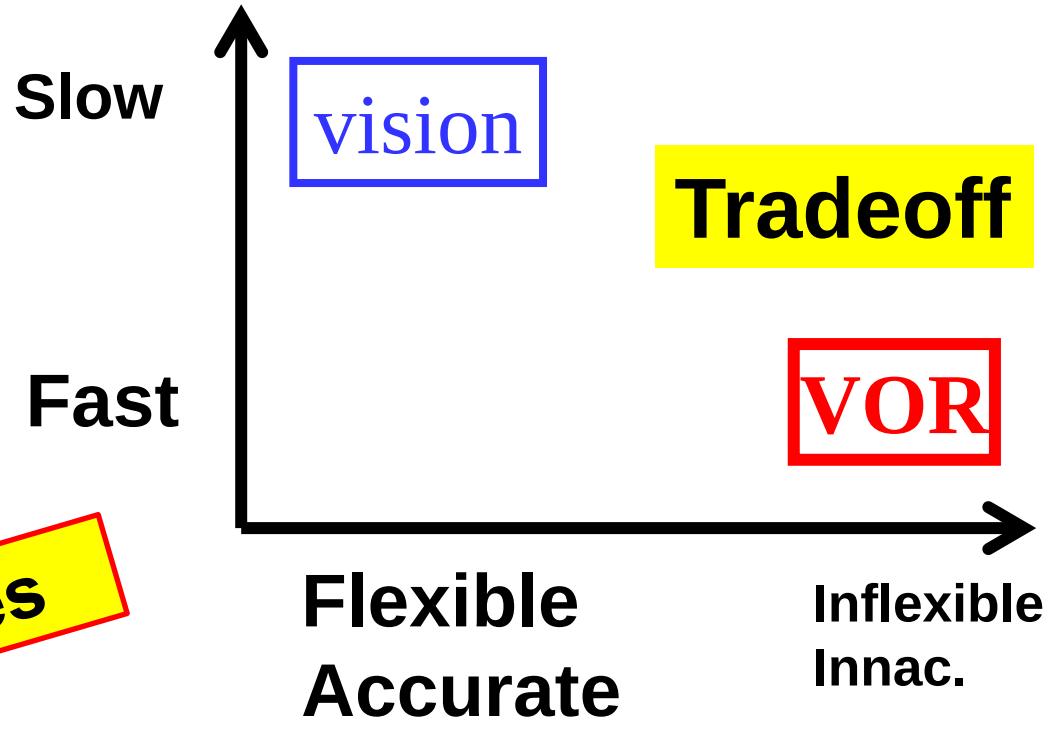




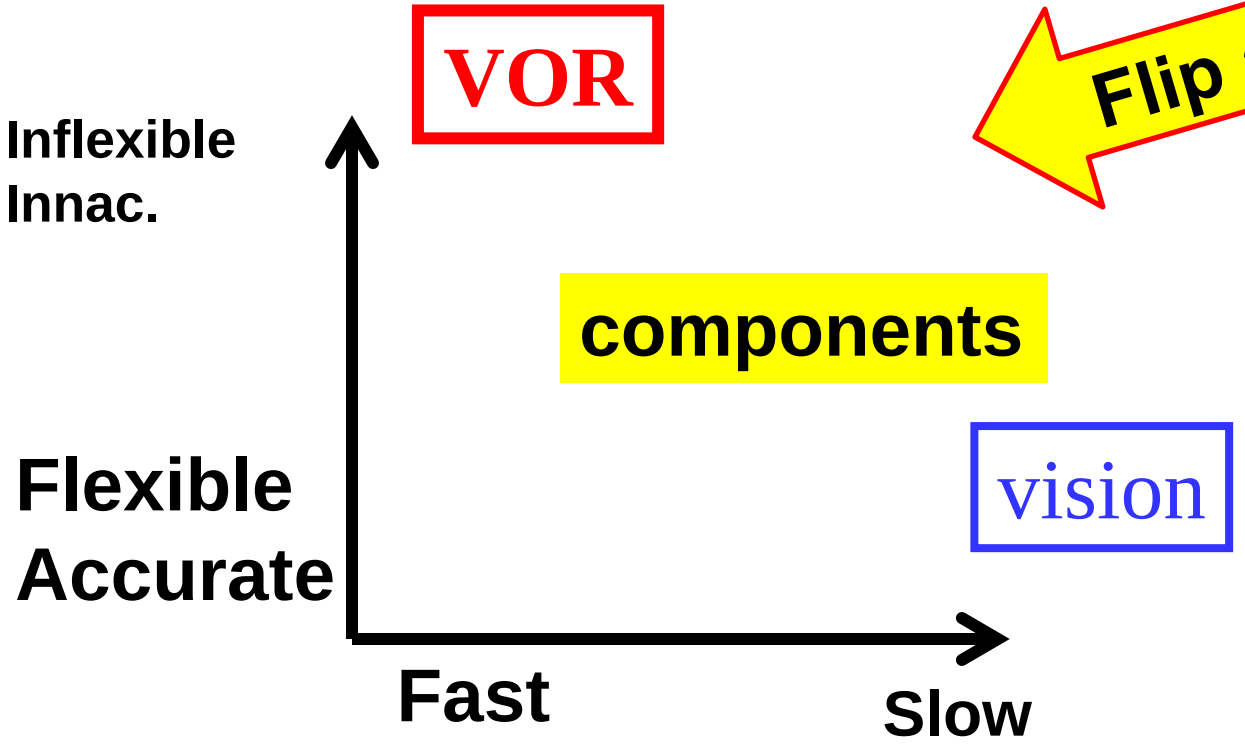
Mechanism



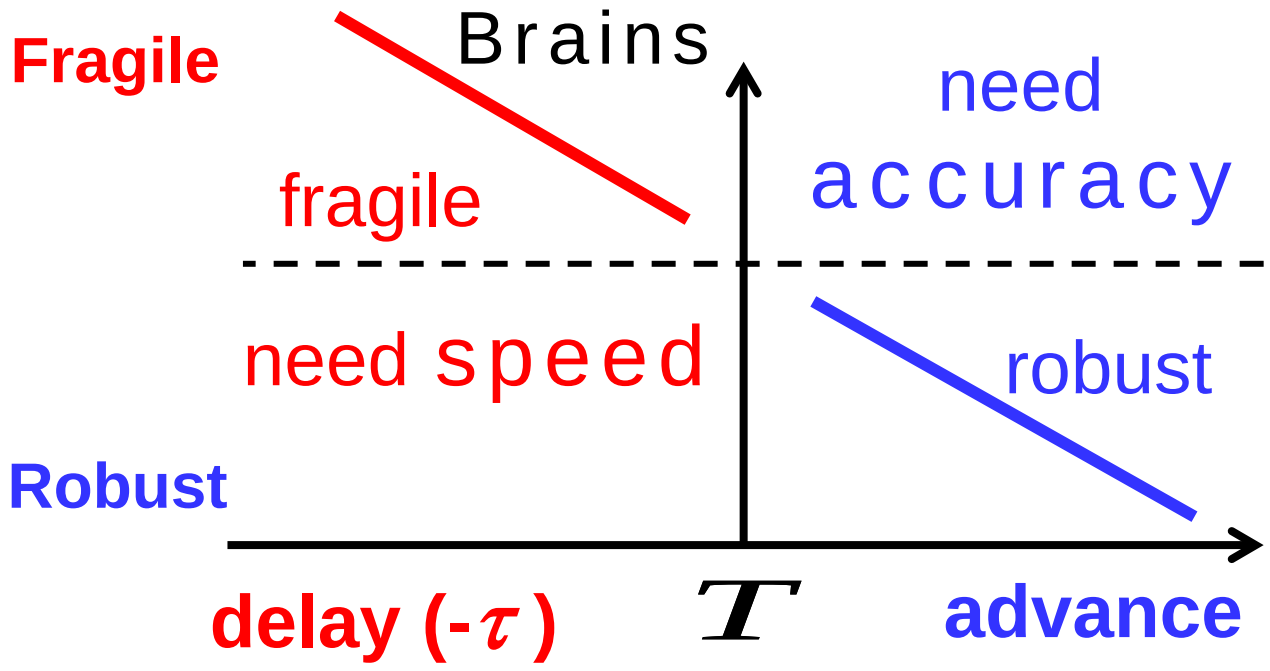
mal cartoon

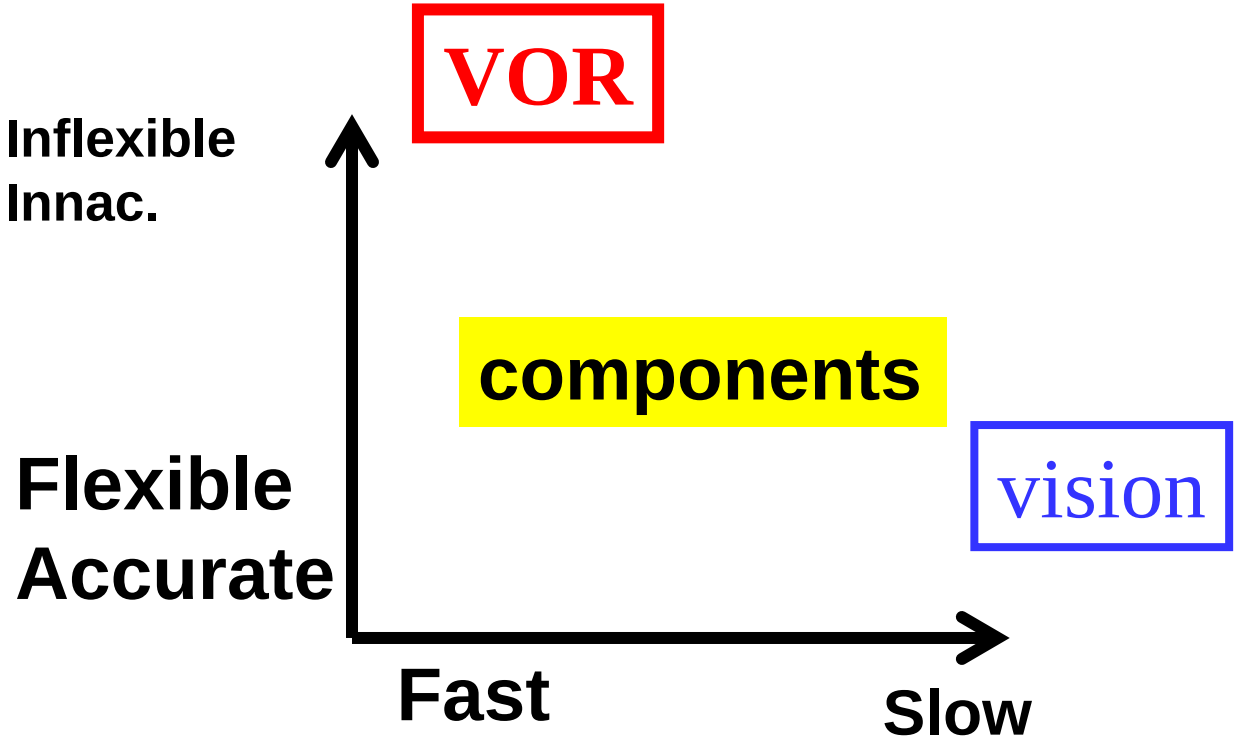

Slow
vision
Tradeoff
Fast
VOR
Flexible
Accurate
Inflexible
Innac.

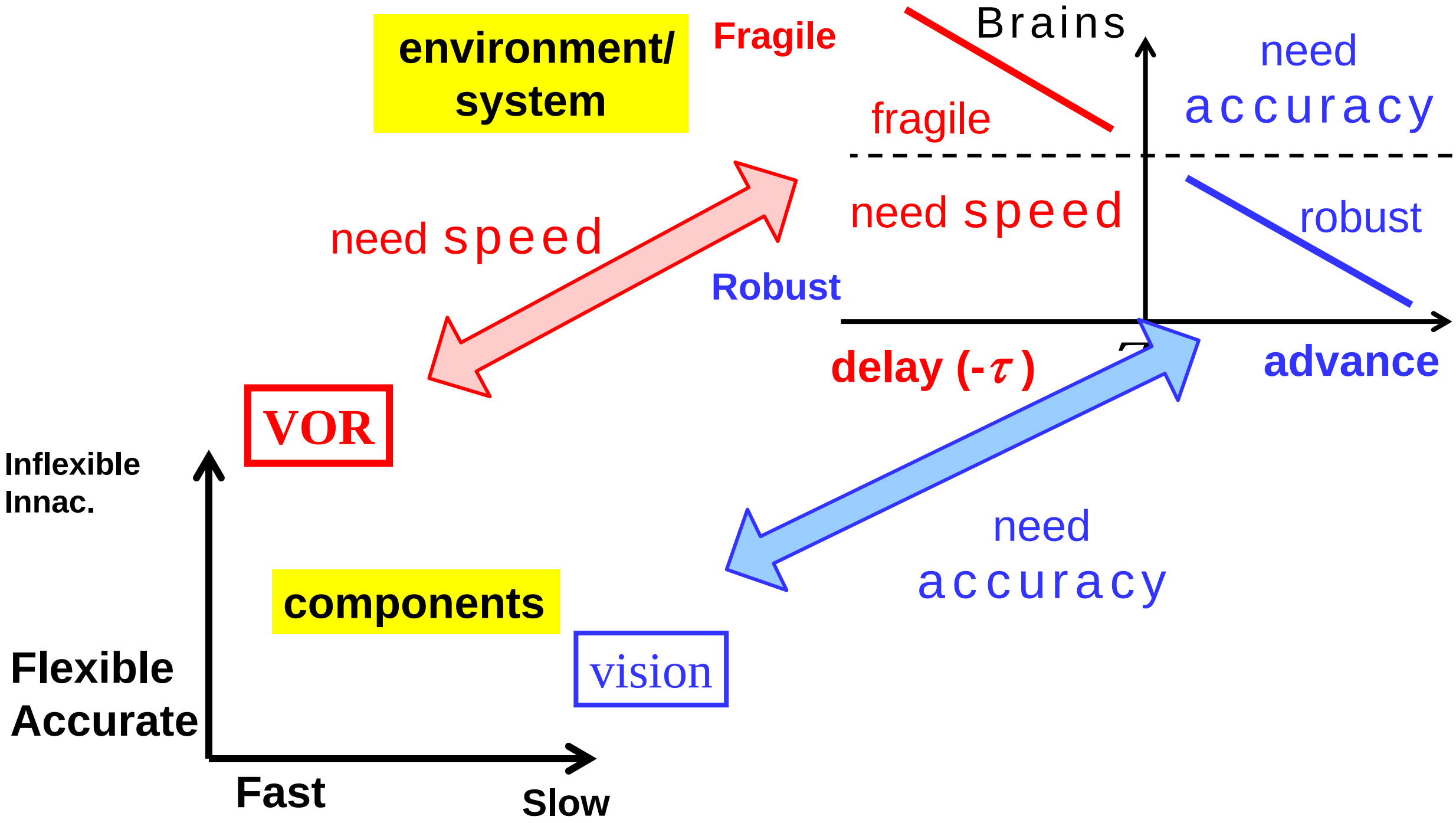
Flip axes

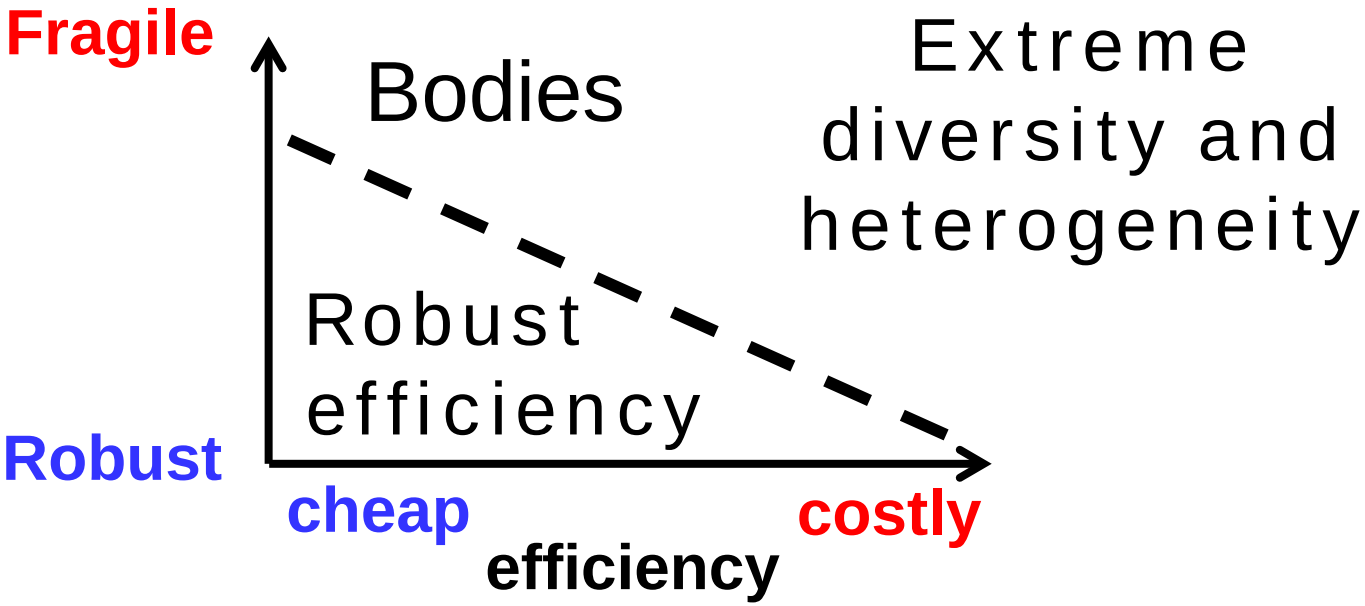

VOR
Inflexible
Innac.
components
vision
Flexible
Accurate
Fast
Slow

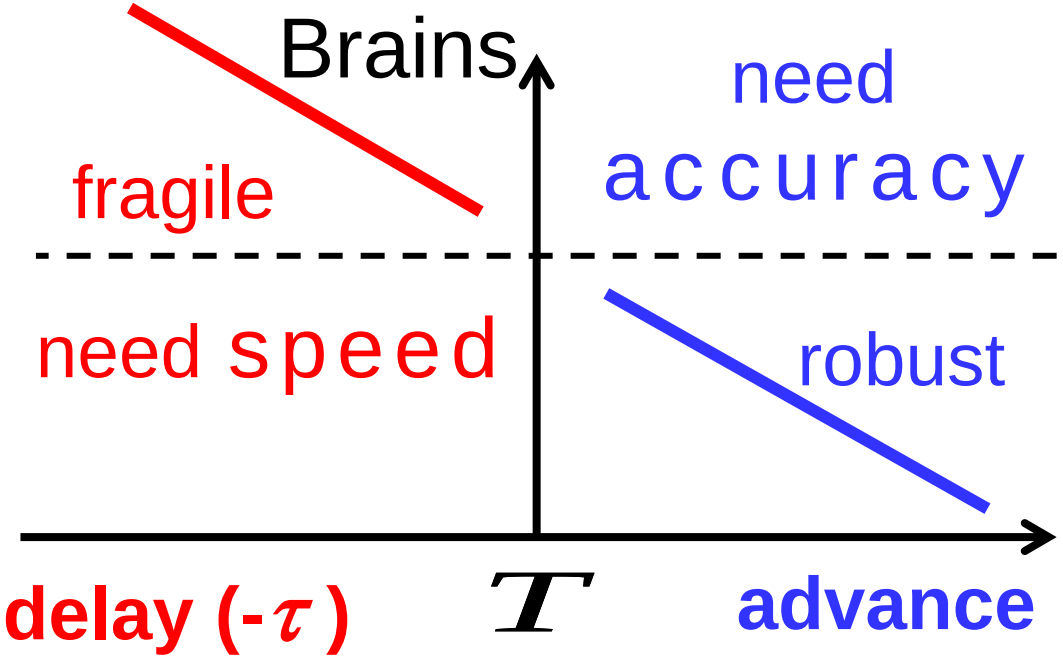
environment/ system behavior

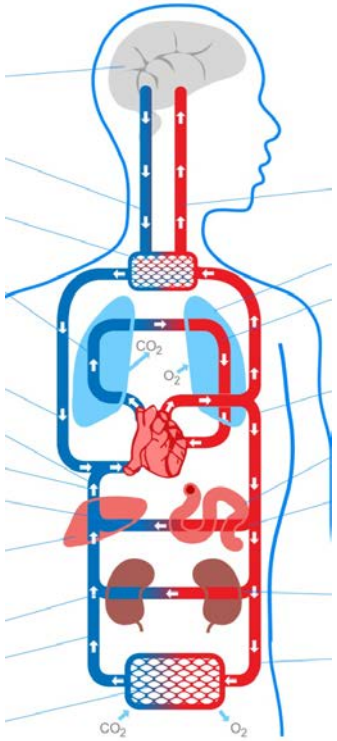




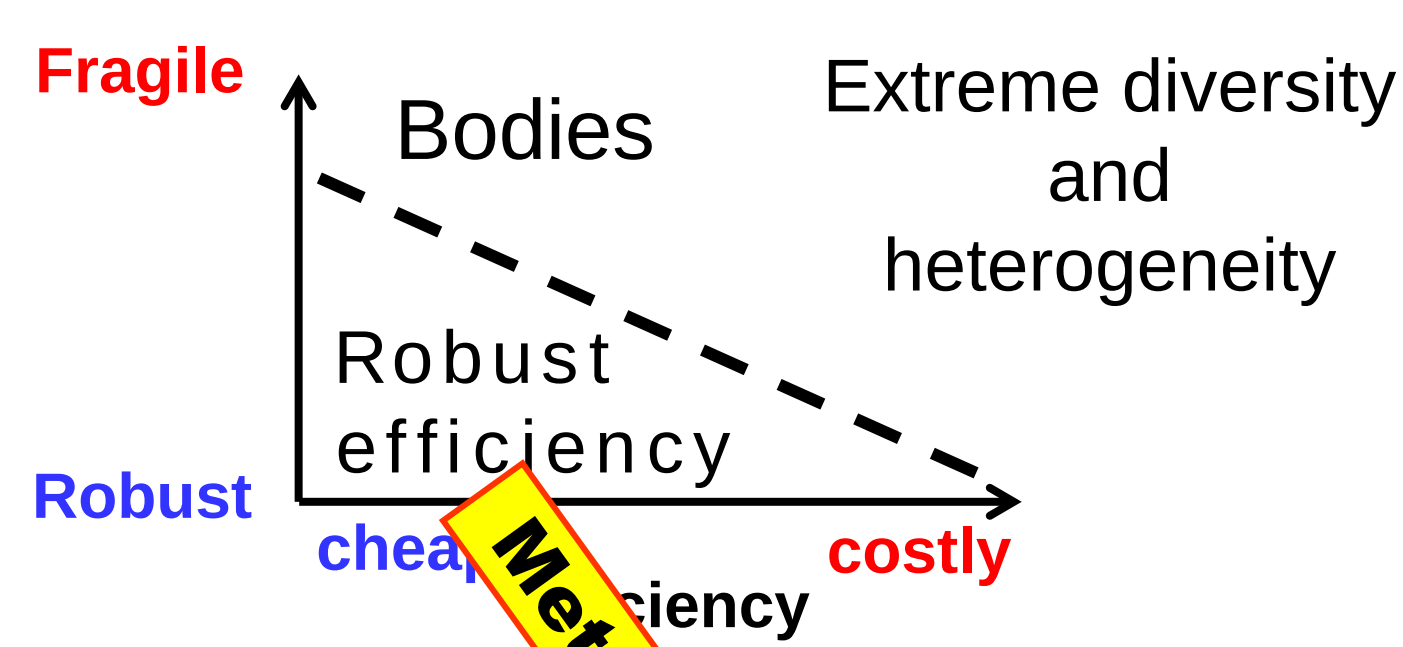

environment/
system
Fragile
Brains
need
accuracy
fragile
need speed
robust
need speed
Robust
delay (-τ)
advance
VOR
Inflexible
Innac.
need
accuracy
components
vision
Flexible
Accurate
Fast
Slow


Fragile
Bodies
Extreme diversity and heterogeneity
Robust efficiency
Robust
cheap
costly
efficiency

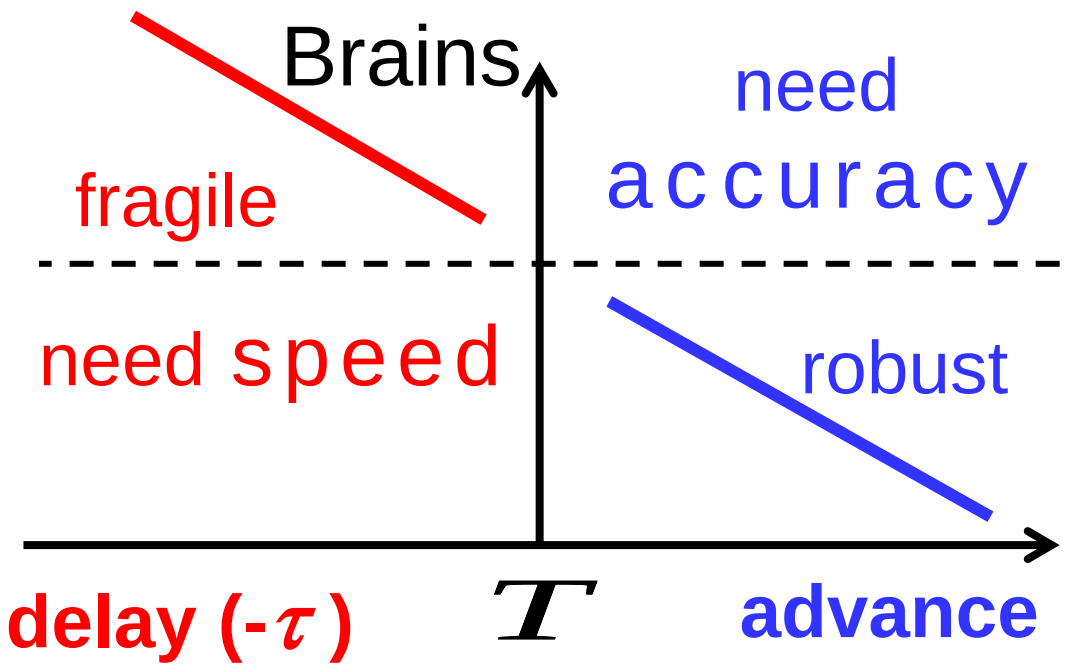

Brains
need accuracy
fragile
need speed
robust
delay (-τ)
T
advance

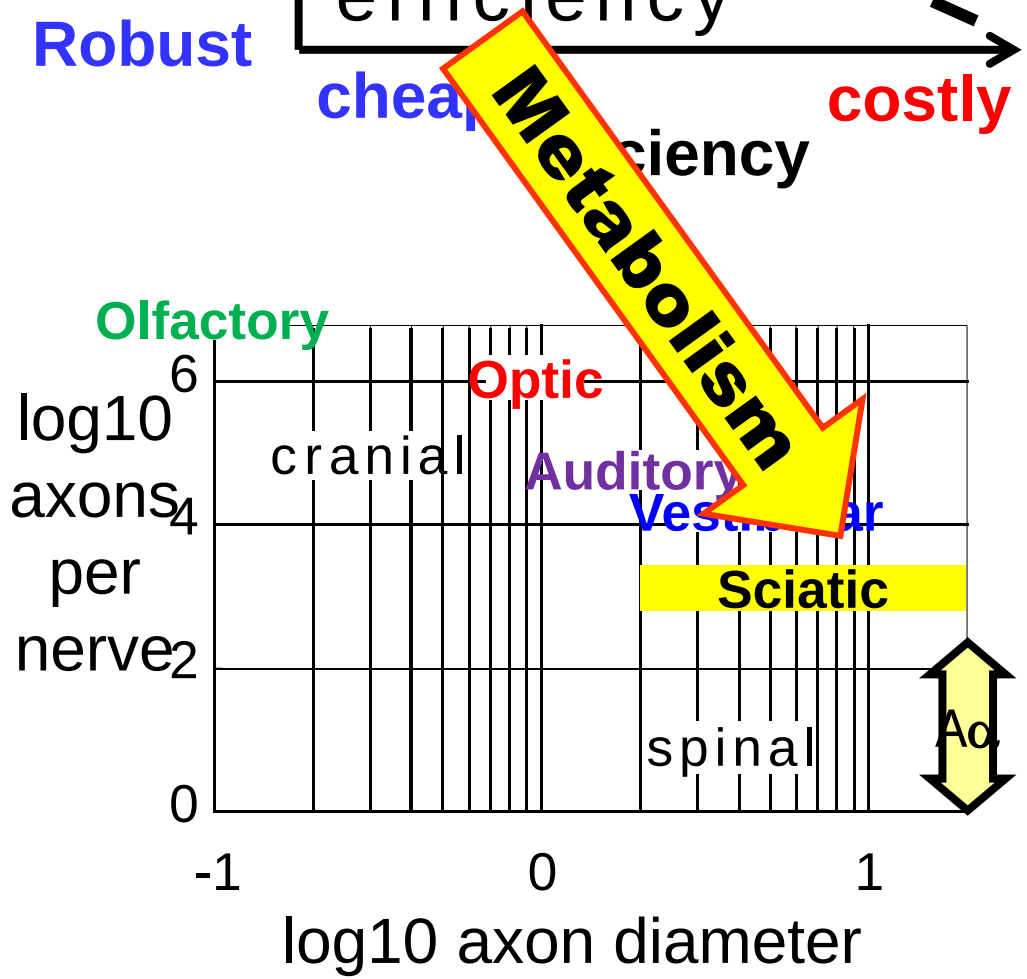

CO2
O2
CO2
O2

GREAT BRITAIN
FARAH
KENYA

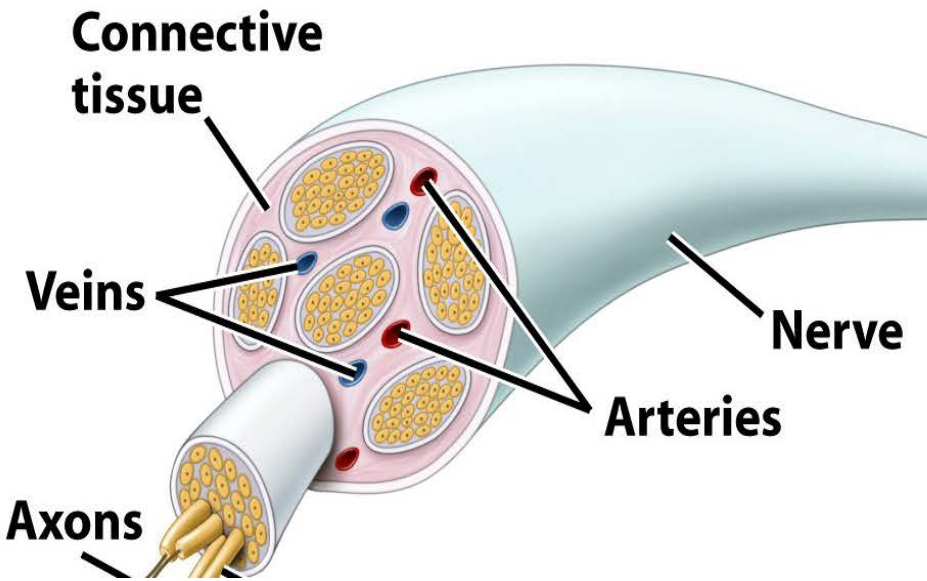


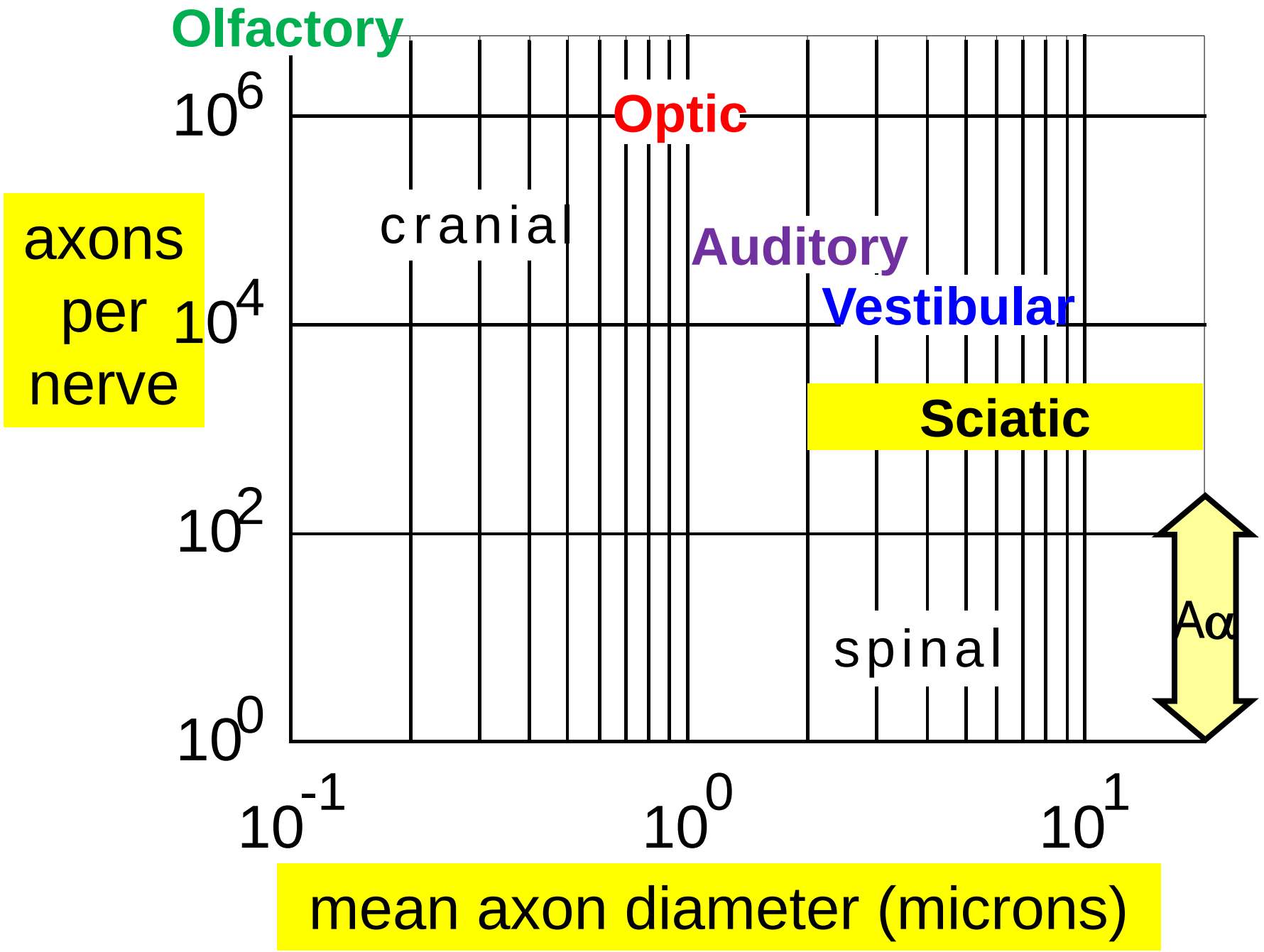
Extreme diversity and heterogeneity

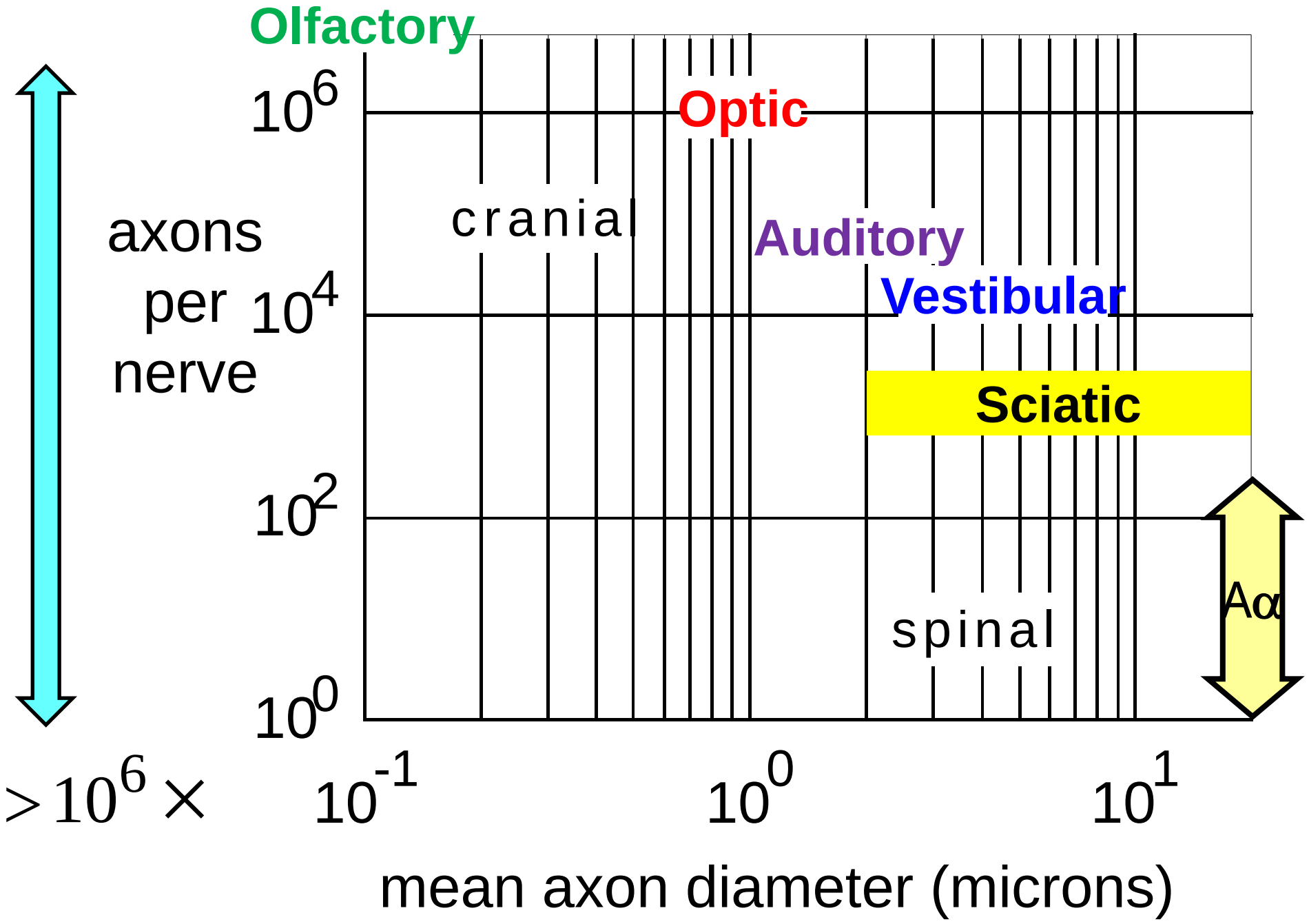




Extreme component level heterogeneity




Olfactory
Optic
cranial
Auditory
Vestibular
Sciatic
spinal
Aα
axons per nerve
10^6
10^4
10^2
10^0
10^{-1}
10^0
10^1
mean axon diameter (microns)


Olfactory
Optic
cranial
Auditory
Vestibular
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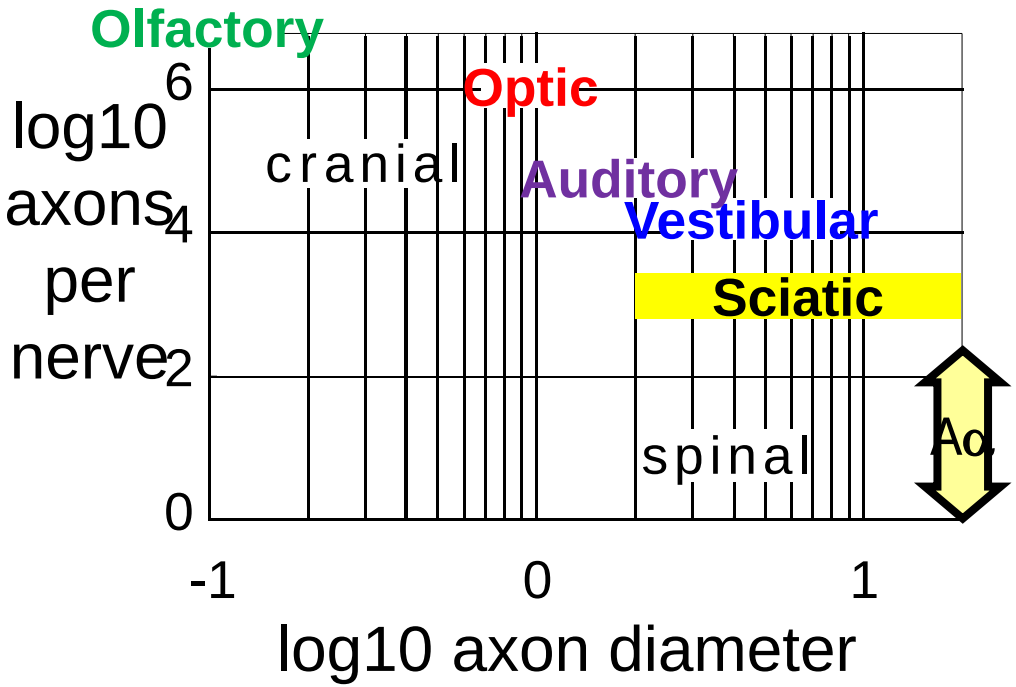
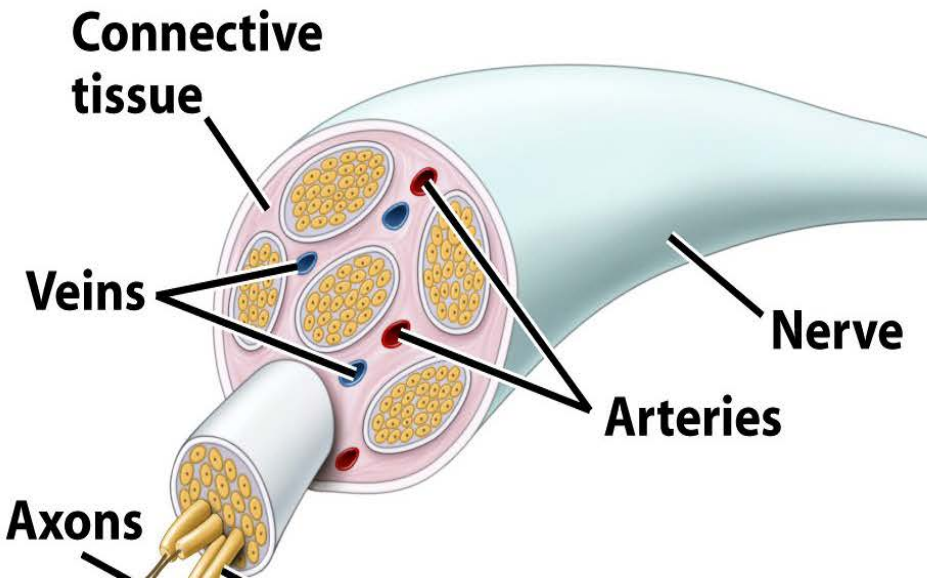
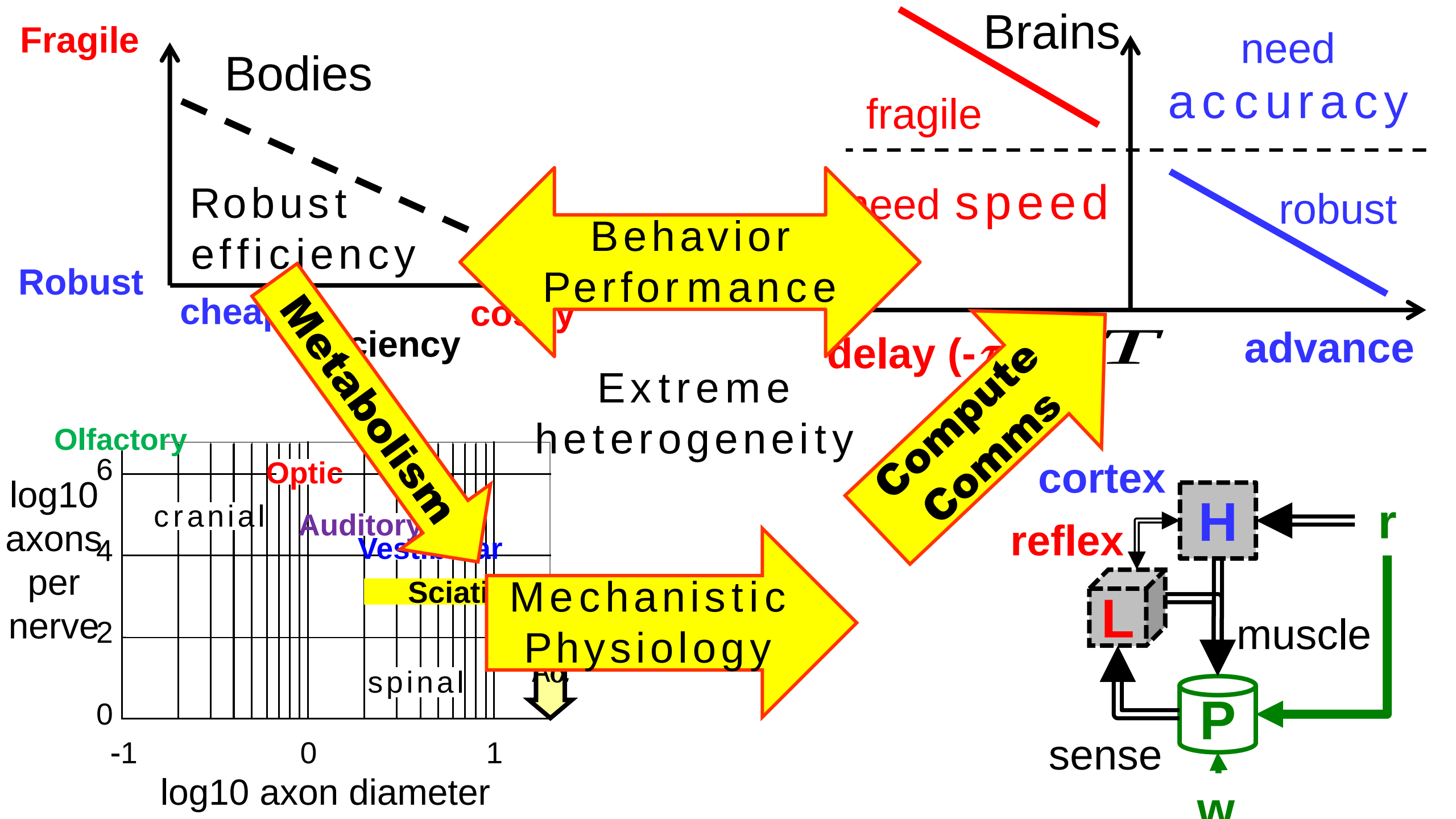
spinal
Aα
axons per nerve
10^6
10^4
10^2
10^0
$> 10^6 \times$
10^{-1}
10^0
10^1
mean axon diameter (microns)



Extreme component level heterogeneity




Fragile
Bodies
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need speed
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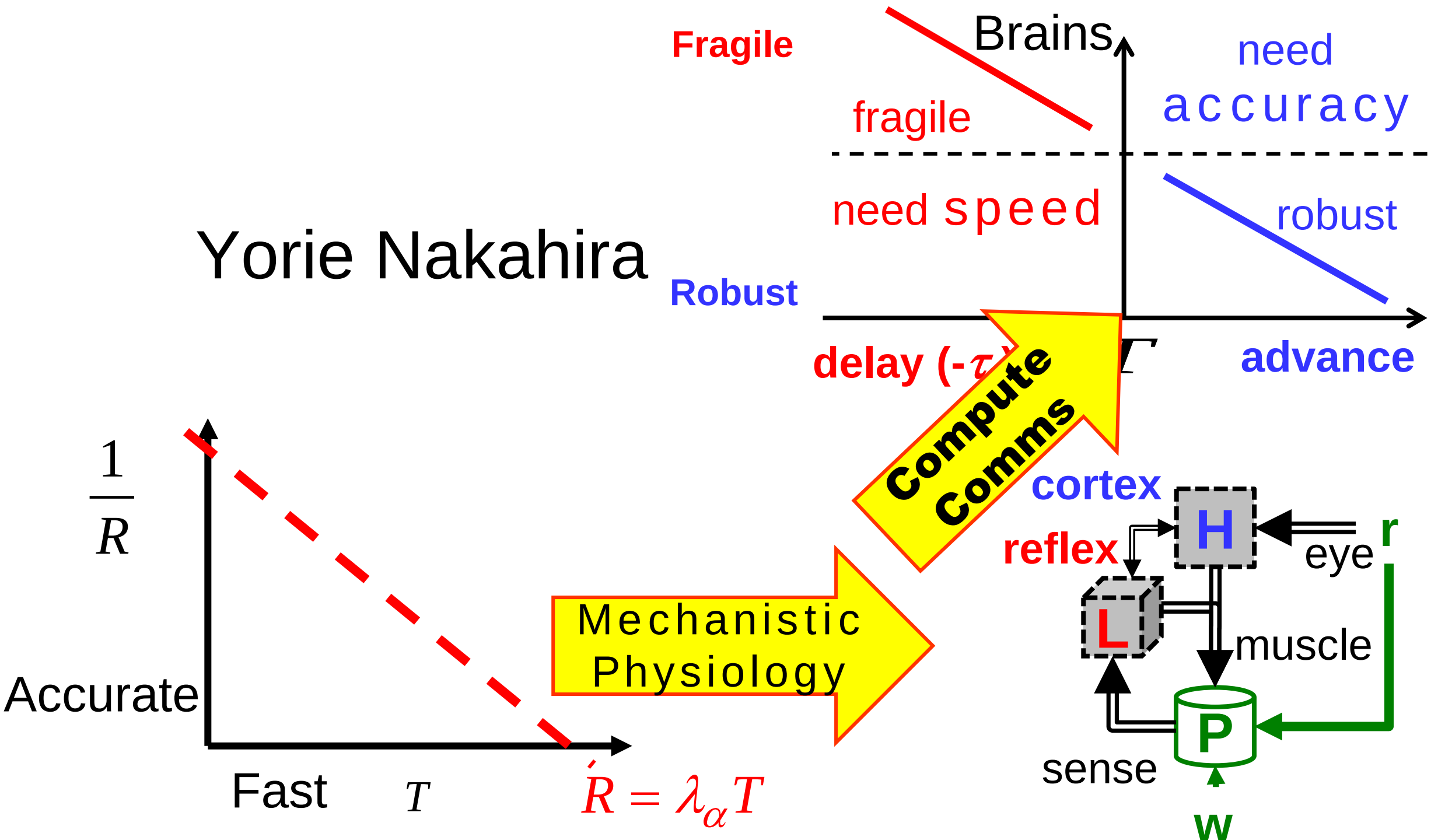
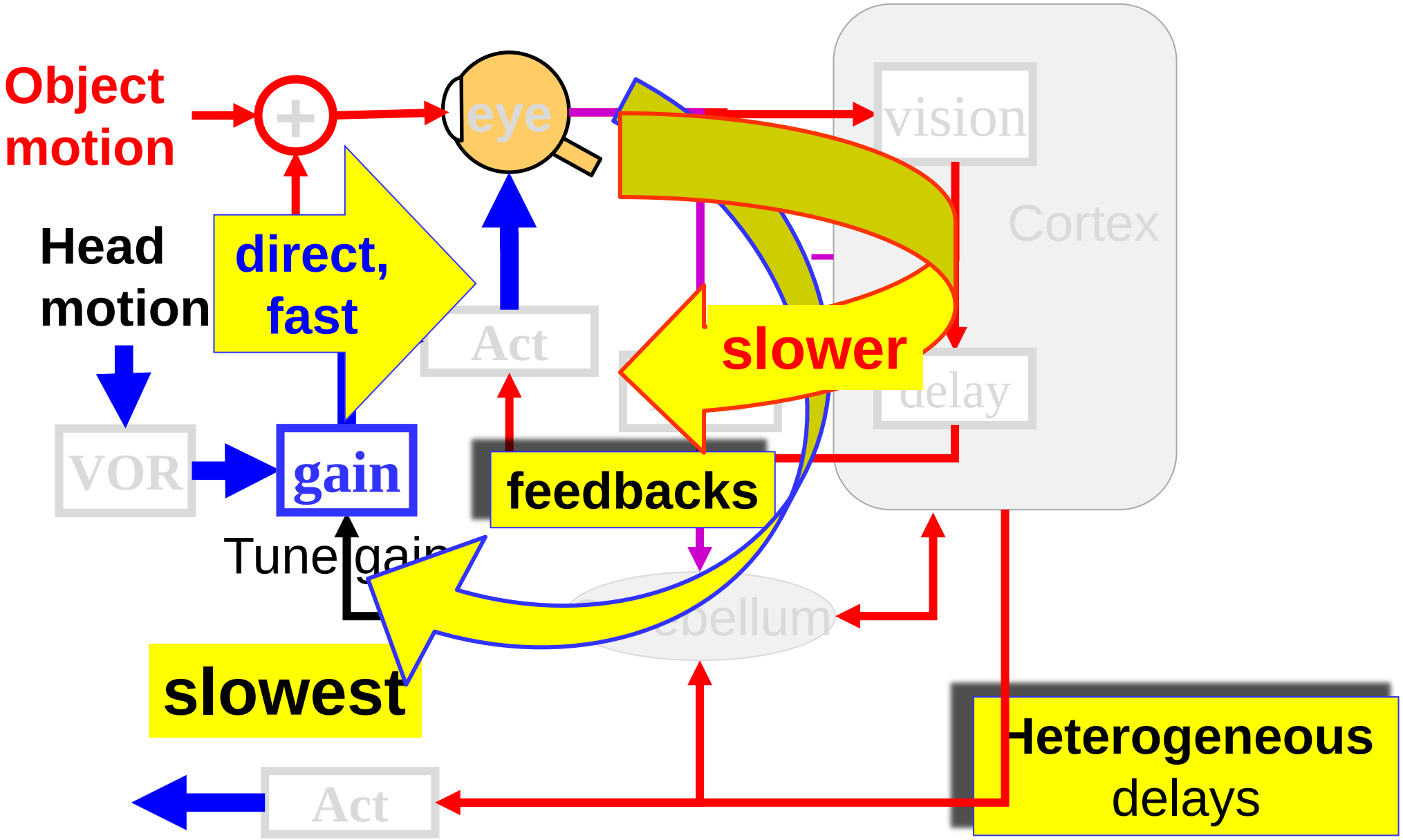
delay (-τ
T
advance
Behavior
Performance
Metabolism
Compute
Comms
Extreme
heterogeneity
Olfactory
Optic
cranial
Auditory
Sciati
spinal
log10
axons
per
nerve
6
4
2
0
-1
0
1
log10 axon diameter
Mechanistic
Physiology
cortex
reflex
H
L
r
muscle
P
sense
w

Yorie Nakahira




Object motion
+
eye
vision
Cortex
Head motion
direct, fast
Act
slower
delay
VOR
gain
feedbacks
Tune gain
cebellum
slowest
Act
Heterogeneous delays

Robust to
"noise" model

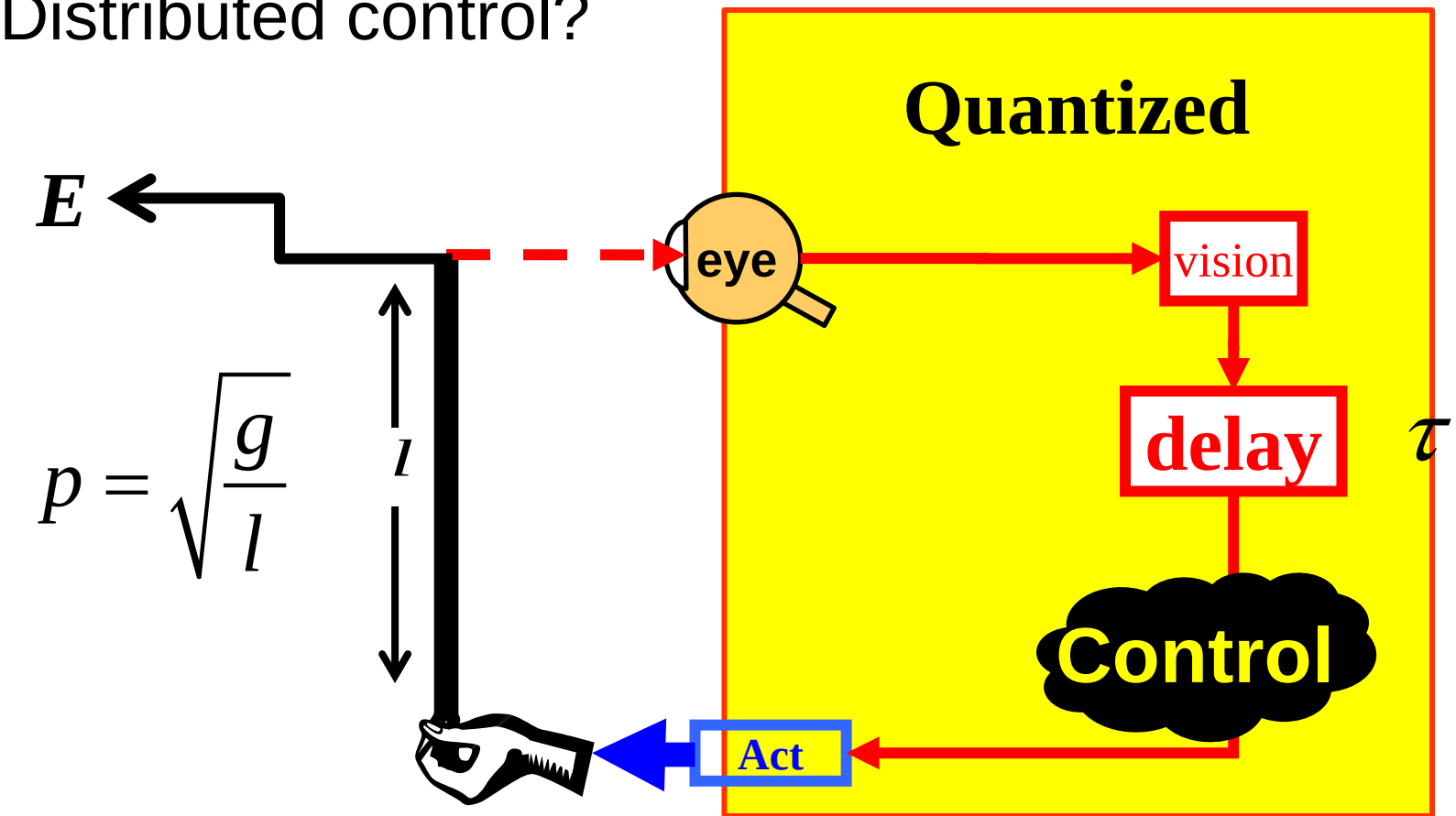
Spiking neurons?
Discrete axons?
Distributed control?

Entropy rate $\exp\left(\int \ln|T|\right)$

Energy (L2) $\|T\|_\infty$

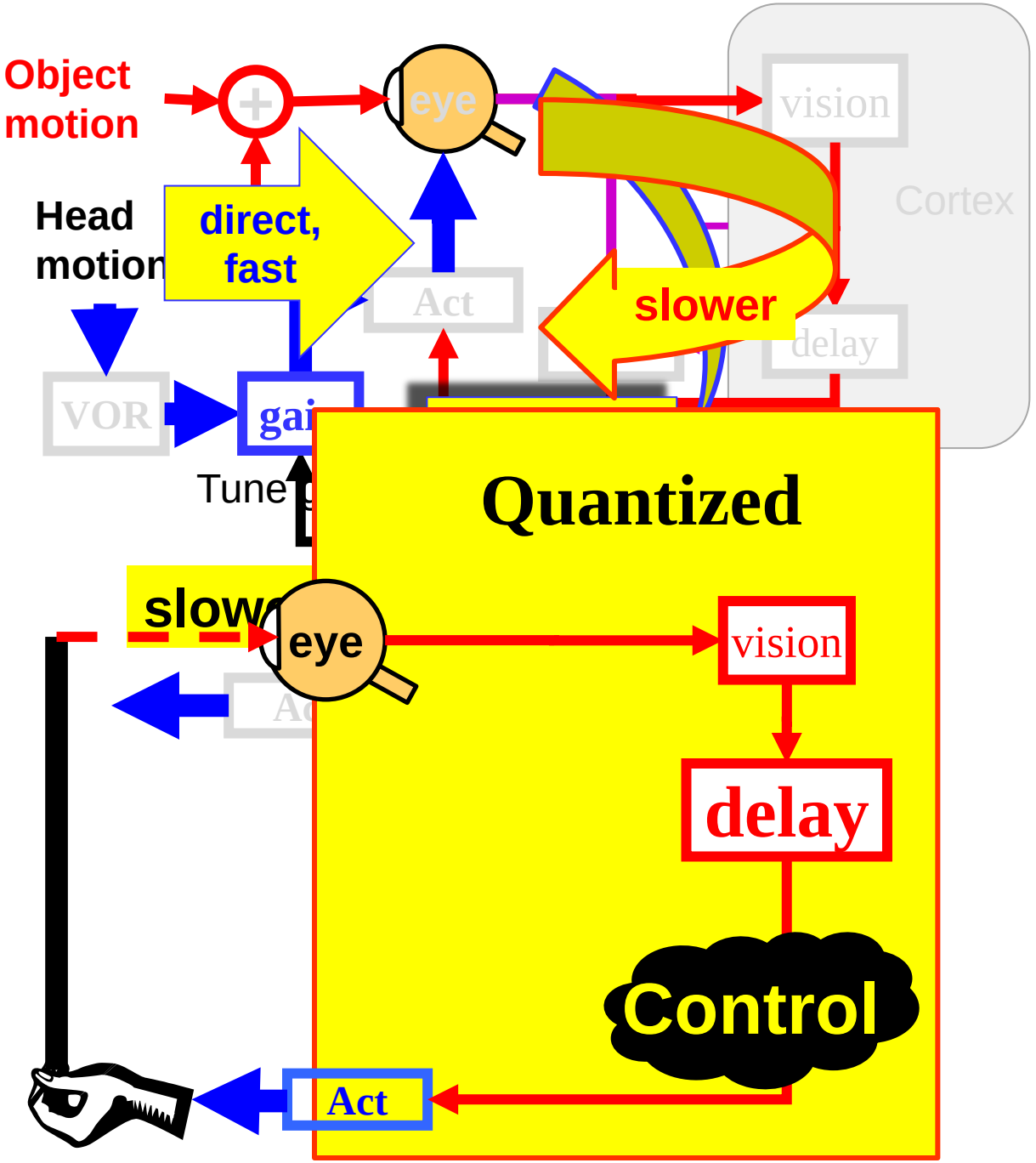
Quantized

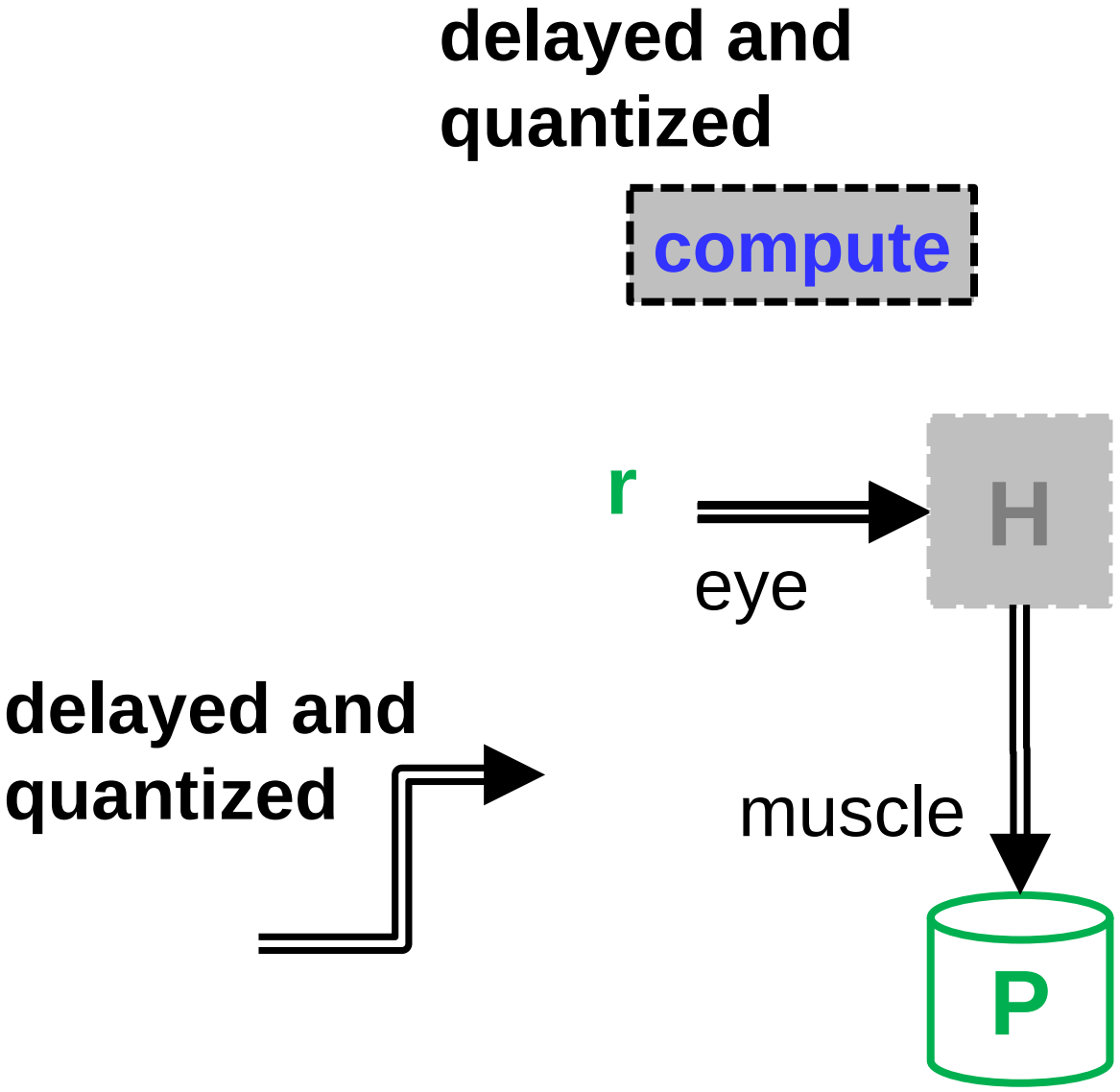
$\propto \exp(p\tau)$

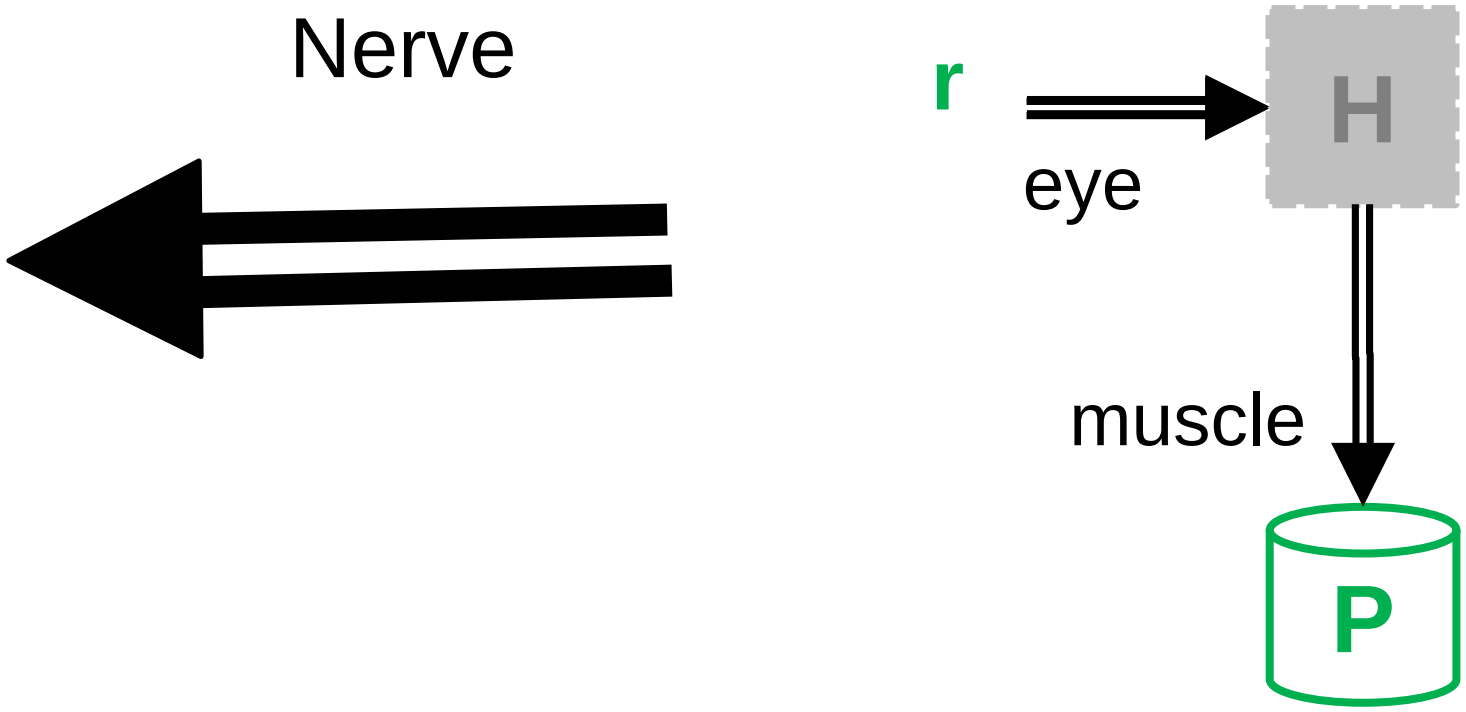


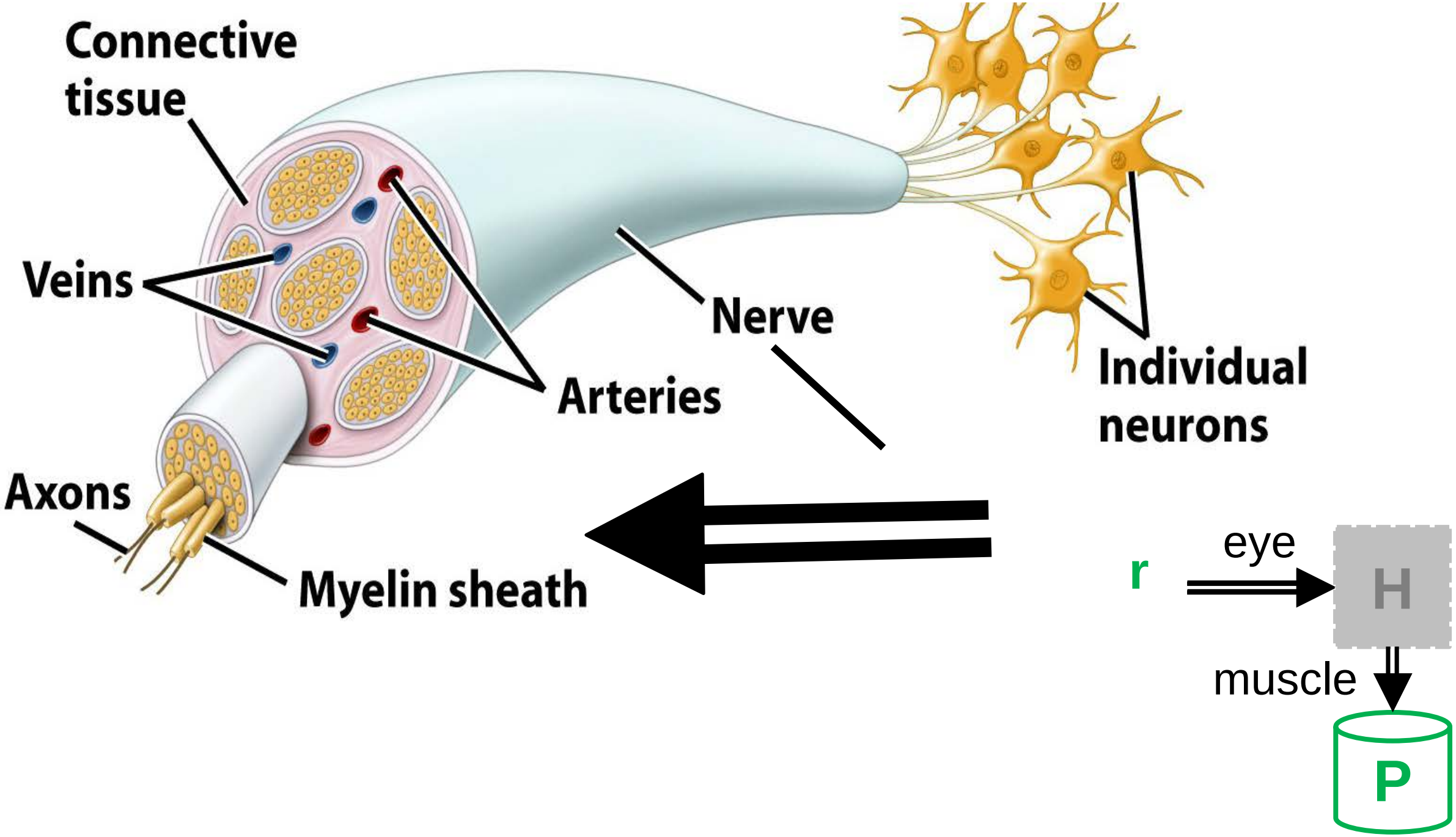
$$p = \sqrt{\frac{g}{l}}$$

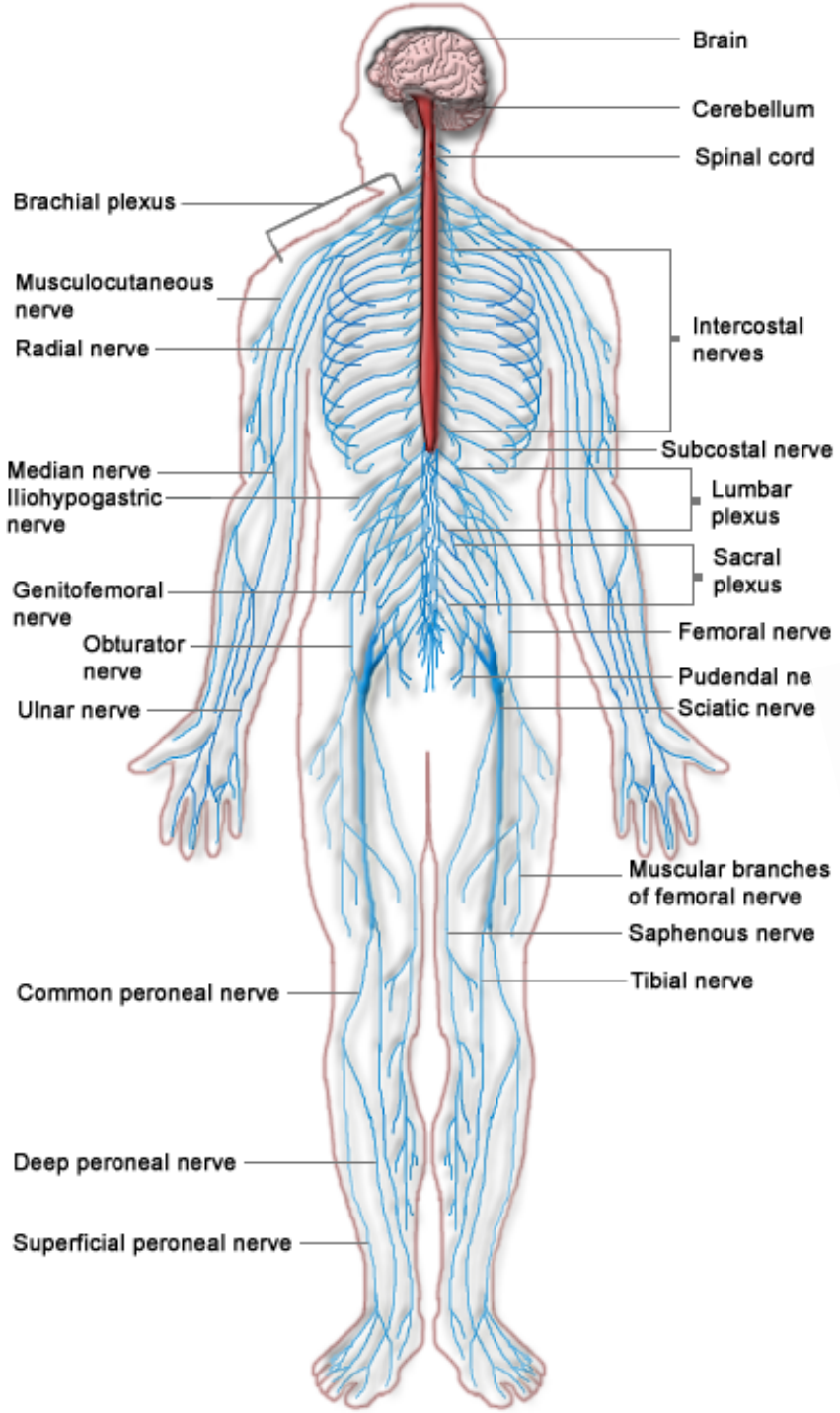
$$\tau \approx .3s$$


Object motion
Head motion
direct, fast
eye
vision
Cortex
Act
slower
delay
VOR
gai
Tune
Quantized
slow
eye
vision
delay
Control
Act


delayed and quantized
compute
r
H
eye
muscle
P
delayed and quantized


Nerve
r
H
eye
muscle
P


Connective tissue
Veins
Arteries
Nerve
Individual neurons
Axons
Myelin sheath
r
eye
H
muscle
P

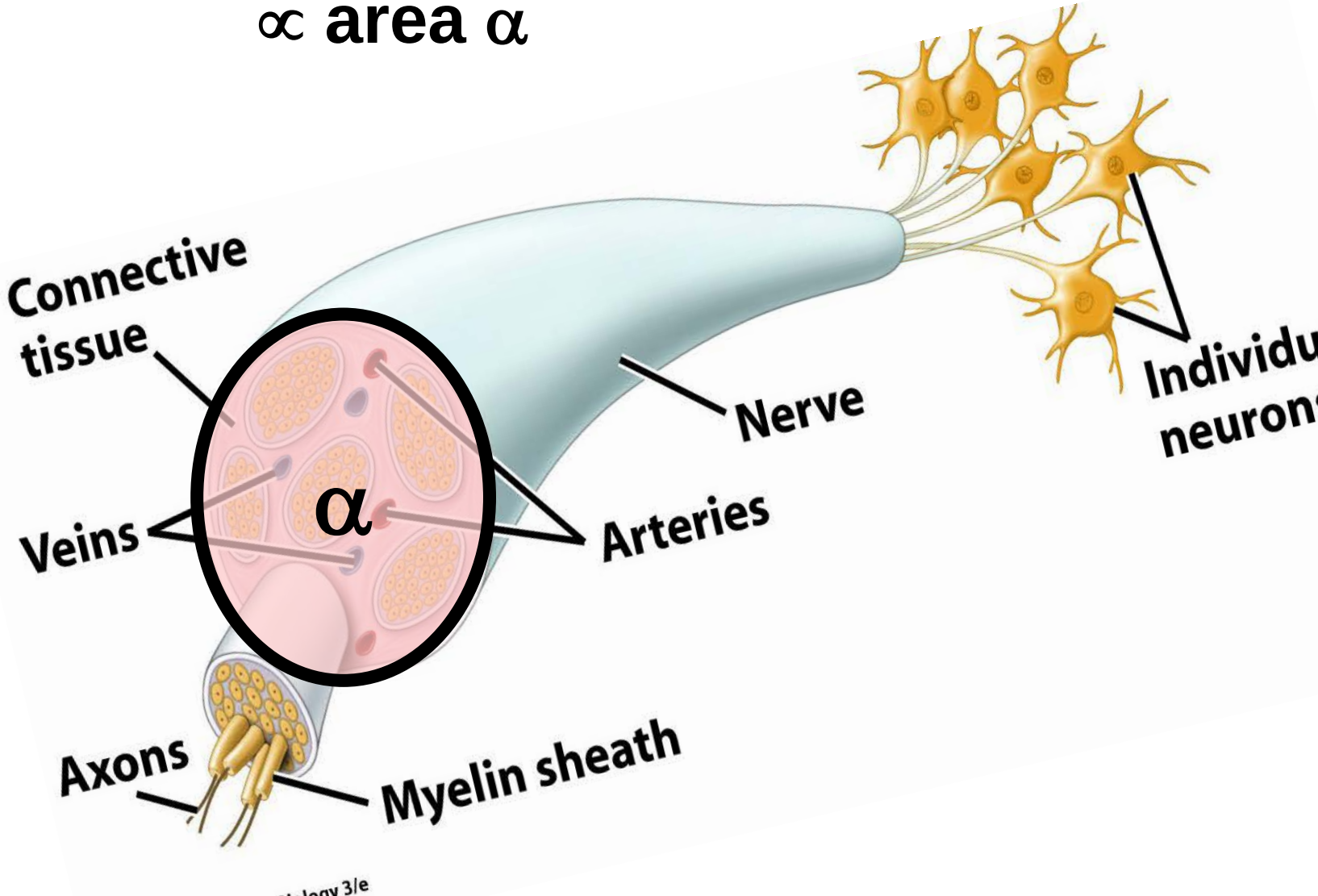


Assume:

resource (cost)

(to build and maintain nerve)

$\propto$ **area α**



Assume:

resource (cost)
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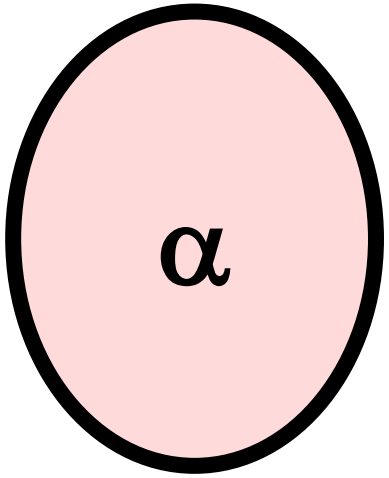
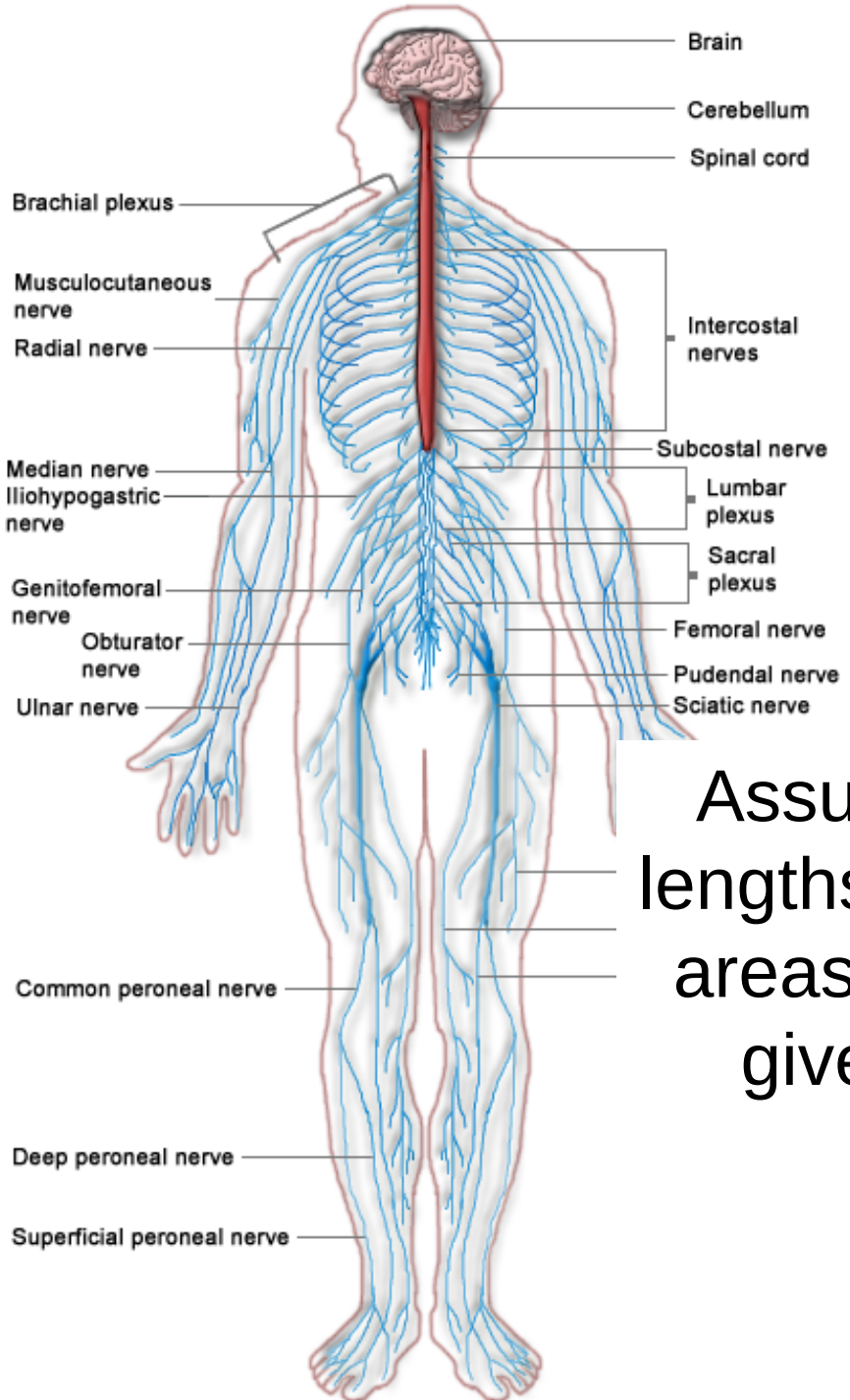


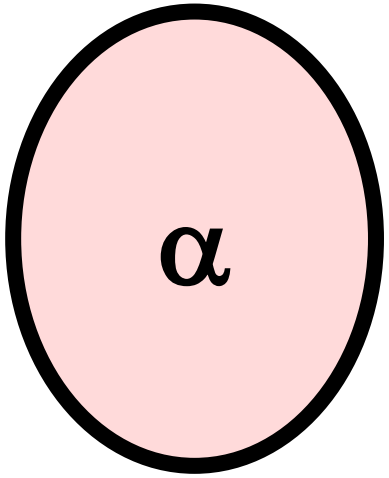


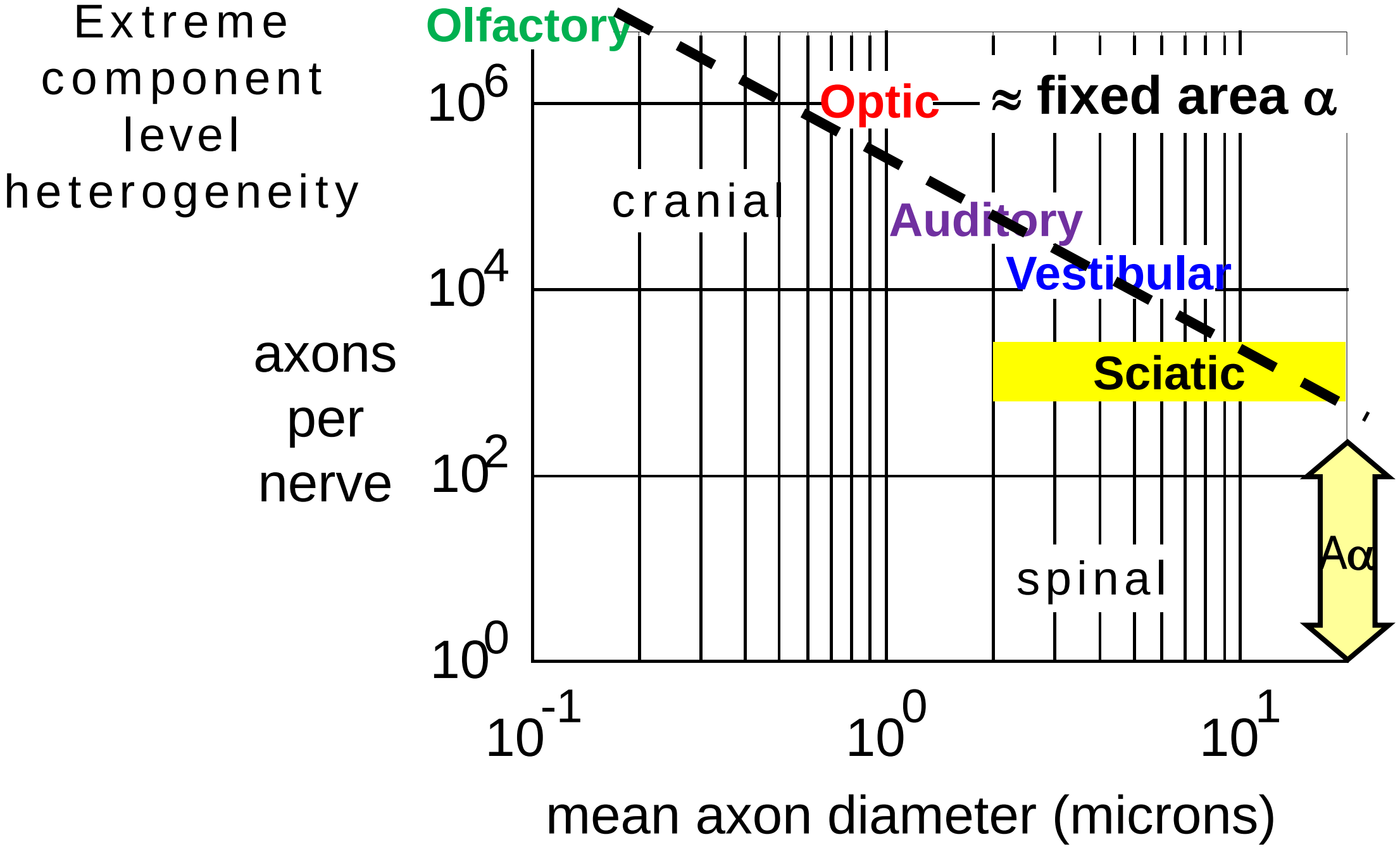
Assume lengths and areas are given

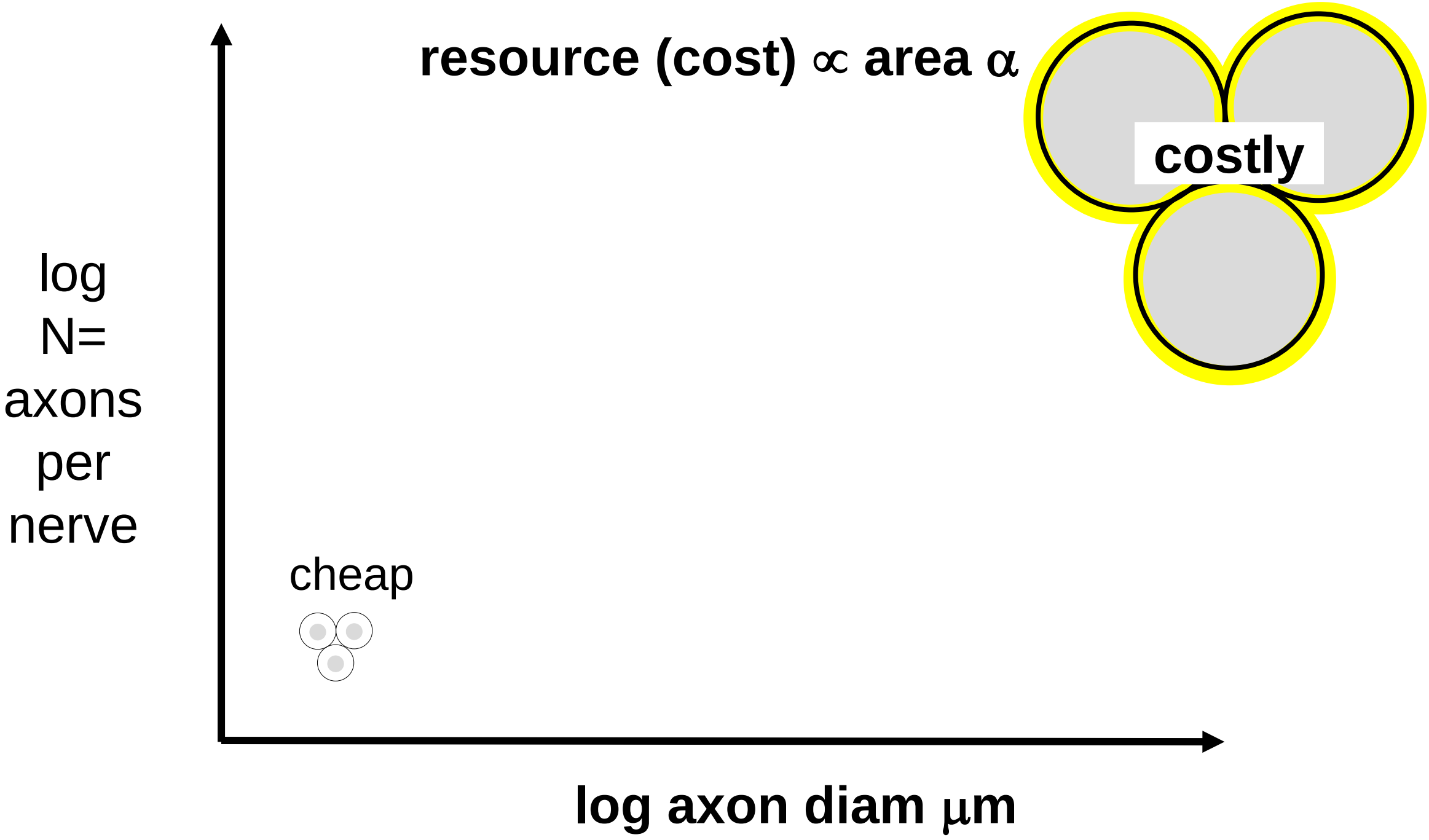
Assume:

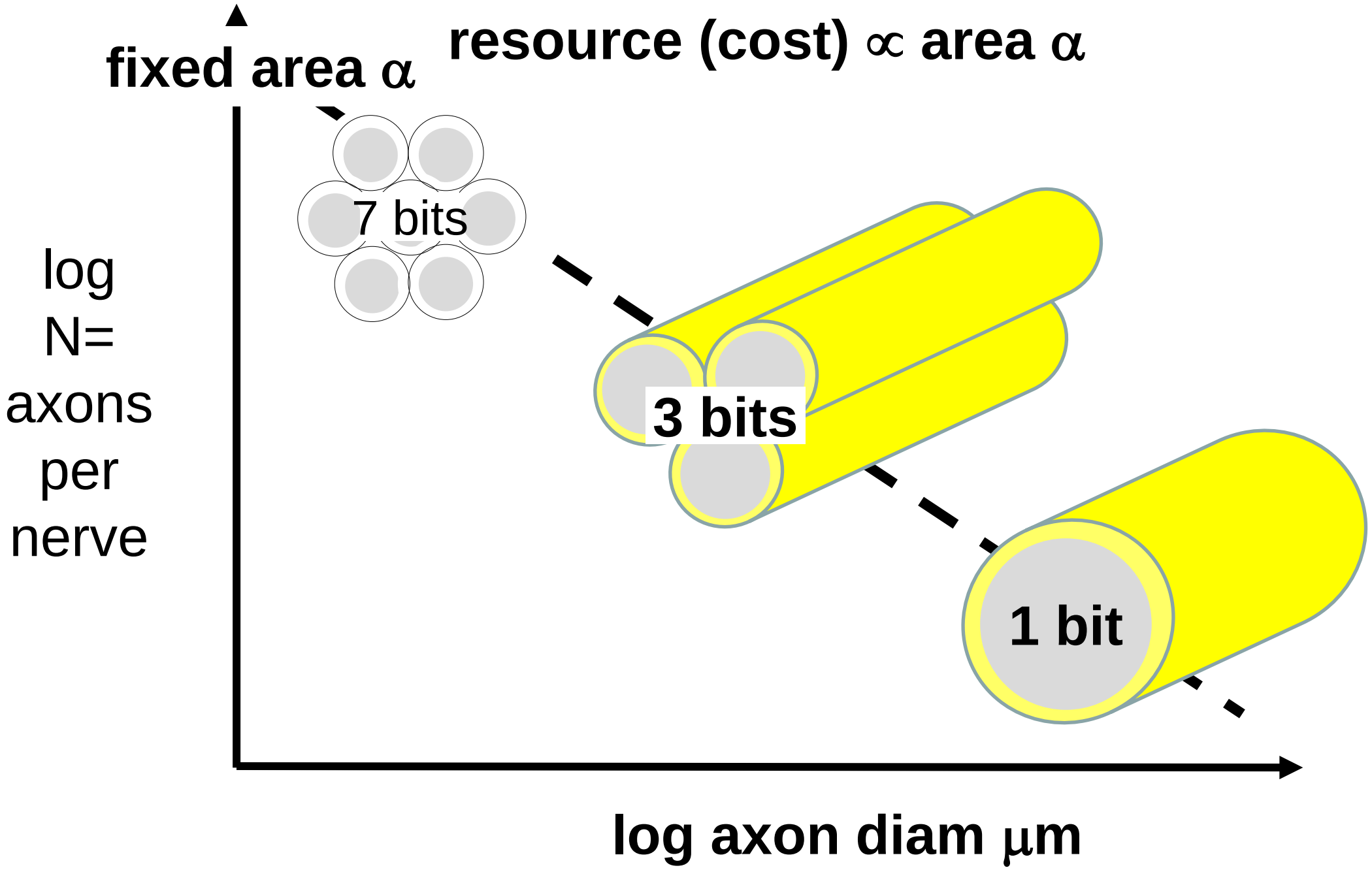
resource (cost)
(to build and maintain nerve)

$\propto$ **area** α

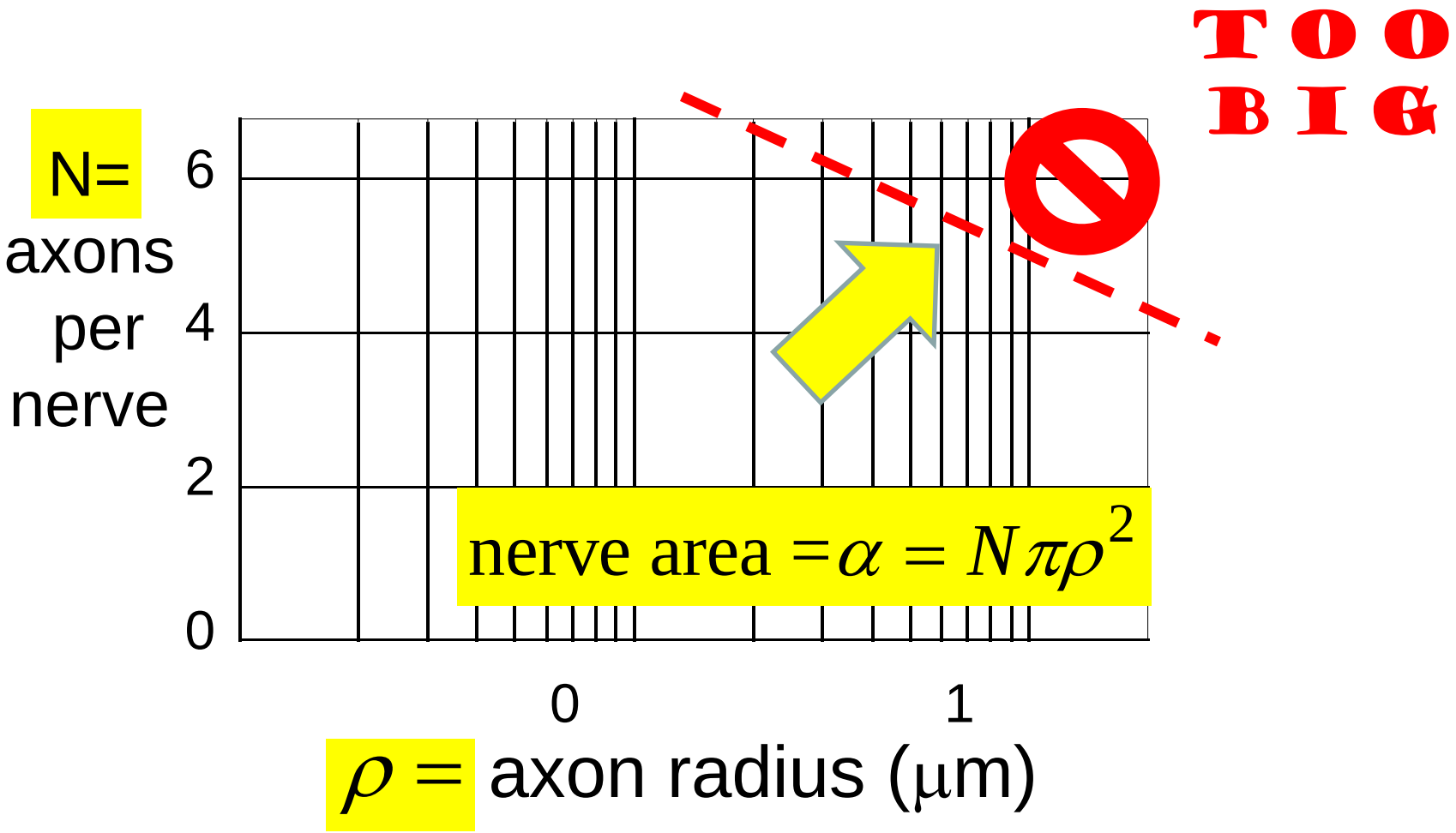



Extreme component level heterogeneity
Olfactory
Optic
≈ fixed area α
cranial
Auditory
Vestibular
Sciatic
spinal
Aα
axons per nerve
10^6
10^4
10^2
10^0
10^{-1}
10^0
10^1
mean axon diameter (microns)

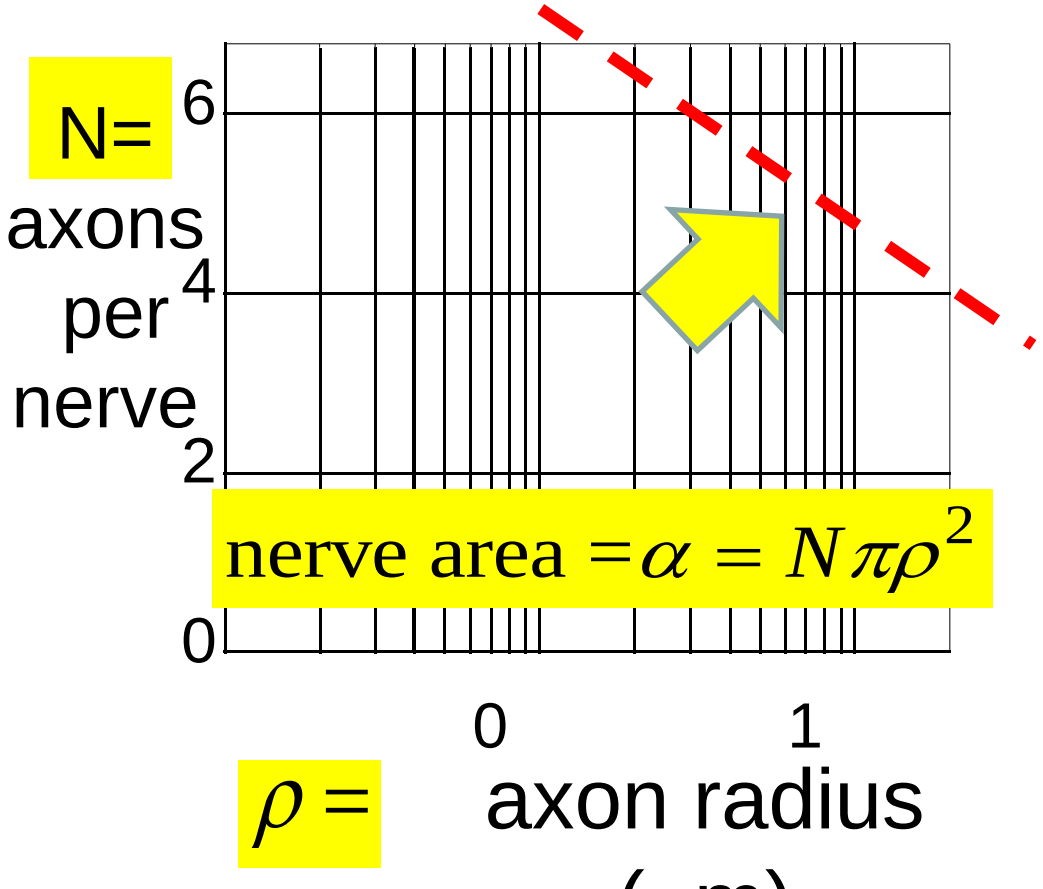

resource (cost) ∝ area α
costly
log
N=
axons
per
nerve
cheap
log axon diam μm


fixed area α
resource (cost) ∝ area α
7 bits
3 bits
1 bit
log
N=
axons
per
nerve
log axon diam μm

resource (cost) ∝ area α



resource (cost) ∝ area α



$$\text{spike speed} \propto \rho$$

(propagation)

$$\frac{1}{T} = \frac{1}{delay} \propto \rho$$

Tradeoffs in spiking neurons

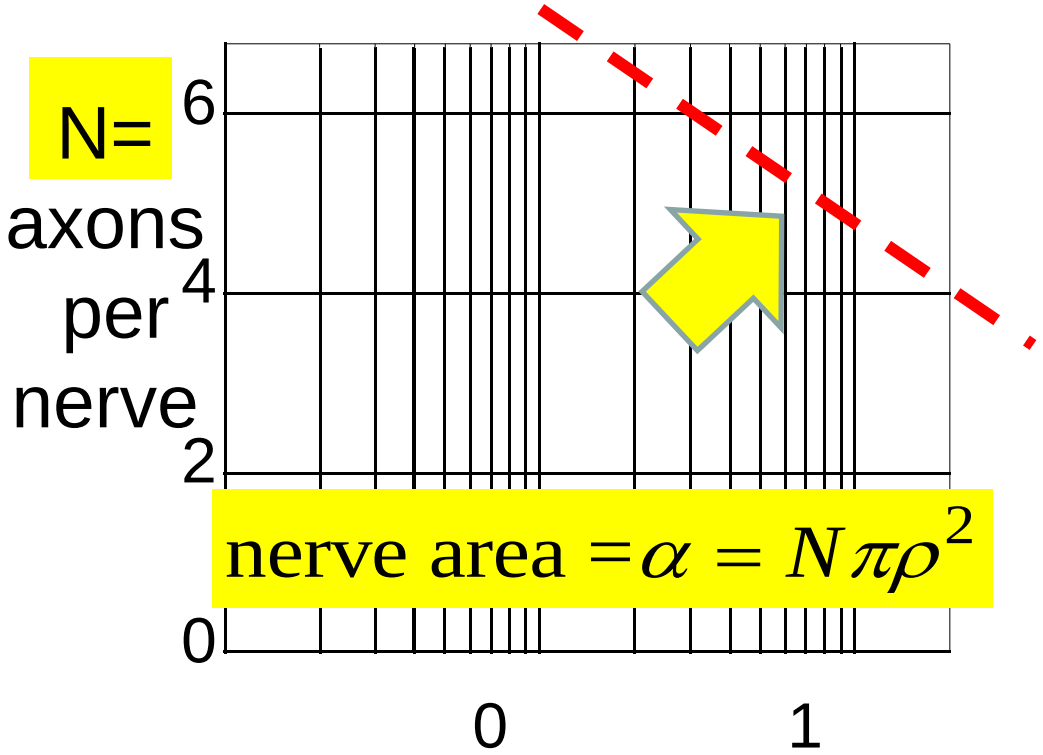
spike rate $\propto \rho$

spike speed $\propto \rho$

(propagation)

$$\frac{1}{T} = \frac{1}{delay} \propto \rho$$

resource (cost) ∝ area α



Tradeoffs in spiking neurons

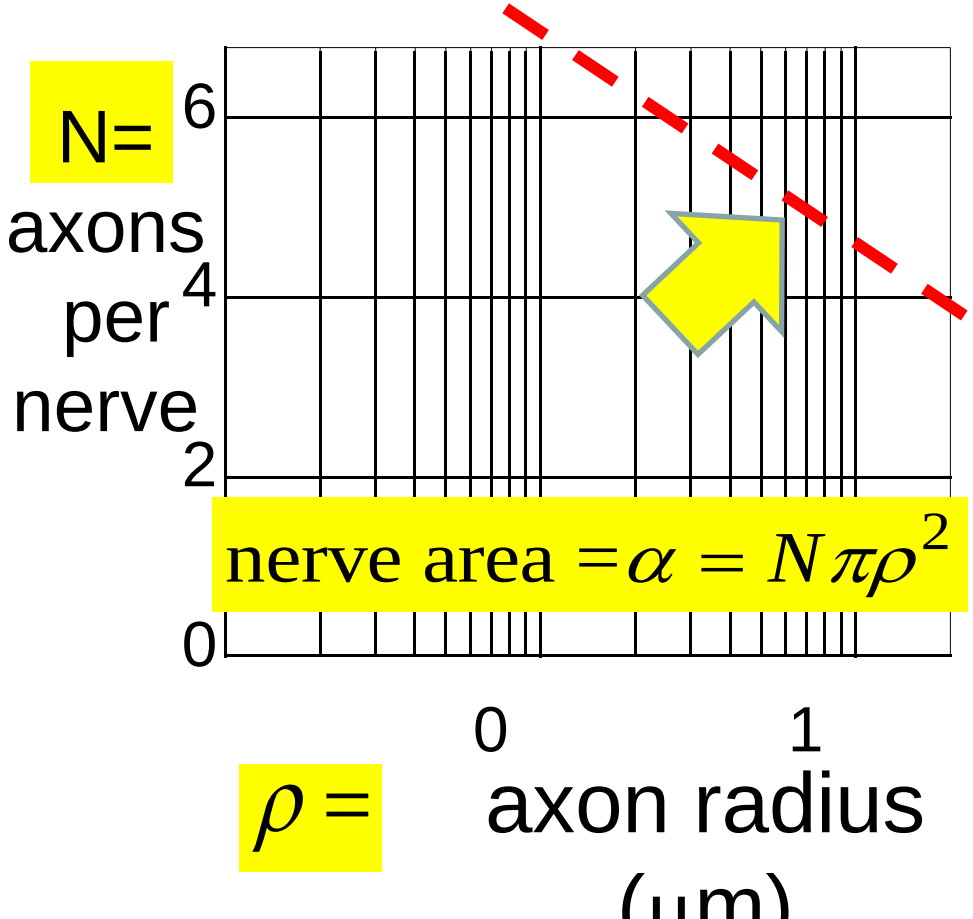
spike rate $\propto \rho$

bandwidth R (bits/time) $\propto N\rho$

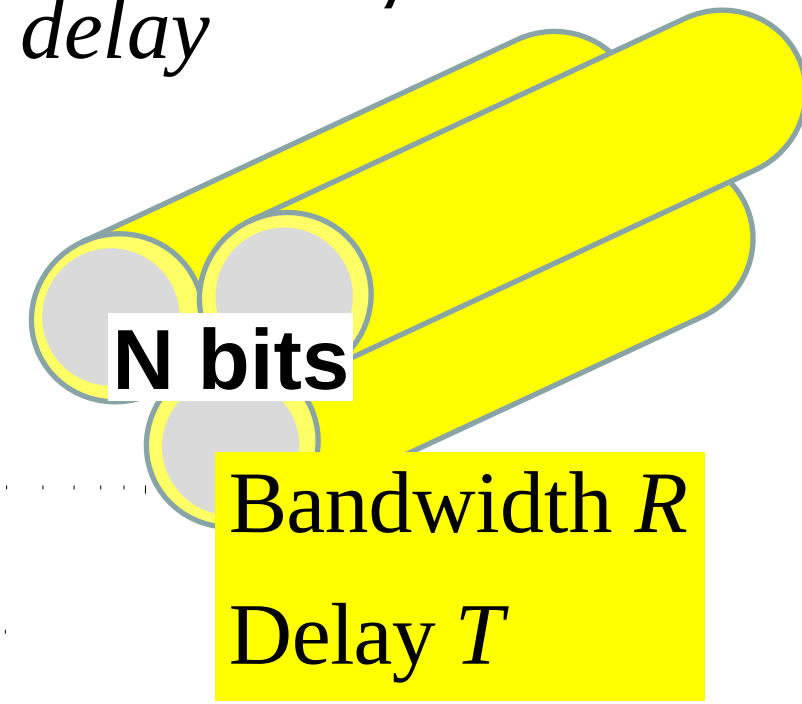
spike speed $\propto \rho$

(propagation)

$$\frac{1}{T} = \frac{1}{delay} \propto \rho$$



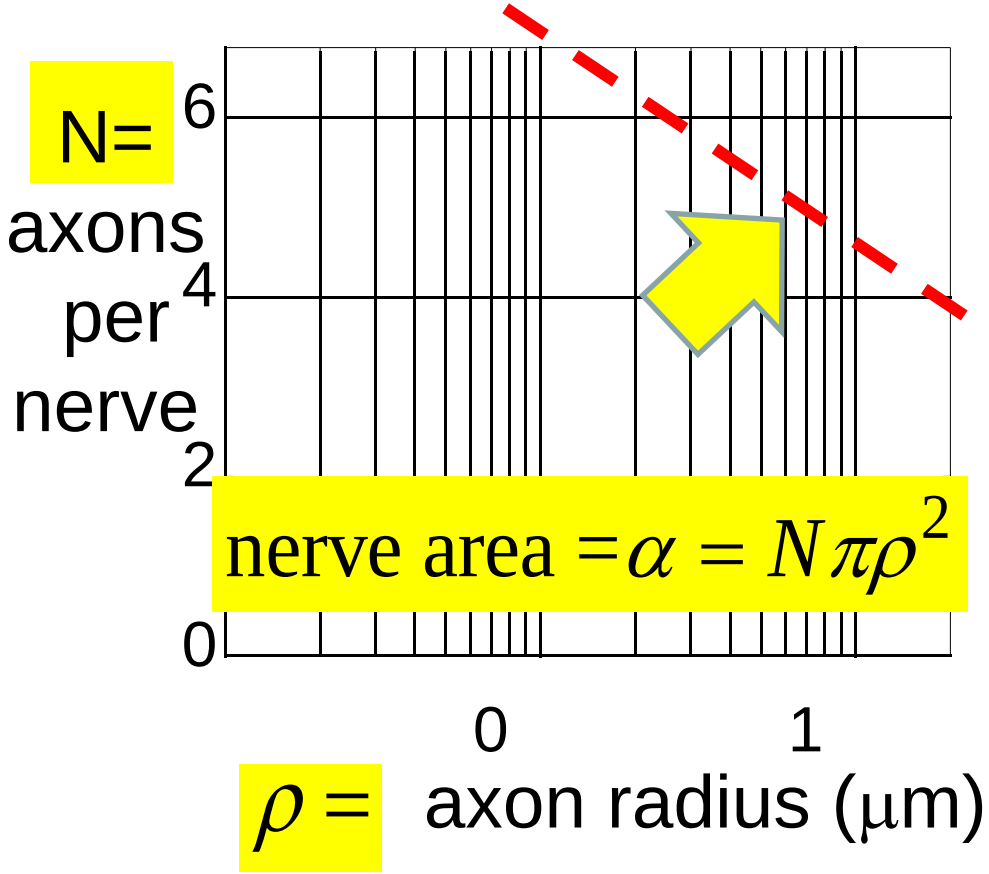
Digital
tradeoffs
in
spiking
neurons



Terry Sejnowski
Simon Laughlin

spike rate $\propto \rho$

bandwidth R (bits/time) $\propto N\rho$



$$R \propto \left(\frac{\alpha}{\rho^2}\right)\rho \propto \left(\frac{\alpha}{\rho}\right)$$

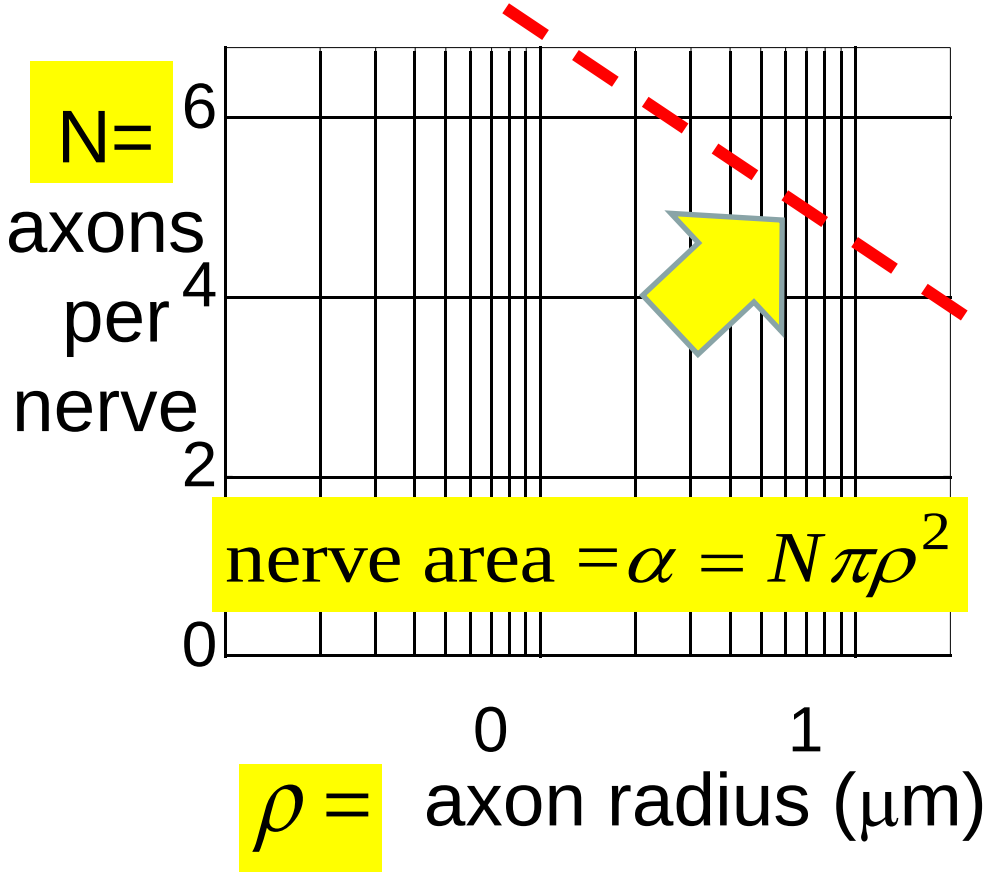
$$N \propto \frac{\alpha}{\rho^2}$$

spike rate $\propto \rho$

R (bits/time) $\propto N\rho$

spike speed $\propto \rho$

$\therefore$ delay $T_s \propto \dfrac{1}{\rho}$

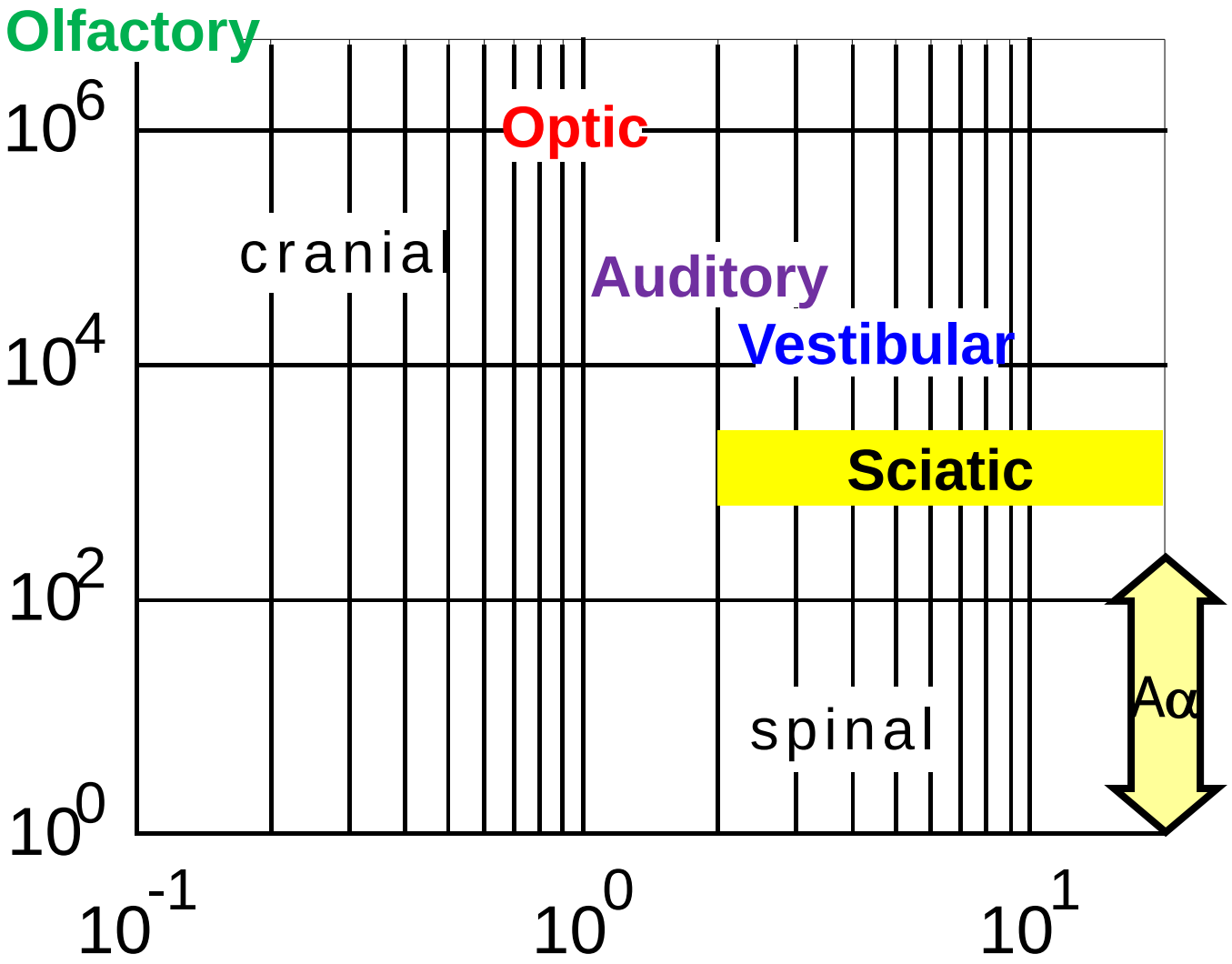


$$R \propto \left(\frac{\alpha}{\rho^2}\right)\rho \propto \left(\frac{\alpha}{\rho}\right) \propto \alpha T$$

$$\therefore R = \lambda_\alpha T$$

$$\lambda_\alpha \propto \alpha$$

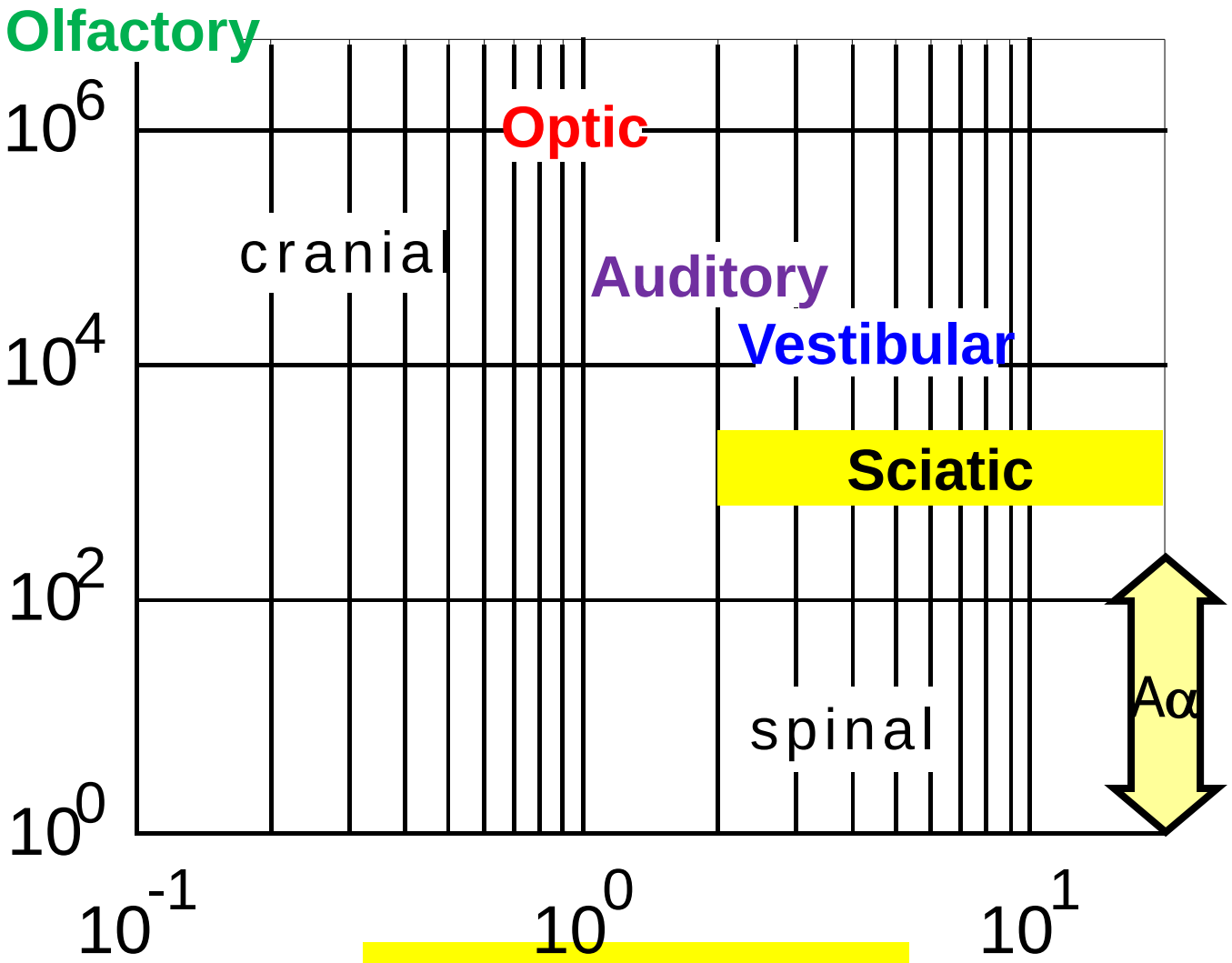
N= axons per nerve



$$diam = 2\rho$$

$$R = \lambda_\alpha T\ ?$$

N= axons per nerve



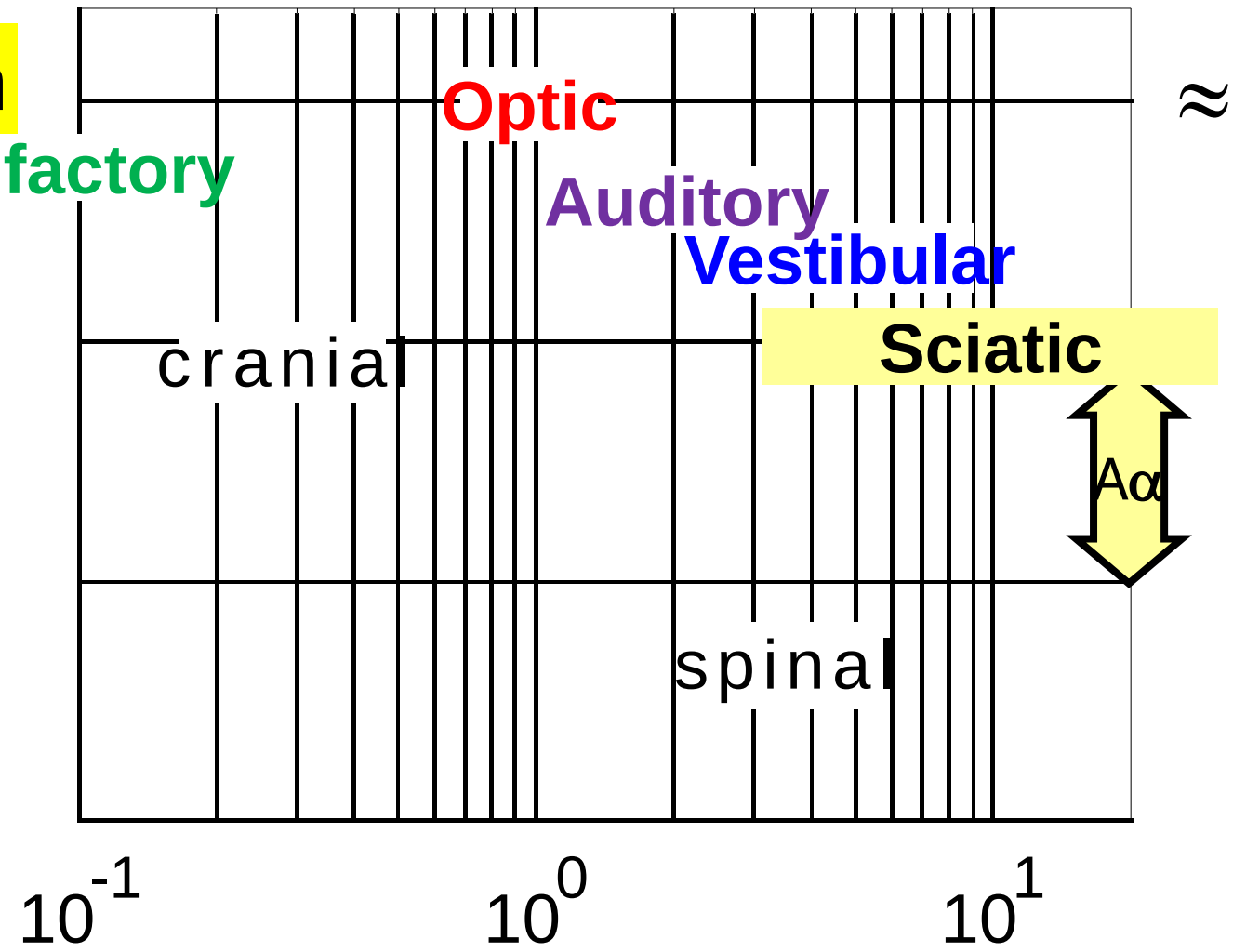
$$diam = 2\rho \propto \frac{1}{T}$$

$$R = \lambda_\alpha T\ ?$$

bandwidth

$R \propto$

$N\rho$



$\approx 10^7$ bits/s

≈ 1 bit/s

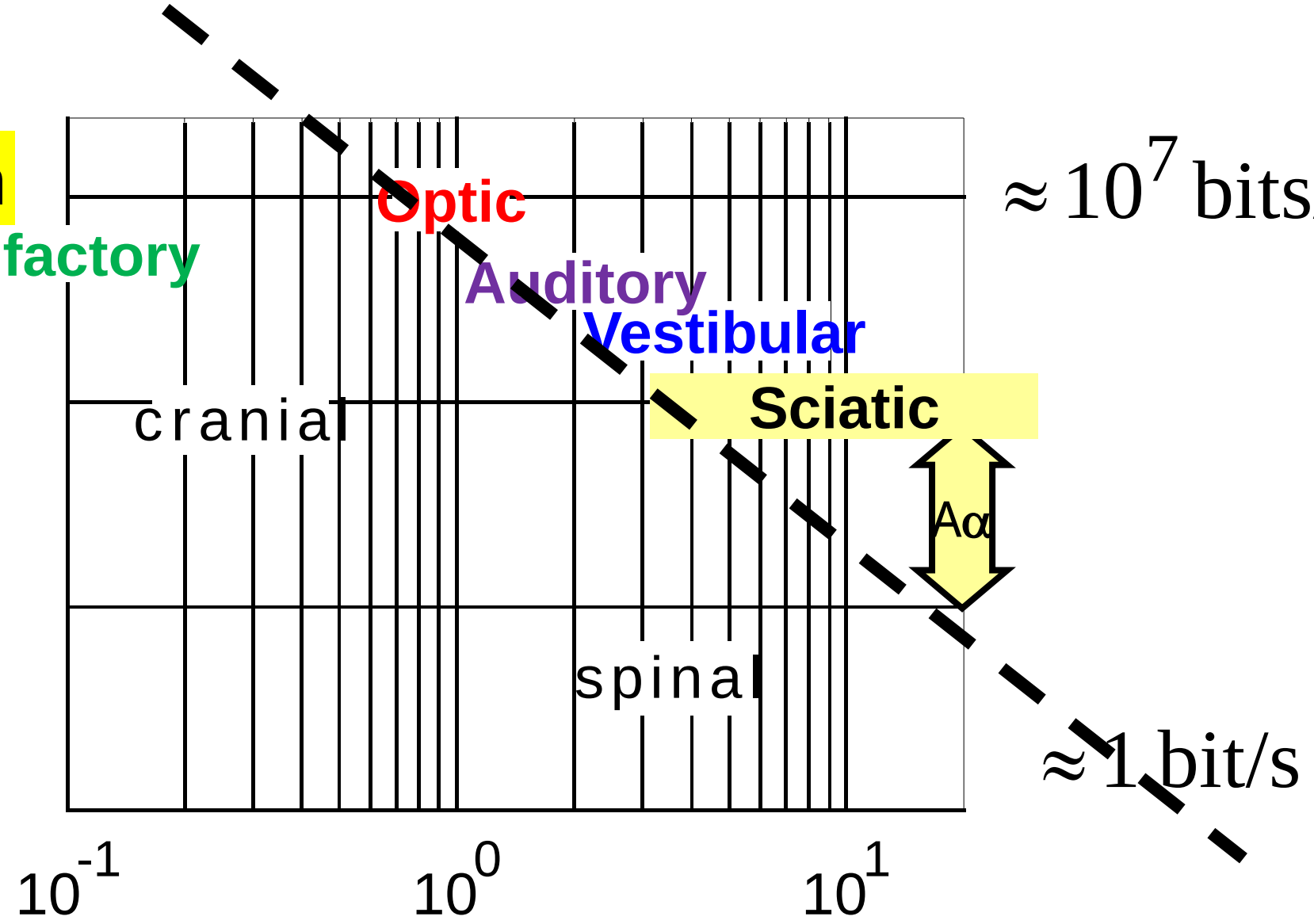
$> 10^7 \times$

$$diam = 2\rho \propto \frac{1}{T}$$

$$R = \lambda_\alpha T\,?$$

bandwidth

$$R \propto N\rho$$

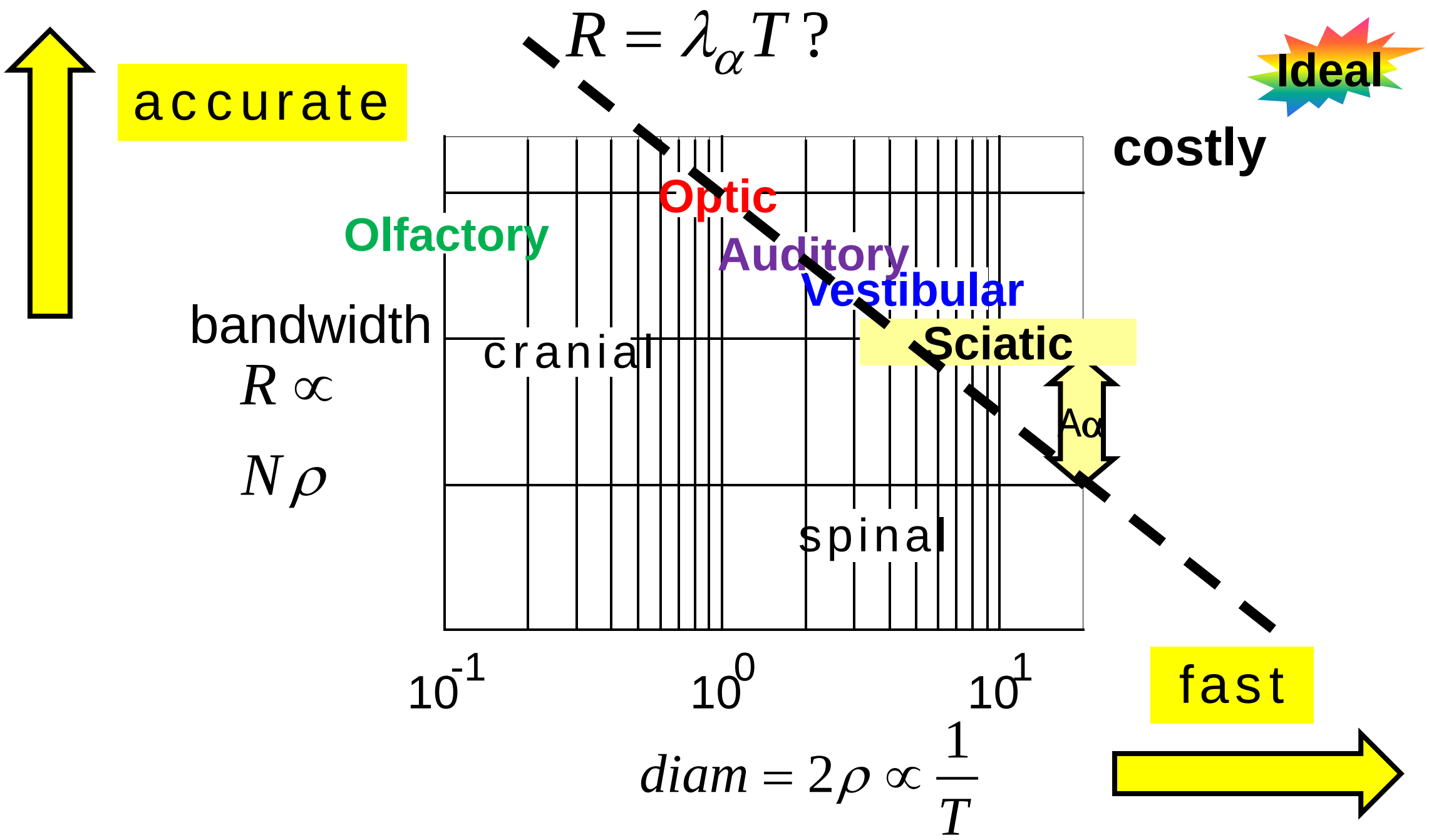


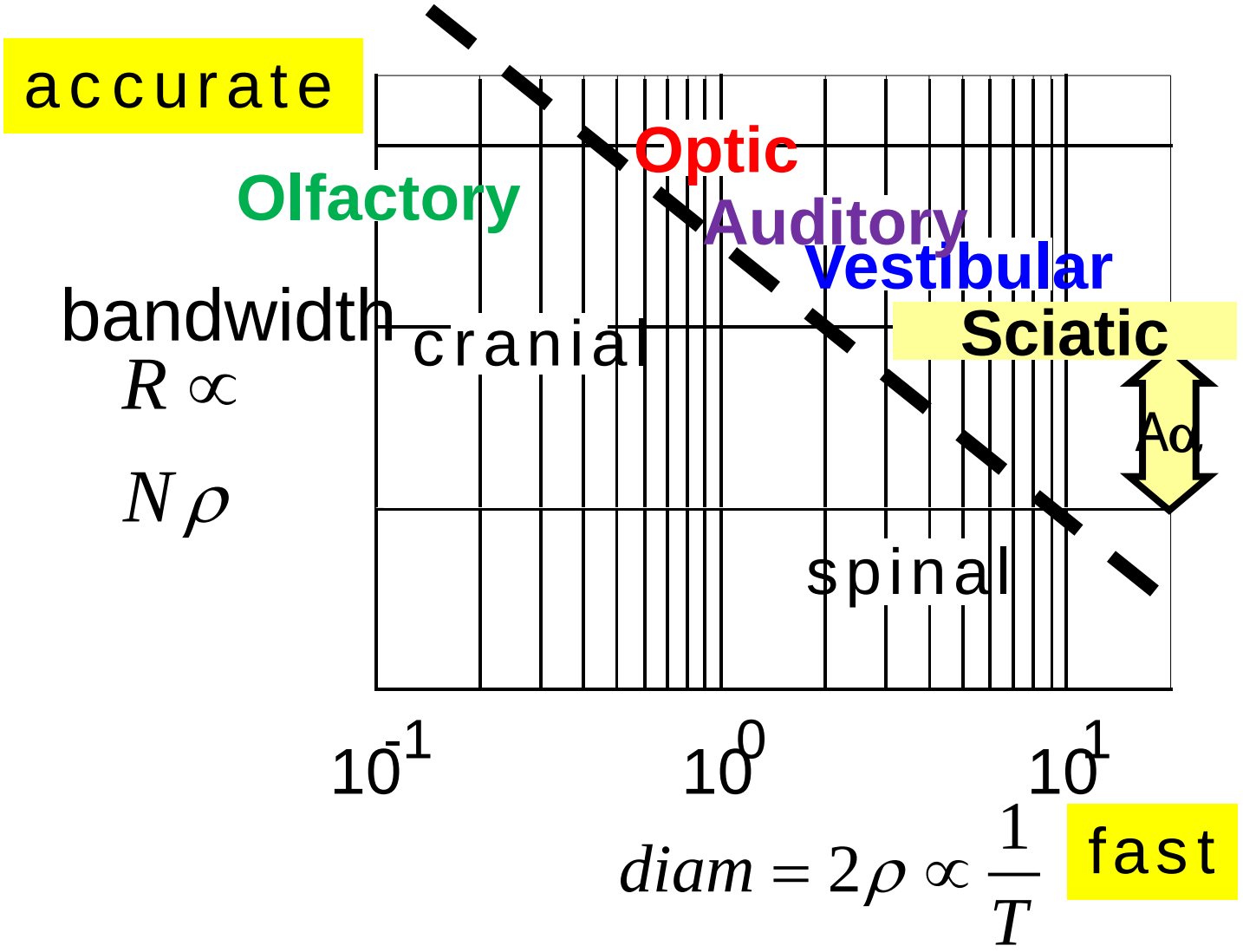
$\approx 10^{7}$ bits/s

≈ 1 bit/s

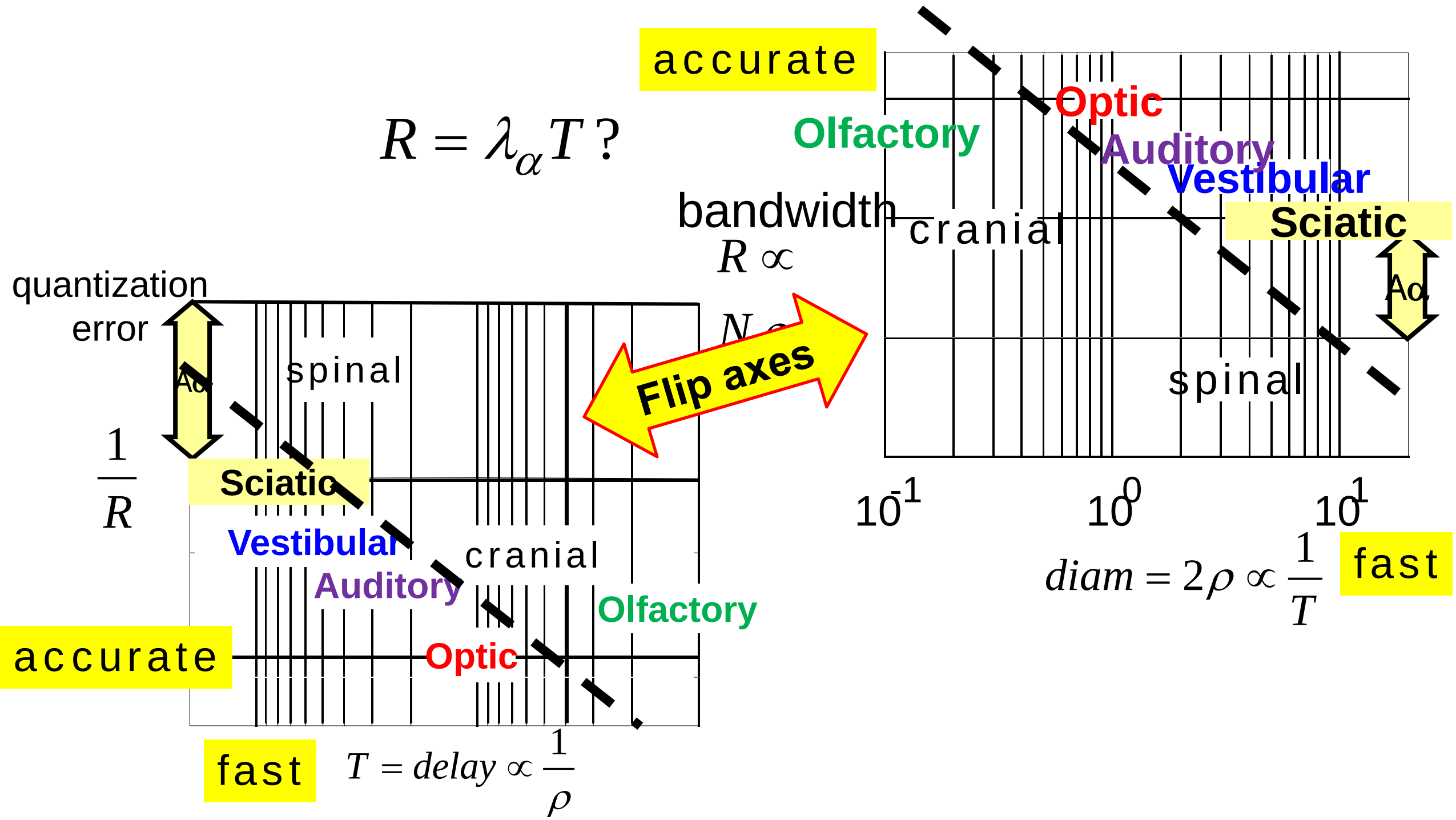
$$diam = 2\rho \propto \frac{1}{T}$$

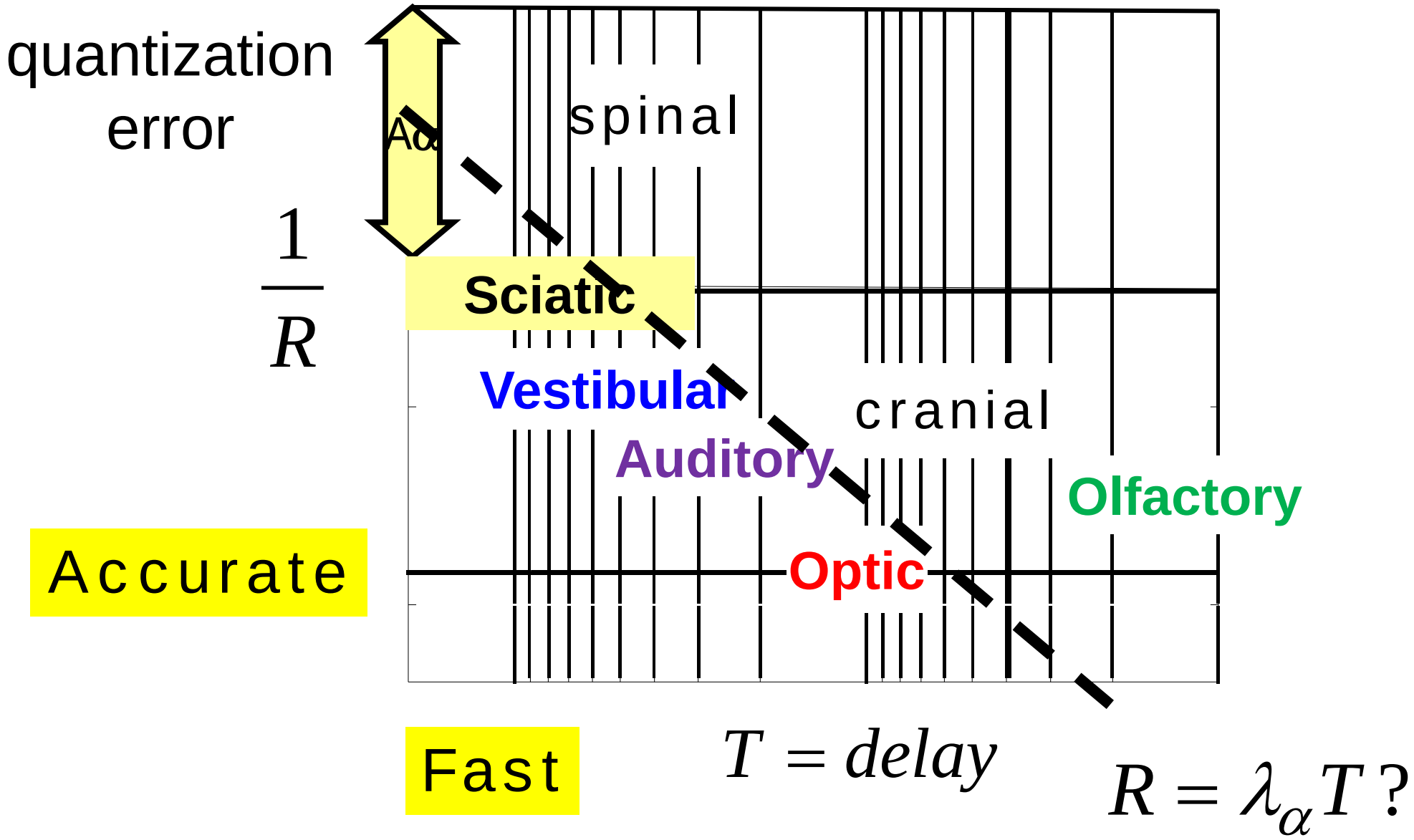
$$R = \lambda_{\alpha} T\,?$$


$R = \lambda_\alpha T$?
Ideal
accurate
costly
Optic
Olfactory
Auditory
Vestibular
bandwidth
$R \propto$
$N \rho$
cranial
Sciatic
Aα
spinal
10^{-1}
10^{0}
10^{1}
fast
$diam = 2\rho \propto \frac{1}{T}$

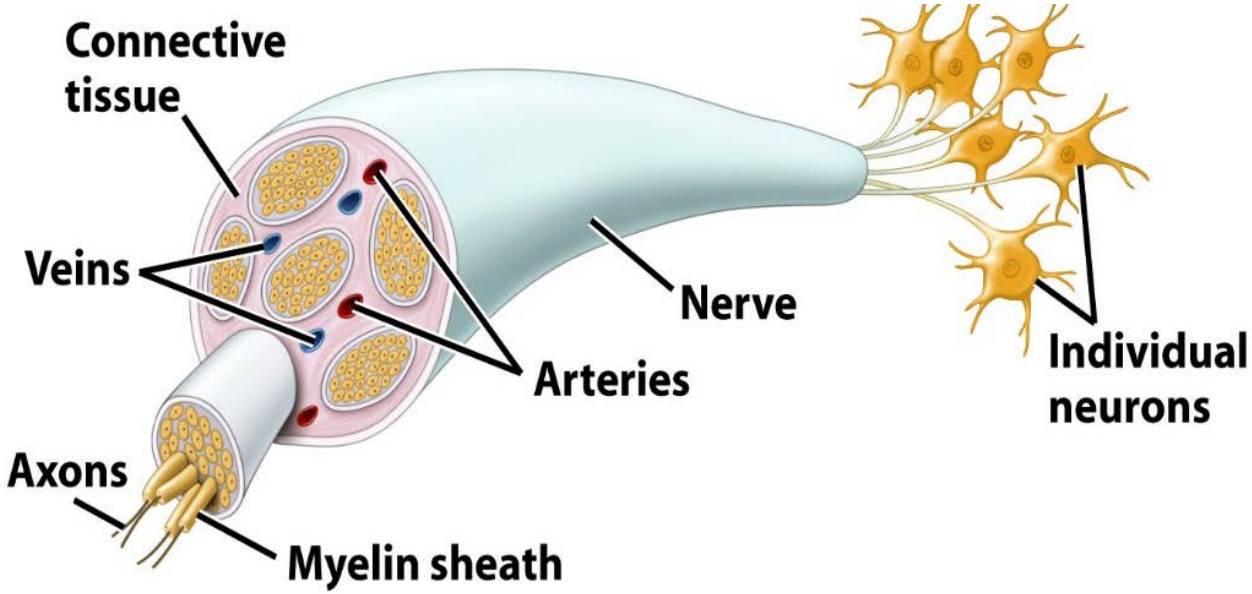

accurate
Optic
Olfactory
Auditory
Vestibular
bandwidth
$R \propto$
$N\rho$
cranial
Sciatic
Ad
spinal
10^{-1}
10^{0}
10^{1}
$diam = 2\rho \propto \frac{1}{T}$
fast

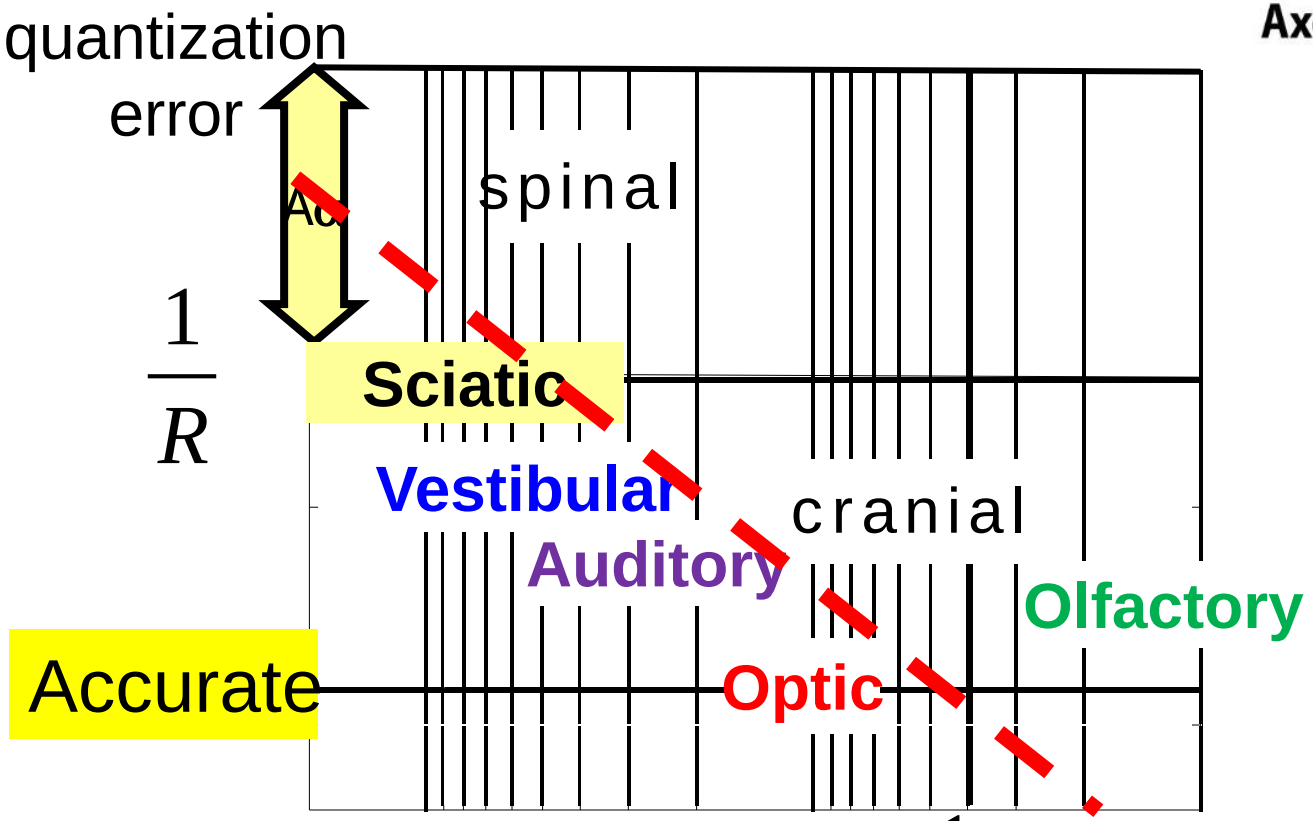
$$R = \lambda_\alpha T\,?$$




quantization error
Aα
spinal
$\frac{1}{R}$
Sciatic
Vestibular
cranial
Auditory
Olfactory
Accurate
Optic
Fast
$T = delay$
$R = \lambda_\alpha T$?

speed accuracy tradeoff

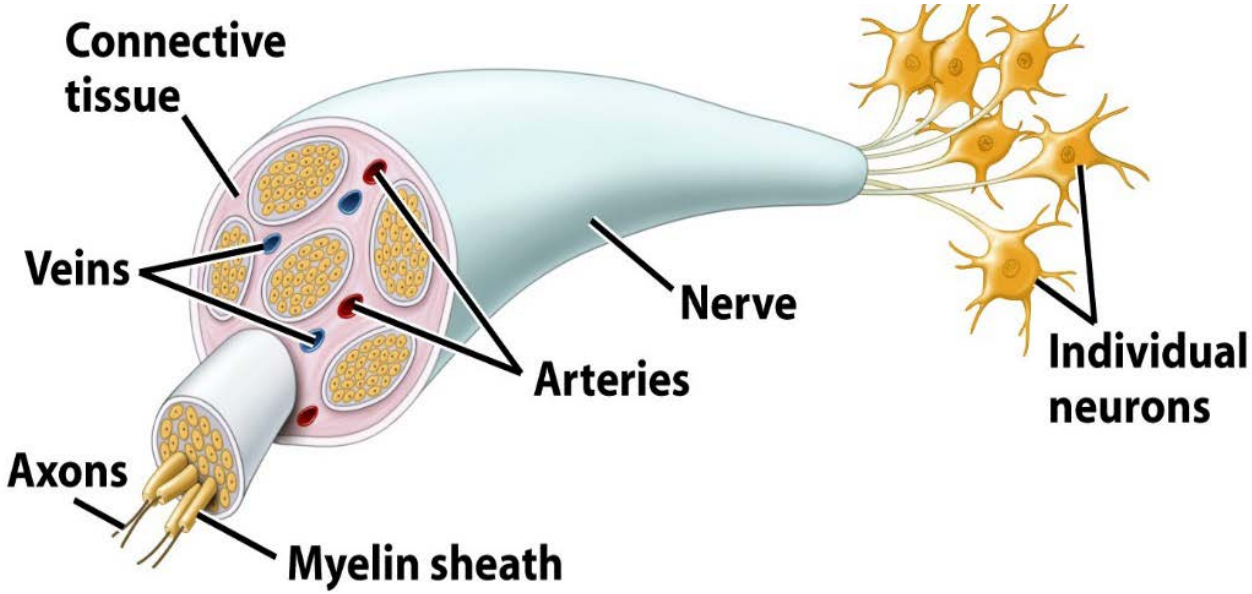


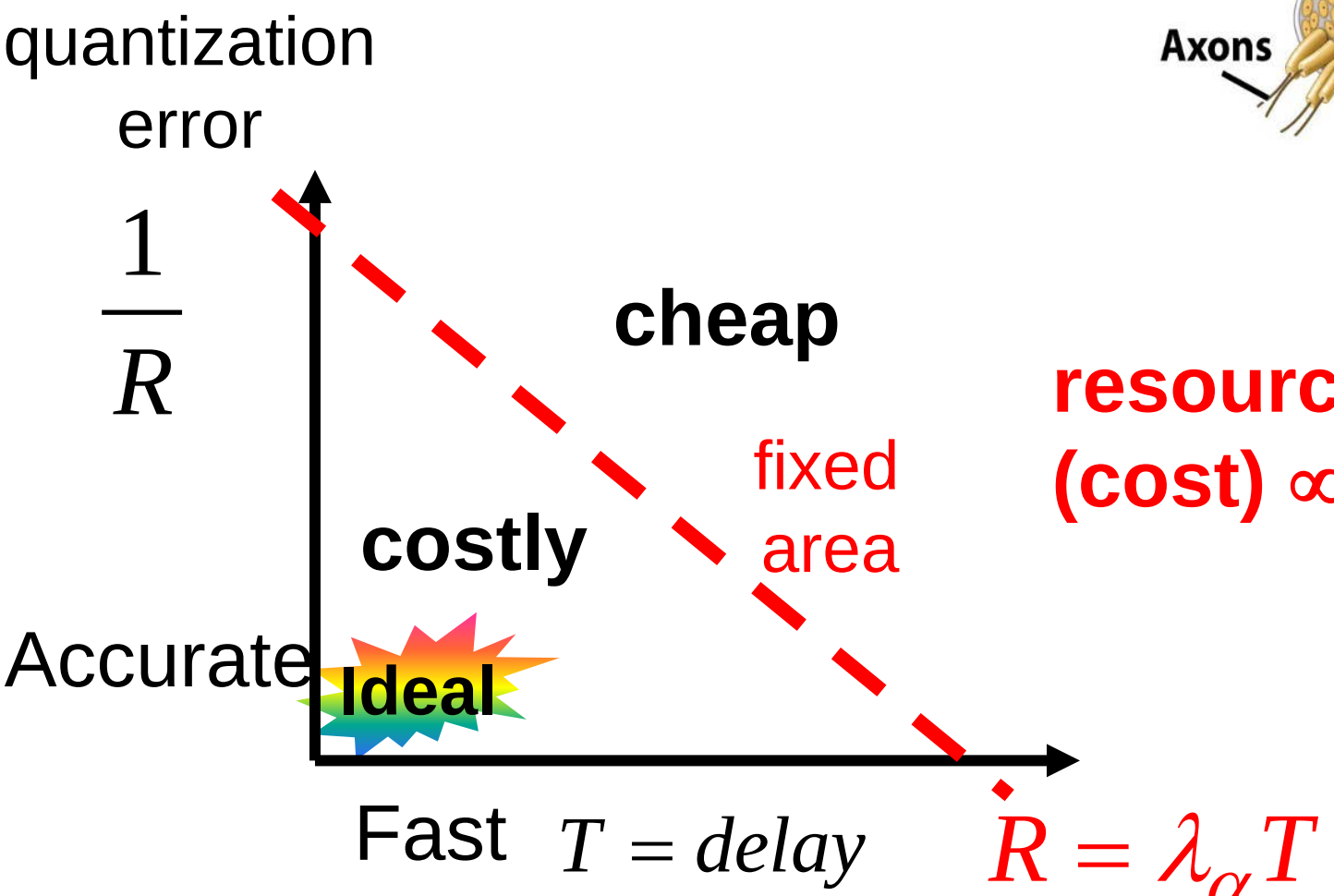


$$T = delay \propto \frac{1}{\rho}$$

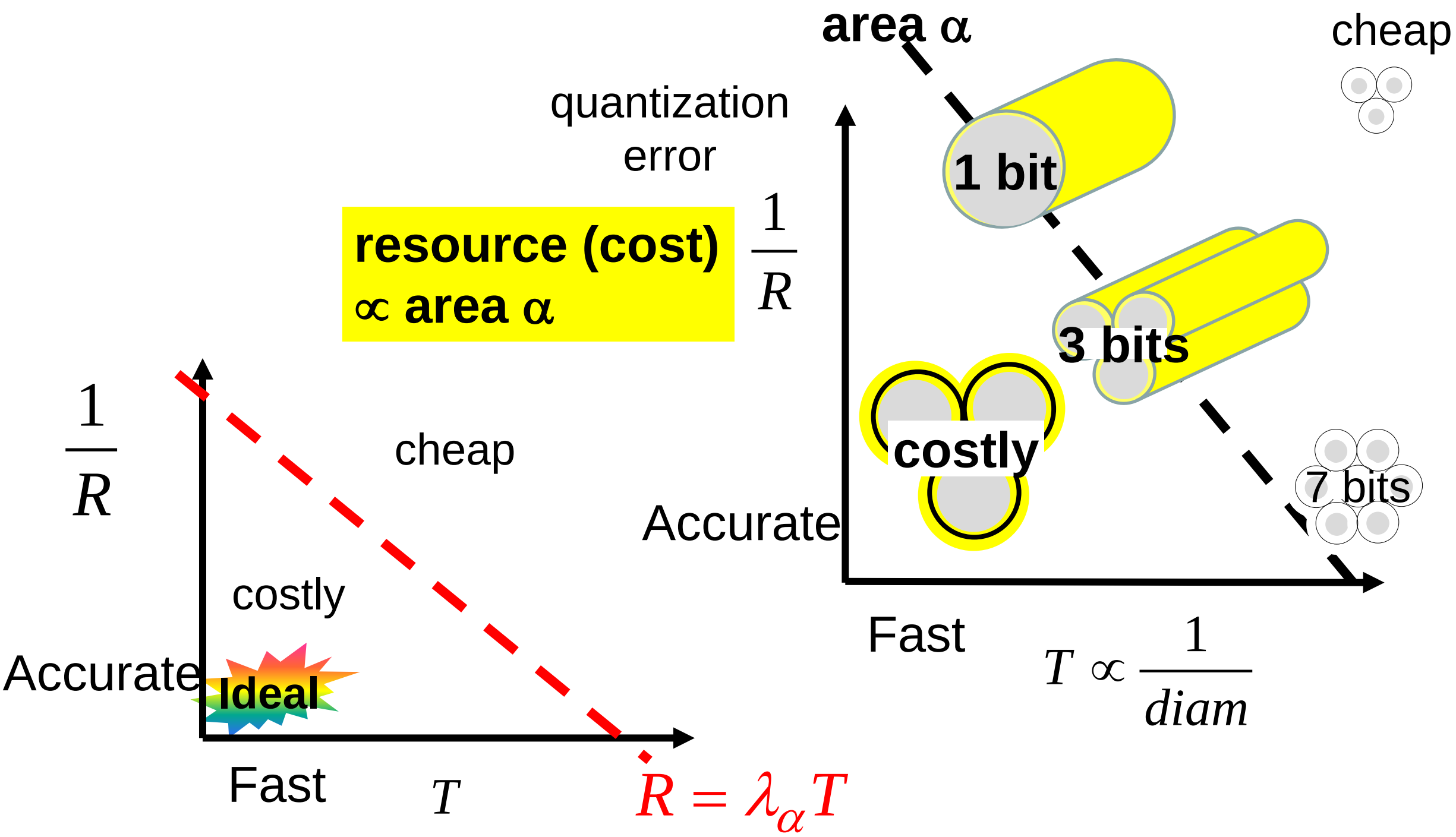
$$R = \lambda_\alpha T$$

speed accuracy tradeoff

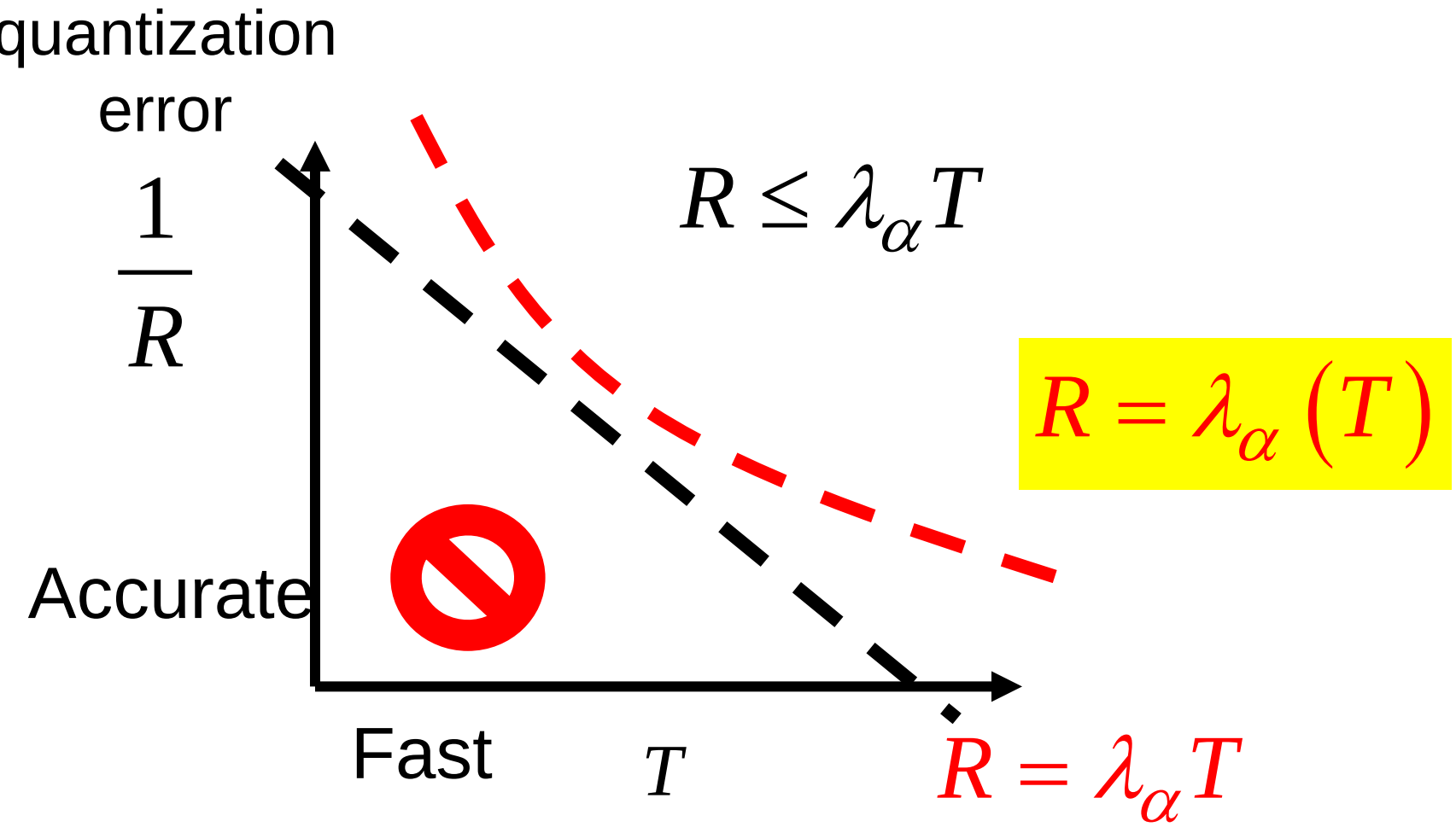


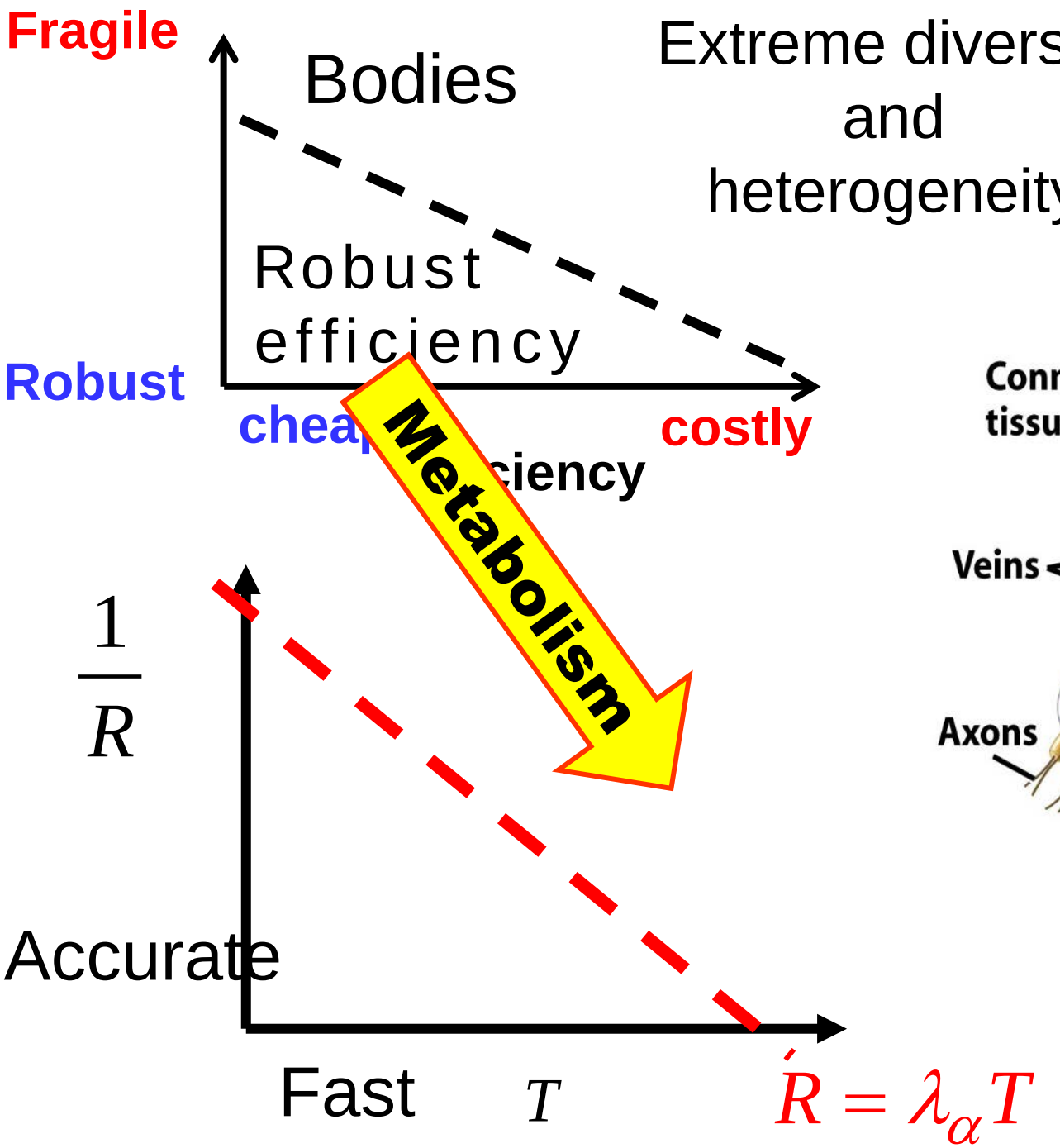


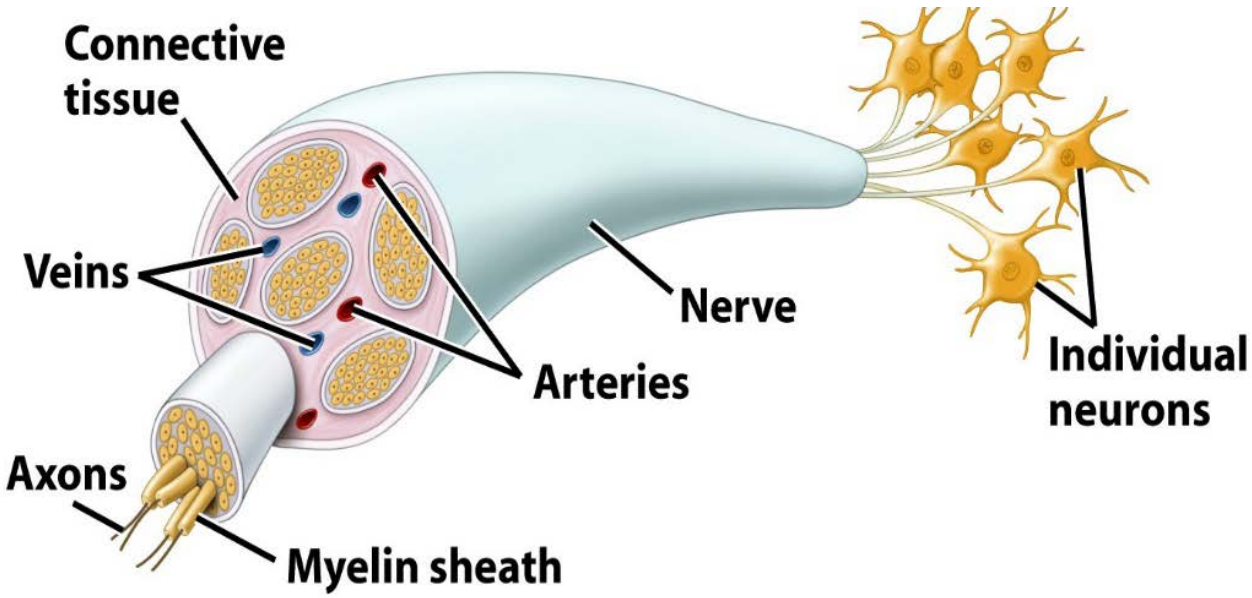
resource (cost) ∝ area α

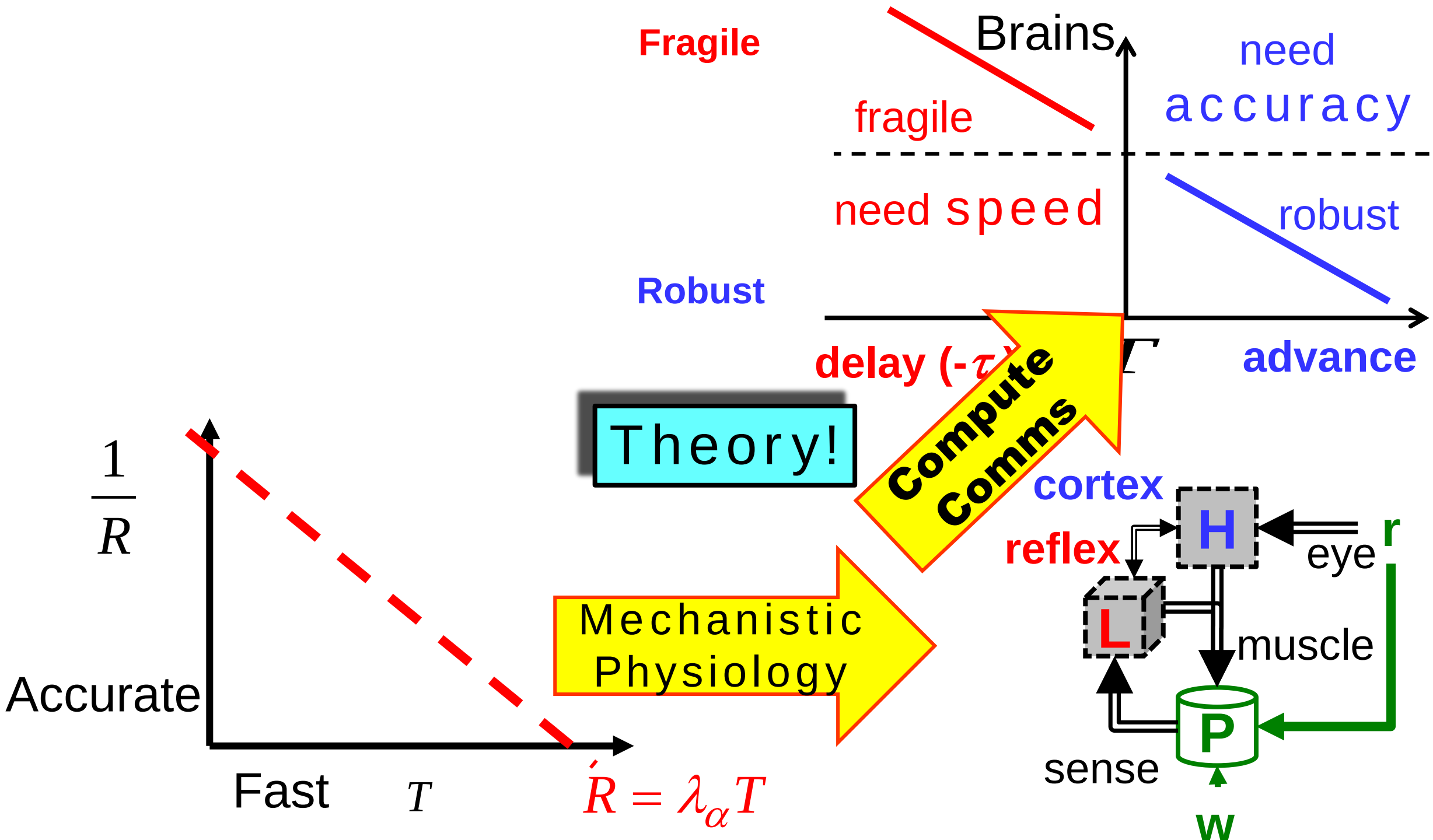

area α
cheap
quantization error
resource (cost) ∝ area α
$\frac{1}{R}$
1 bit
3 bits
costly
7 bits
Accurate
Fast
$T \propto \frac{1}{diam}$
$\frac{1}{R}$
cheap
costly
Accurate
Ideal
Fast
T
$R = \lambda_\alpha T$

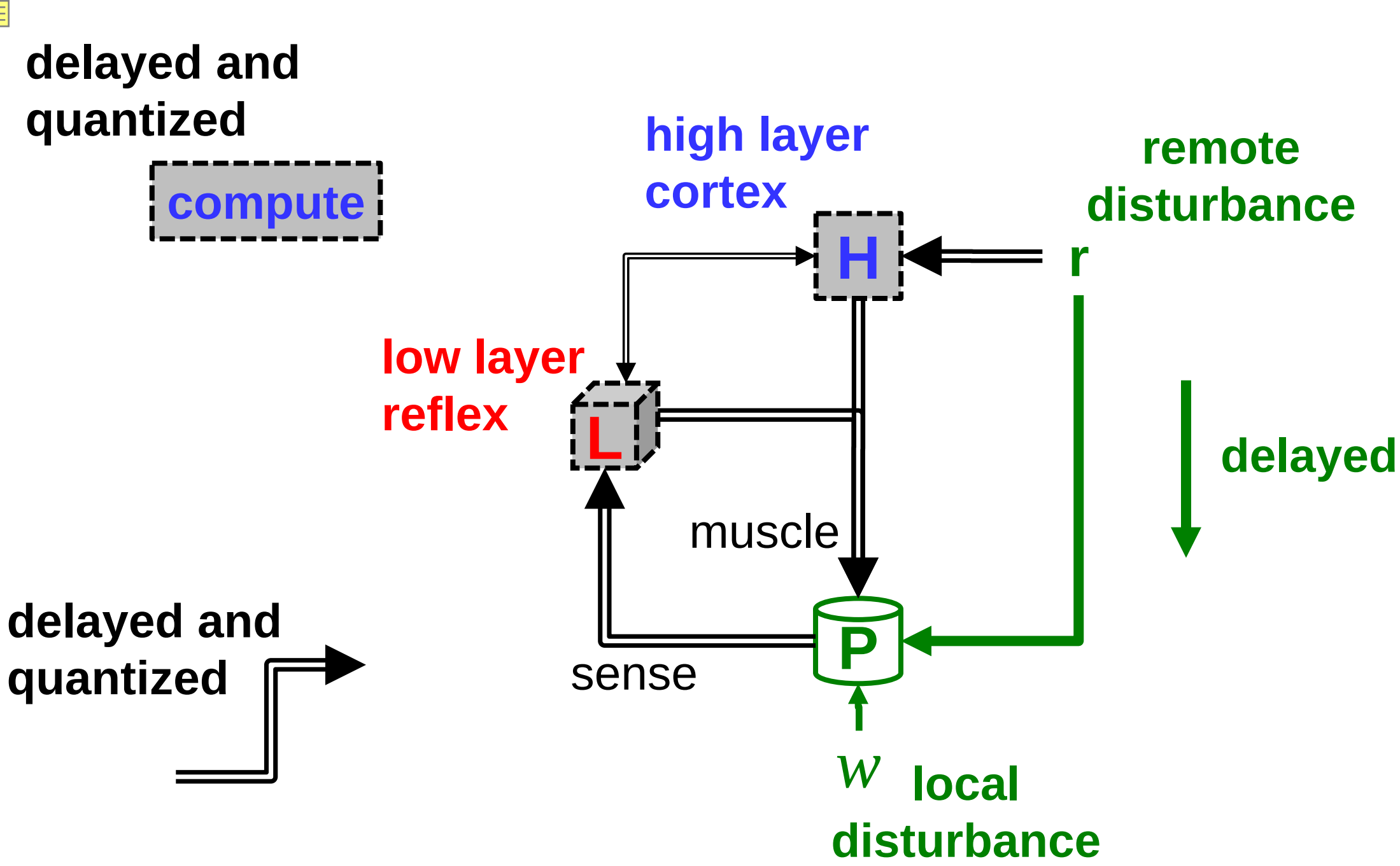
resource (cost)
$\propto$ area α




Fragile
Bodies
Extreme diversity
and
heterogeneity
Robust
efficiency
Robust
cheap
costly
ciency
Metabolism
$\frac{1}{R}$
Accurate
Fast
T
$\dot{R} = \lambda_\alpha T$

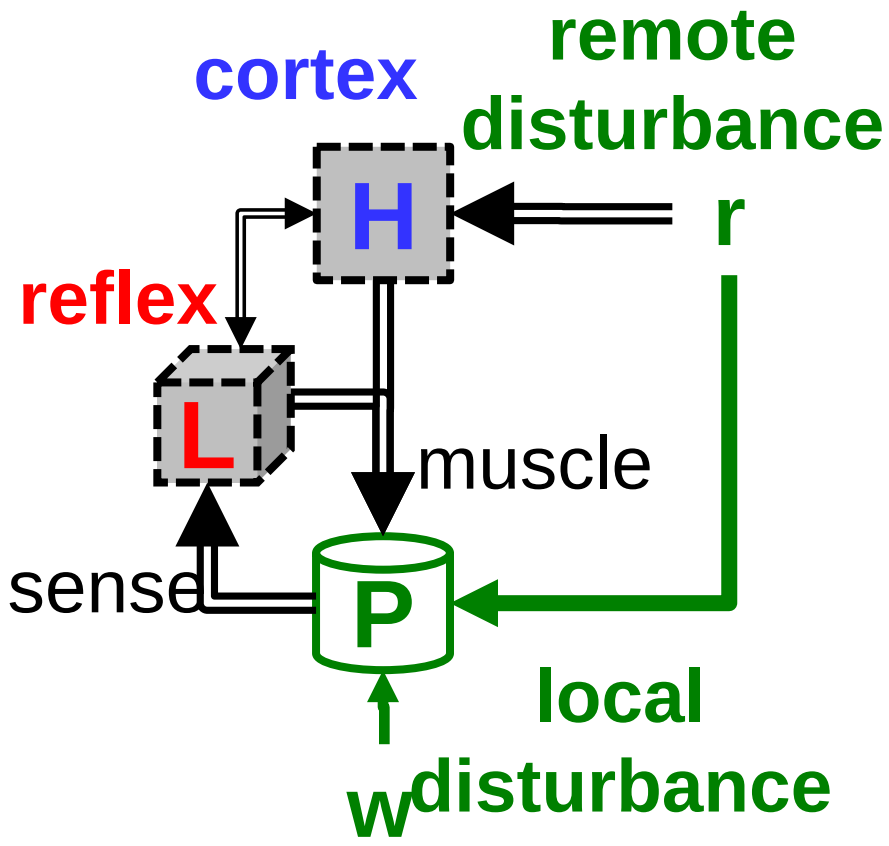

Connective tissue
Veins
Nerve
Arteries
Individual neurons
Axons
Myelin sheath


Fragile
Brains
need
accuracy
fragile
need speed
robust
Robust
delay (-τ)
τ
advance
Theory!
Compute
Comms
cortex
reflex
H
eye
r
L
muscle
P
sense
w
1/R
Accurate
Fast
T
Ṙ = λ_α T
Mechanistic
Physiology


delayed and quantized
compute
high layer cortex
remote disturbance
H
r
low layer reflex
L
delayed
muscle
delayed and quantized
P
sense
w
local disturbance

Low layers
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
unconscious
>>10Mb/sec
distributed
local

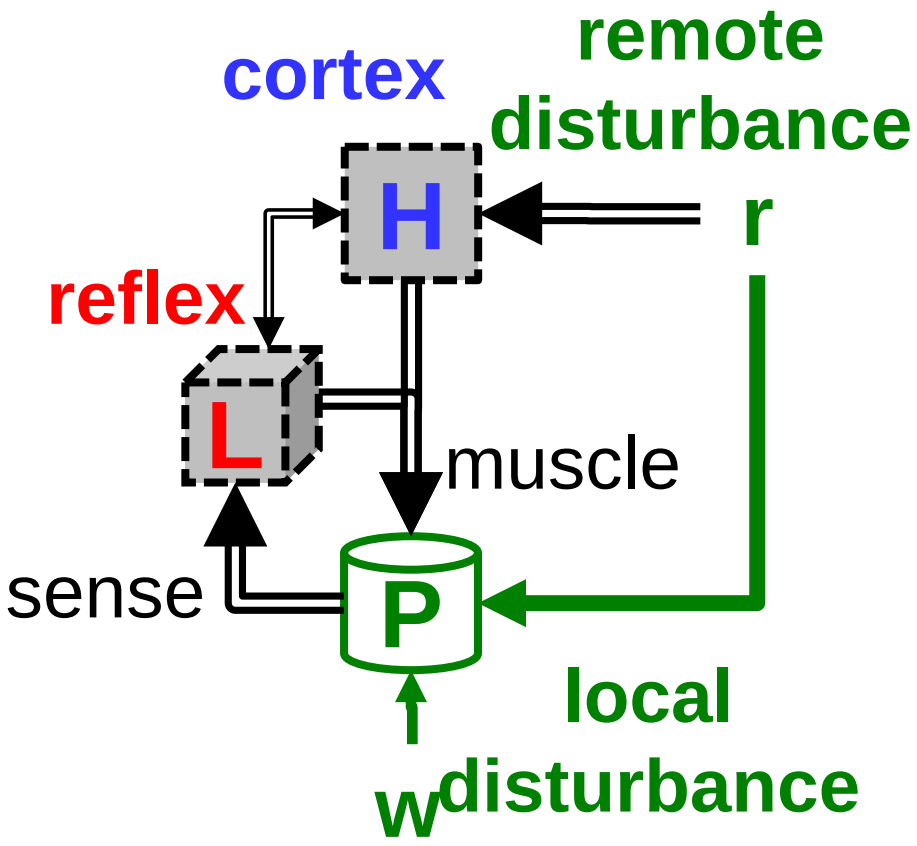
layered/recursive
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$



High layers
future
planning
advance warning
stable (virtual)
large ||r| *but*
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized
global

Low layers
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
~~unconscious~~
~~>>10Mb/sec~~
~~distributed~~
~~local~~

~~layered/recursive~~
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$

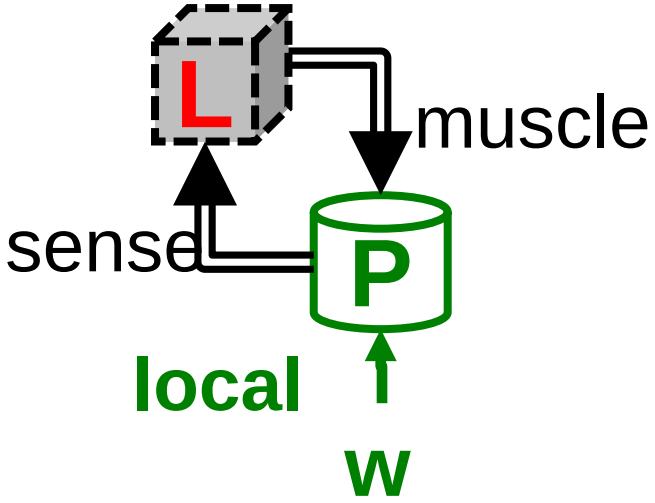


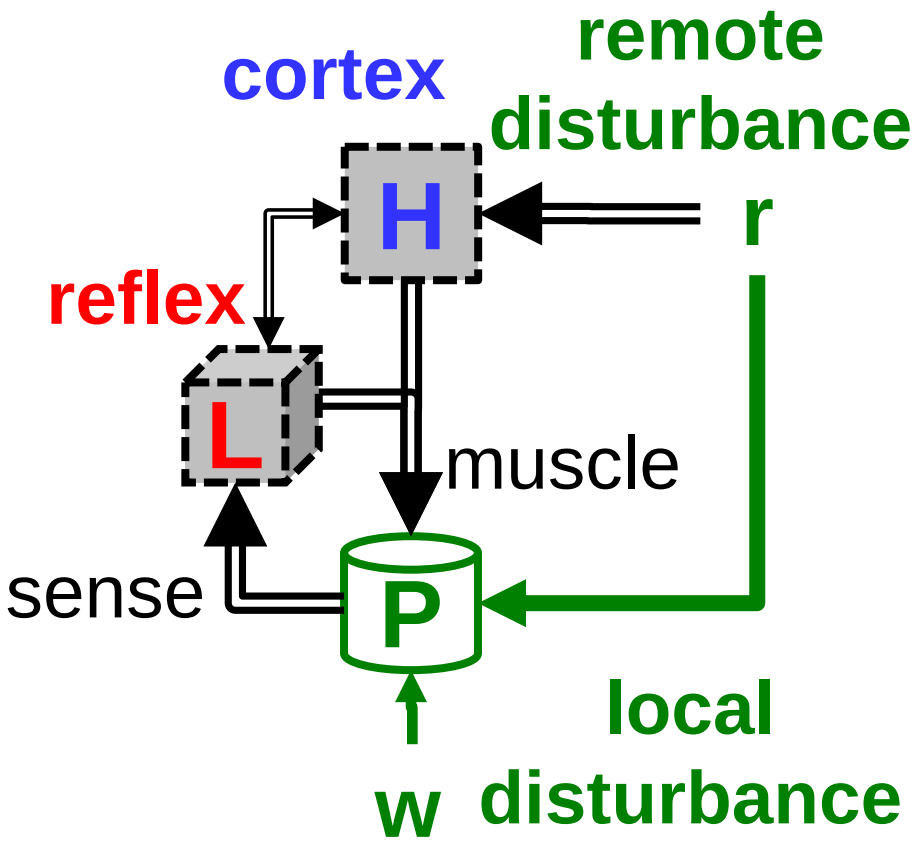
High layers
future
planning
advance warning
stable (virtual)
large ||r| *but*
small ||CL|| <<1
need accuracy
~~conscious~~
~~<100 bits/sec~~
centralized
global

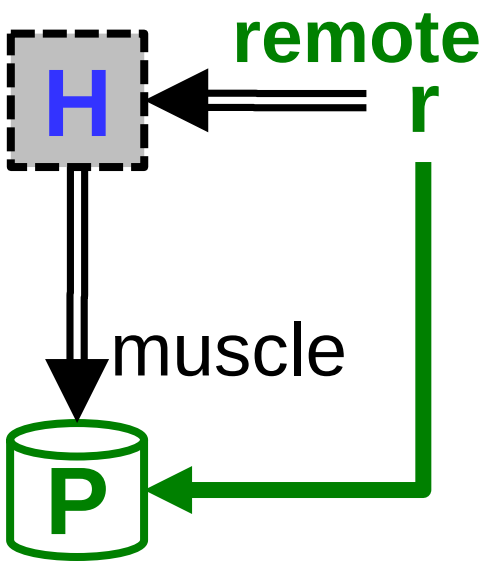
Low layers
past/reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1

dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$

High layers
future/planning
advance warning
stable (virtual)
large ||r| *but*
small ||CL|| <<1

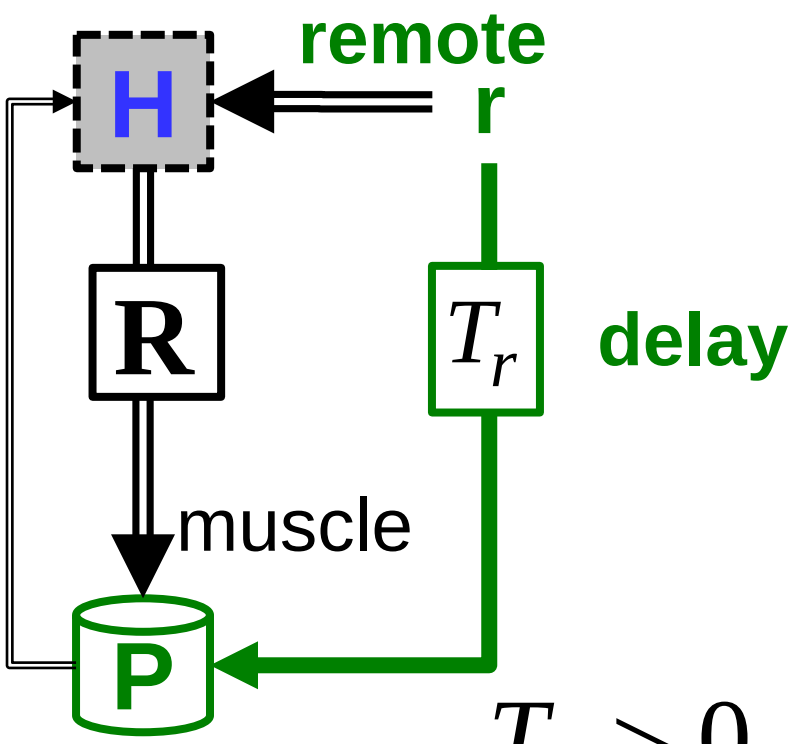






Just quantization (advanced warning)

Comms channel **R** of R bits/time



$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T_r)$$

$$T_r > 0$$

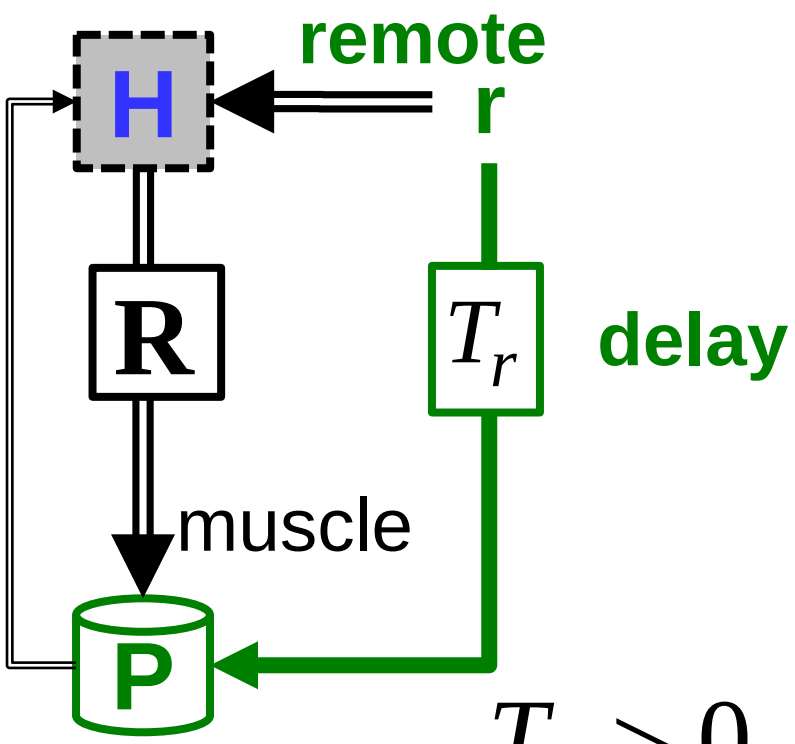
Just quantization (advanced warning)

Comms channel $\mathbf{R}$ of R bits/time

Full information

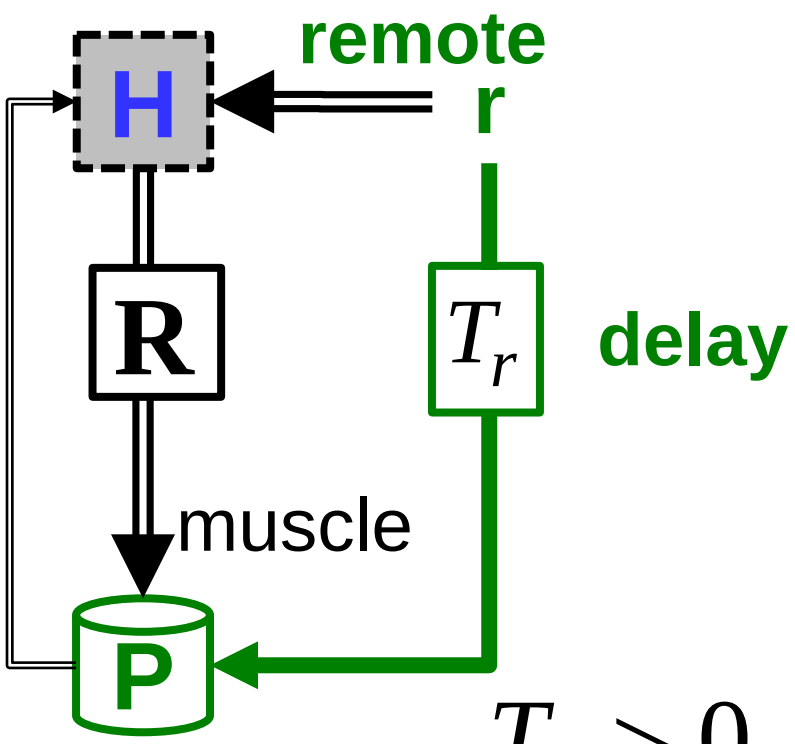
$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - \underbrace{\mathbf{R}[u(t)]}_{\text{quant}} + \underbrace{r(t-T_r)}_{\text{advance}}$$



$T_r > 0$

Just quantization (advanced warning)



Comms channel **R** of R bits/time

Full information

$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T_r)$$

quant advance

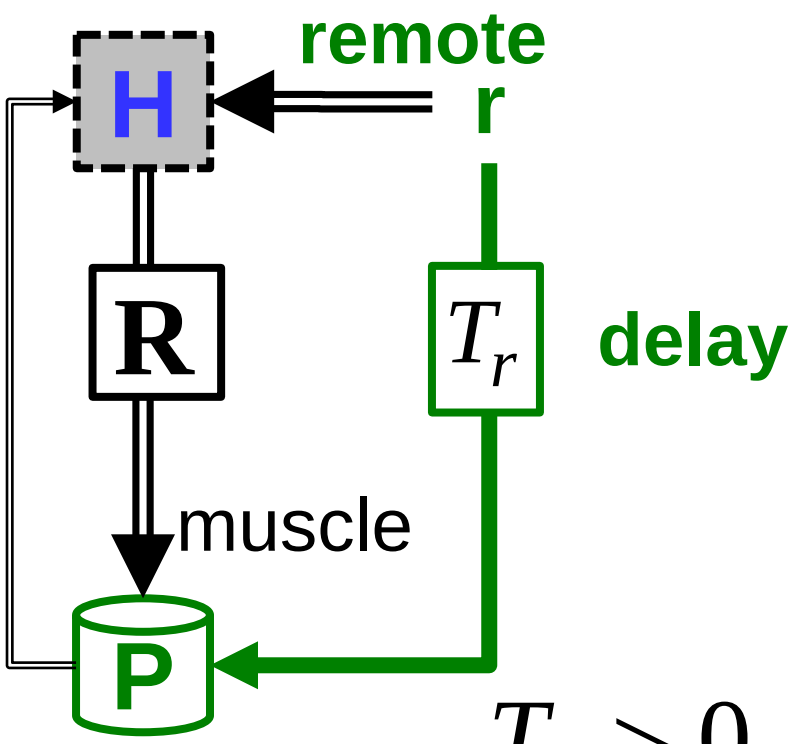
$$T_r > 0$$

$$\min_{\boxed{H}\ \boxed{\mathbf{R}}} \sup_{\|r\|_\infty \le 1} \| x \|_\infty = \left(2^R - |a|\right)^{-1}$$

$$\| x \|_\infty \sup_t |x(t)|$$

Just quantization (advanced warning)

R bits/time



$T_r > 0$

Full information

$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

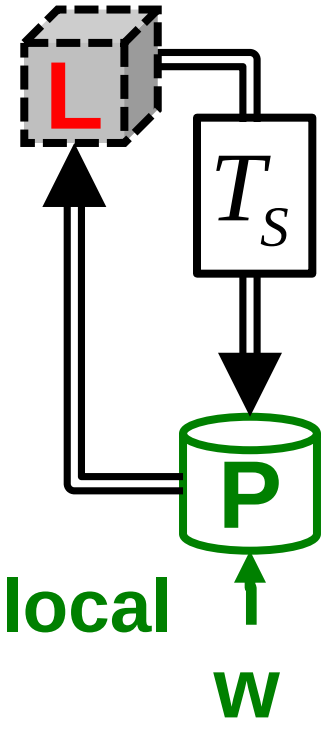
$$x(t+1) = ax(t) - \mathbf{R}\left[u(t)\right] + r(t - T_r)$$

quant advance

$$\min_{\boxed{\mathbf{H}}\ \boxed{\mathbf{R}}} \sup_{\|r\|_\infty \le 1} \|x\|_\infty = \left(2^R - |a|\right)^{-1}$$

R bits $\rightarrow$ 2^R levels

Just delay



Signaling delay ($T_S > 0$)

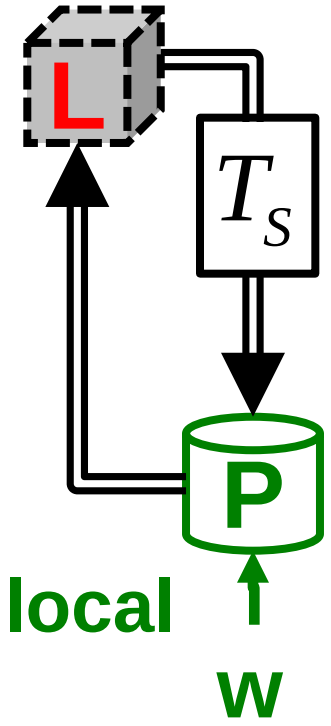
Full information

$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t - T_S) + w(t)$$

delay

Just delay



Signaling delay ($T_S > 0$)

Full information

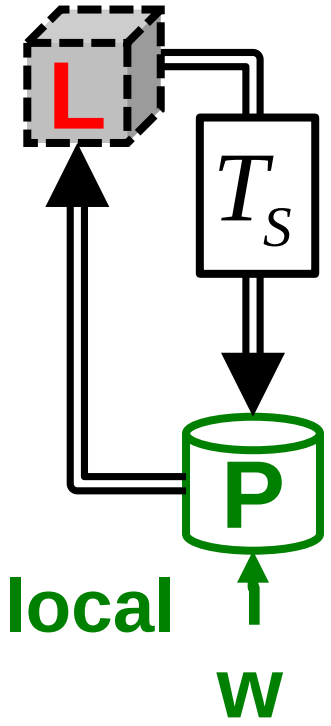
$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t - T_S) + w(t)$$

delay

$$\min_{L} \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \sum_{i=1}^{T_S} \left| a^{i-1} \right|$$

Just delay



$T_S > 0$

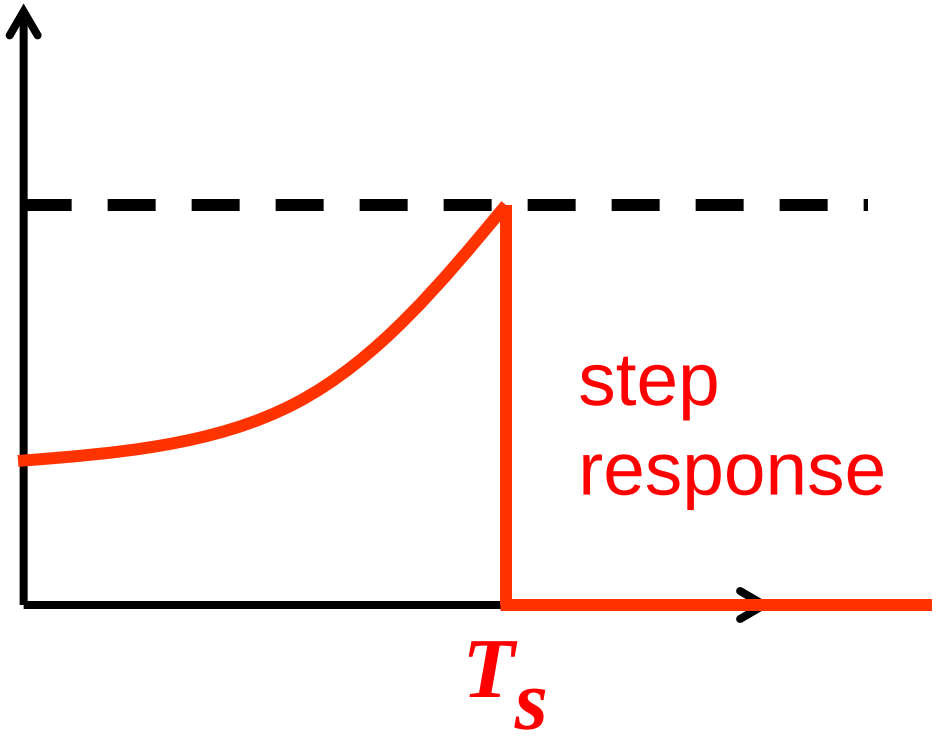
Full information

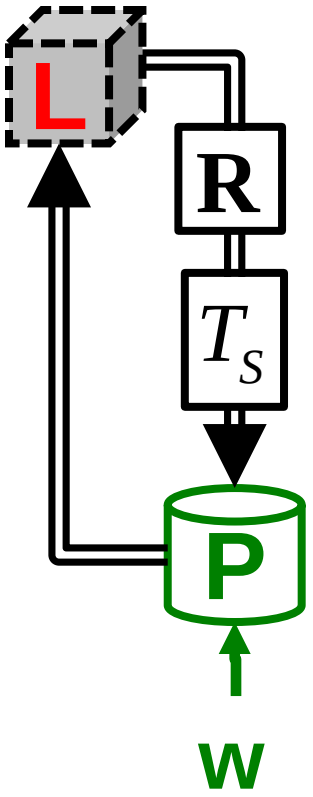
$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - u(t - T_S) + w(t)$$

delay

$$\min_{L} \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \sum_{i=1}^{T_S} \left| a^{i-1} \right|$$





R = R bits/time

Delay ($T_S > 0$)

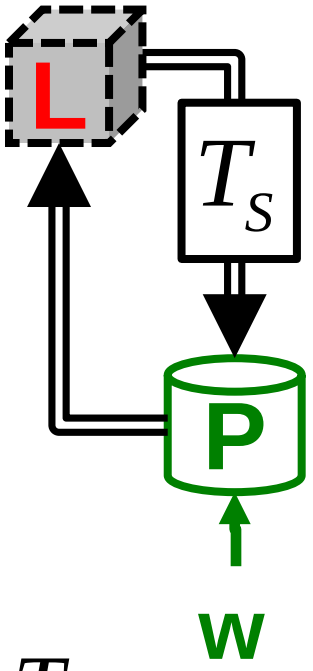
Delay + quantization

Full information

$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

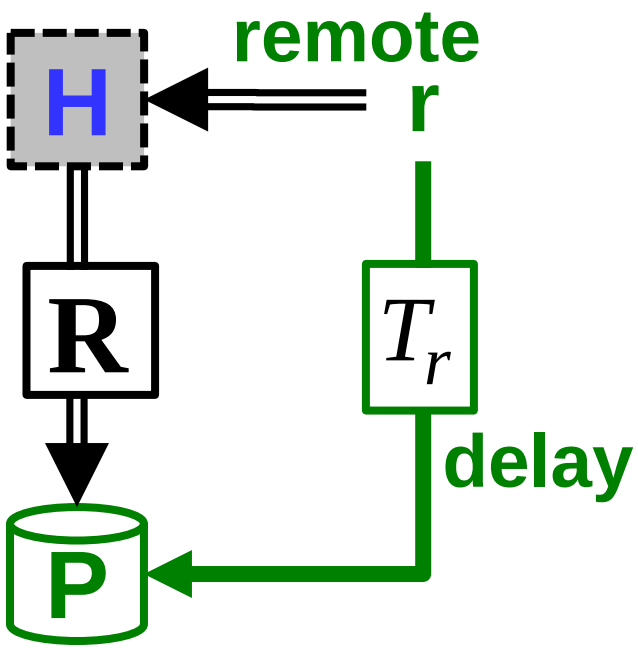
$$x(t+1) = ax(t) - \mathbf{R}\left[u(t - T_S)\right] + w(t)$$

quant delay



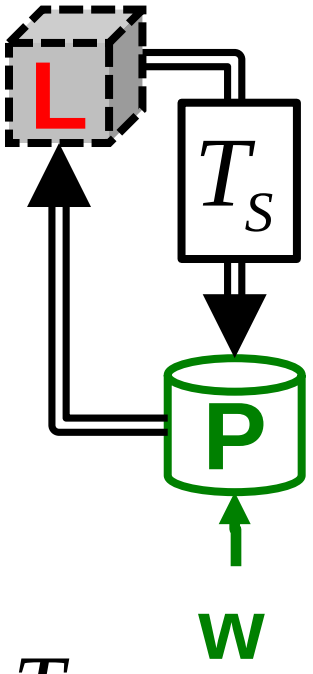
$$\sum_{i=1}^{T_S} \left| a^{i-1} \right|$$

delay



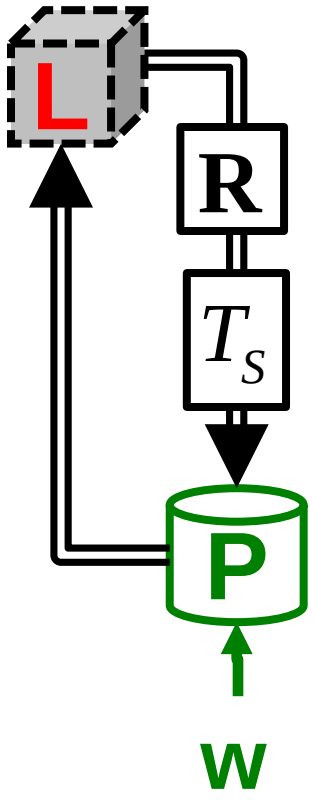
$$\left(2^R - |a|\right)^{-1}$$

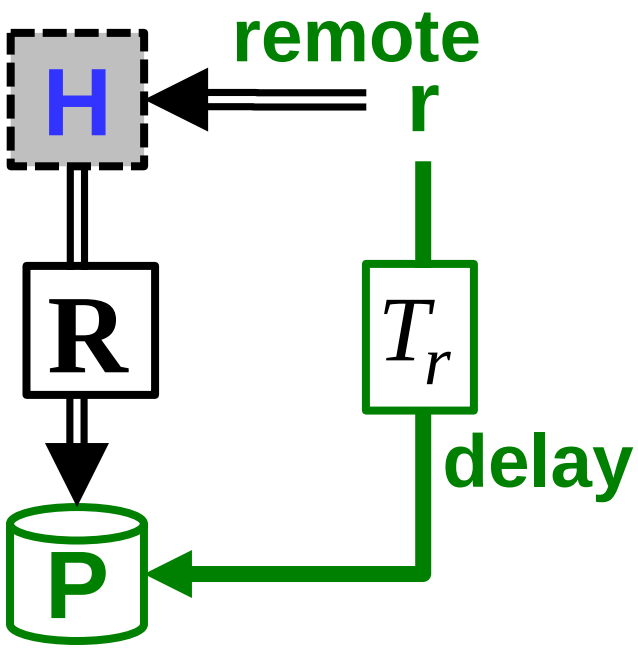
quant



$$\sum_{i=1}^{T_S} \left| a^{i-1} \right|$$

delay



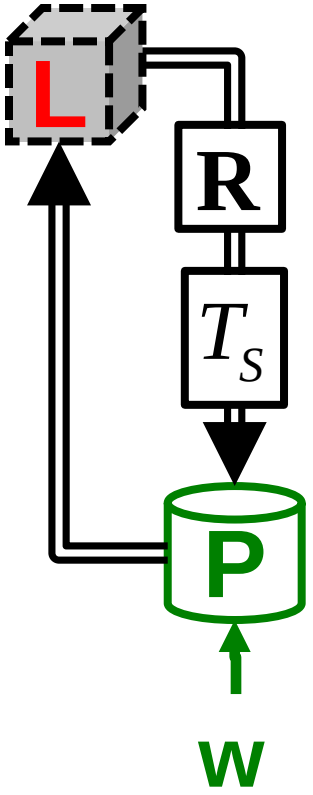


$$\left(2^R - |a|\right)^{-1}$$

quant

$$\sum_{i=1}^{T_S} \left| a^{i-1} \right| \quad + \quad \left| a^{T_S} \right| \left(2^R - |a|\right)^{-1}$$

delay **delay+quant**



R = R bits/time

Delay ($T_S > 0$)

Delay + quantization

Full information

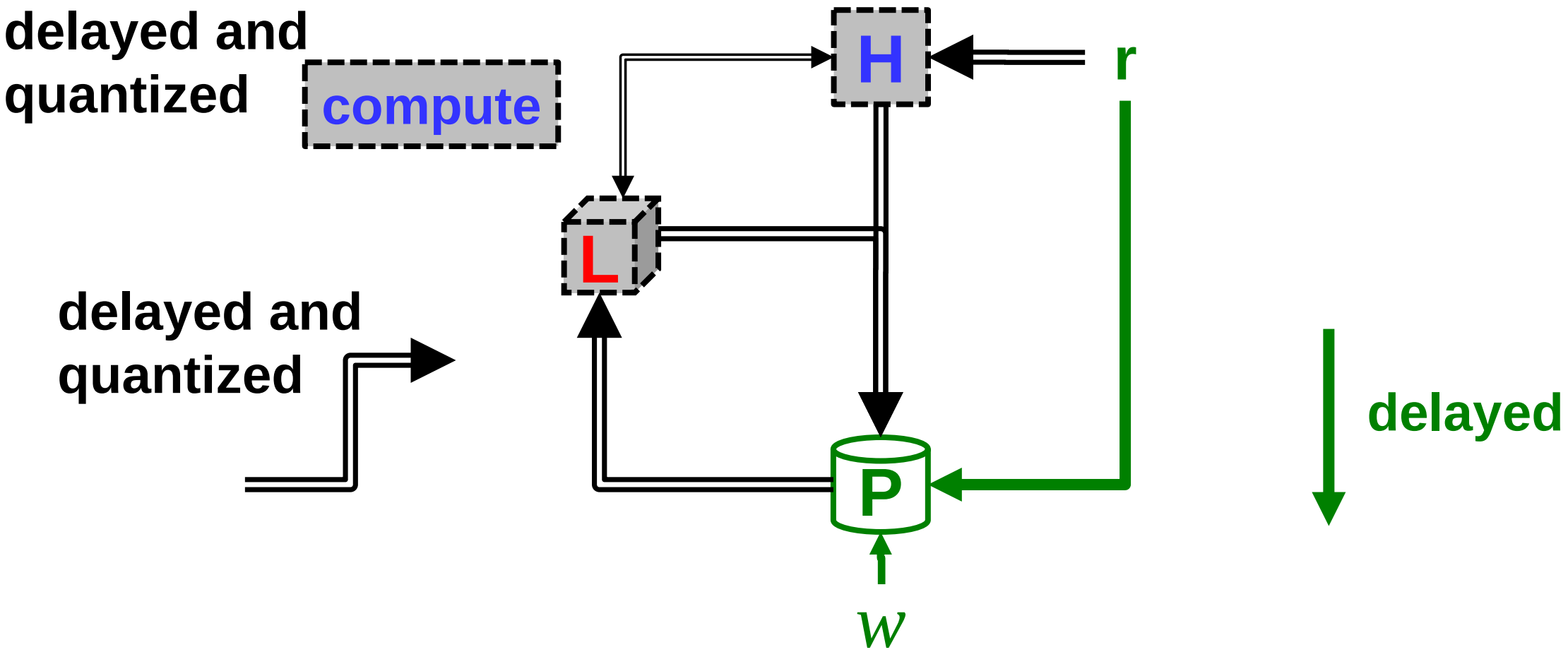
$$u(t) = f(x(0:t), r(0:t), u(0:t-1))$$

$$x(t+1) = ax(t) - \mathbf{R}\left[u(t - T_S)\right] + w(t)$$

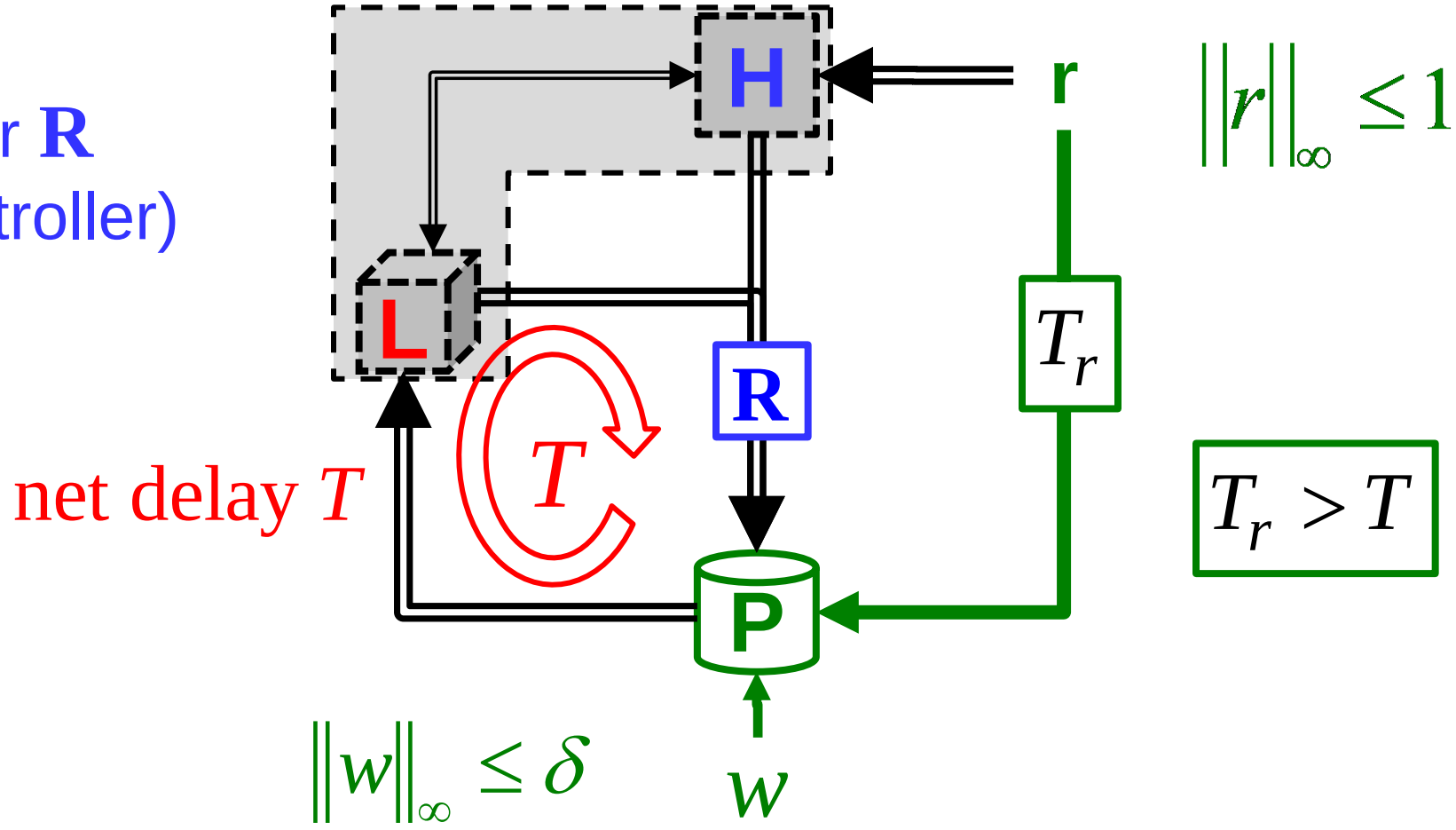
quant delay

$$\min_{\boxed{L}\ \boxed{R}} \sup_{\|w\|_\infty \le 1} \| x \|_\infty = \underbrace{\sum_{i=1}^{T_S} \left| a^{i-1} \right|}_{\text{delay}} + \underbrace{\left| a^{T_S} \right| \left(2^R - |a| \right)^{-1}}_{\text{delay+quant}}$$

G. Nair et al., (2007) Feedback control under data rate constraints: An overview


delayed and
quantized
compute
H
r
L
delayed and
quantized
P
delayed
w

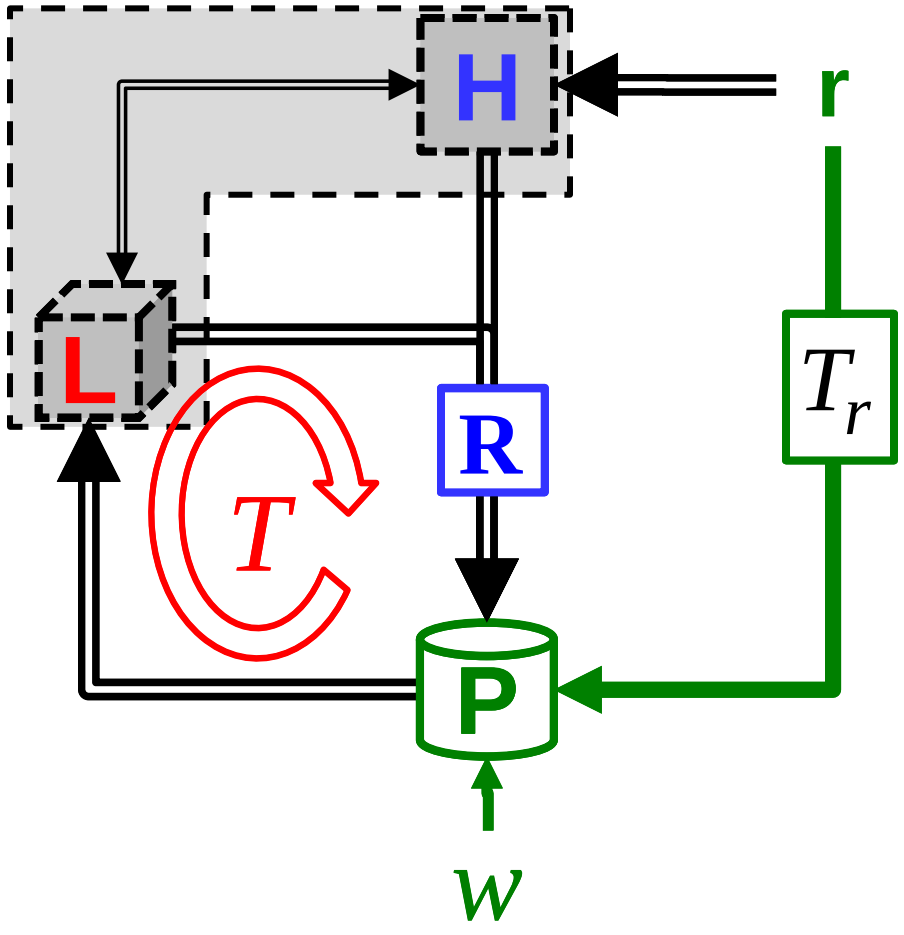
One quantizer **R**
(and one controller)



$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right| + \left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right) + \left(2^{R}-|a|\right)^{-1}$$

delay **delay+quant** **quant**

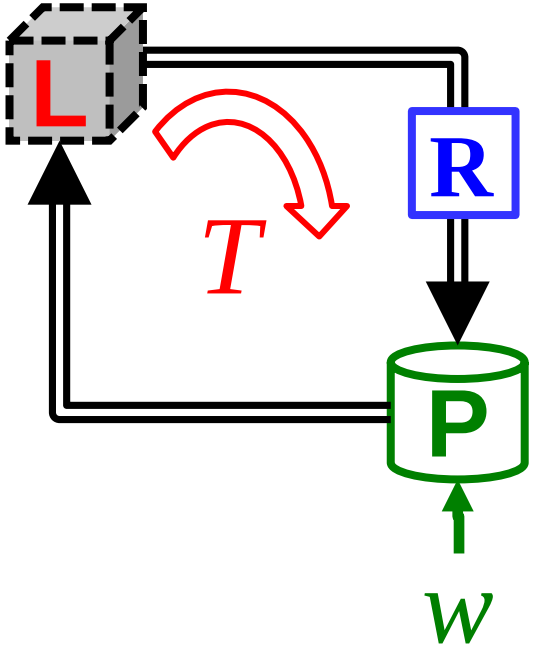
One quantizer
(one controller)



$$\delta\left(\sum_{i=1}^{T}\left|a^{i-1}\right|+\left|a^{T}\right|\left(2^{R}-|a|\right)^{-1}\right)\quad+\left(2^{R}-|a|\right)^{-1}$$

delay **delay+quant** **quant**

$$a = 1 \rightarrow T + \left(2^R - 1\right)^{-1}$$



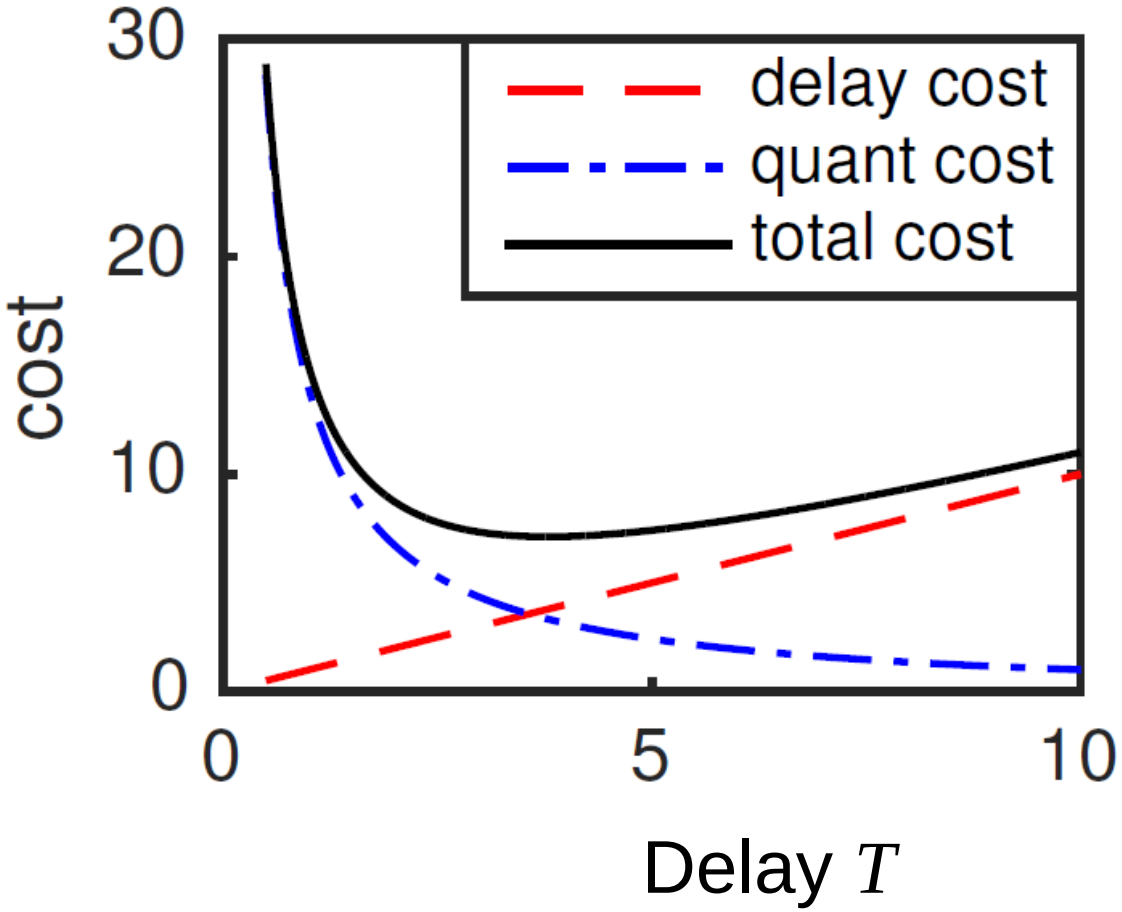
special case

$$\left(\sum_{i=1}^{T} \left|a^{i-1}\right| + \left|a^T\right| \left(2^R - |a|\right)^{-1}\right)$$

Nerve model

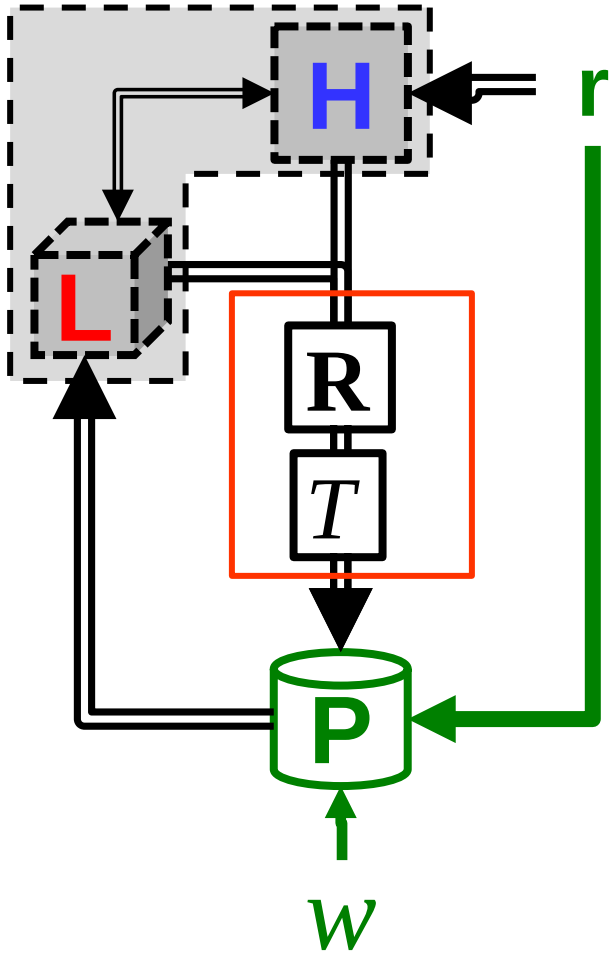
$$R = \lambda T \rightarrow T + \left(2^{\lambda T} - 1\right)^{-1}$$

delay **quant**



One channel with quantizer **R** and delay T

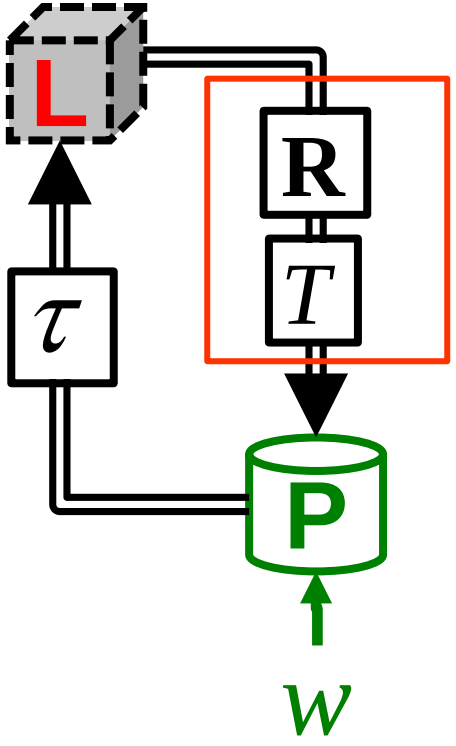
$$R = \lambda T \rightarrow \underbrace{\max\left(T-\tau,0\right)}_{\textbf{delay}} + \underbrace{\left(2^{\lambda T}-1\right)^{-1}}_{\textbf{quant}}$$

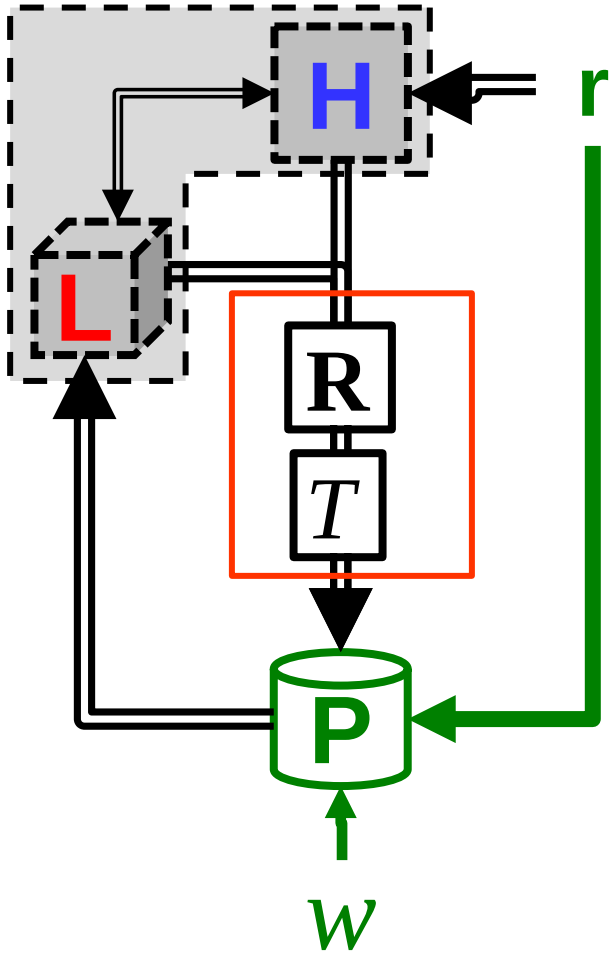


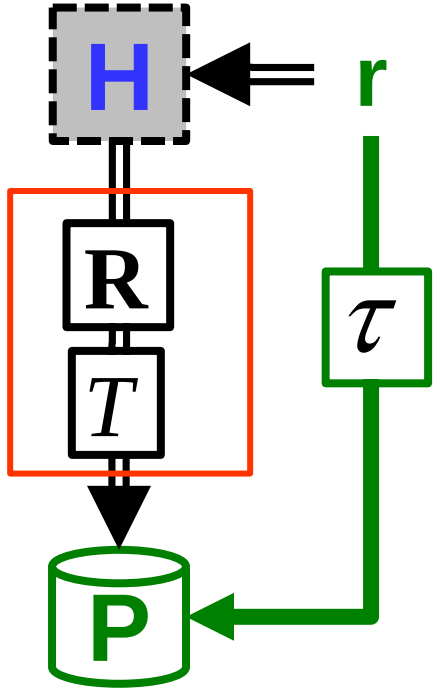
The rest of the system has advance (or delay) = τ

One channel with quantizer **R** and delay T

$$R = \lambda T \rightarrow \underbrace{\max\left(T-\tau,0\right)}_{\textbf{delay}} + \underbrace{\left(2^{\lambda T}-1\right)^{-1}}_{\textbf{quant}}$$



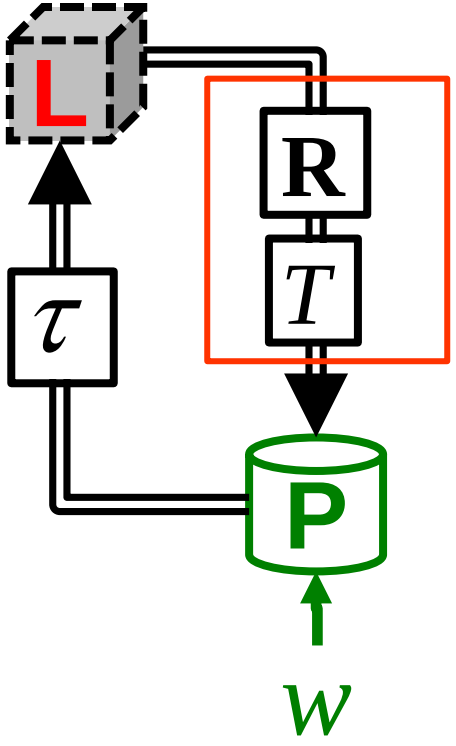




The rest of the system has advance (or delay) = τ

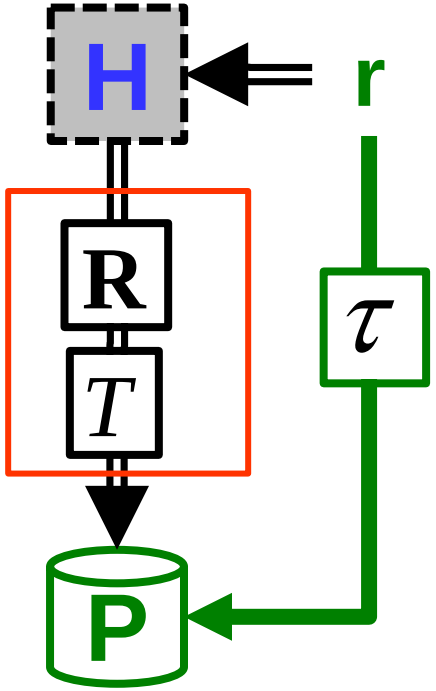
One channel with quantizer **R** and delay T

$$R = \lambda T \rightarrow \underbrace{\max\left(T-\tau,0\right)}_{\textbf{delay}} + \underbrace{\left(2^{\lambda T}-1\right)^{-1}}_{\textbf{quant}}$$



delay (-τ)

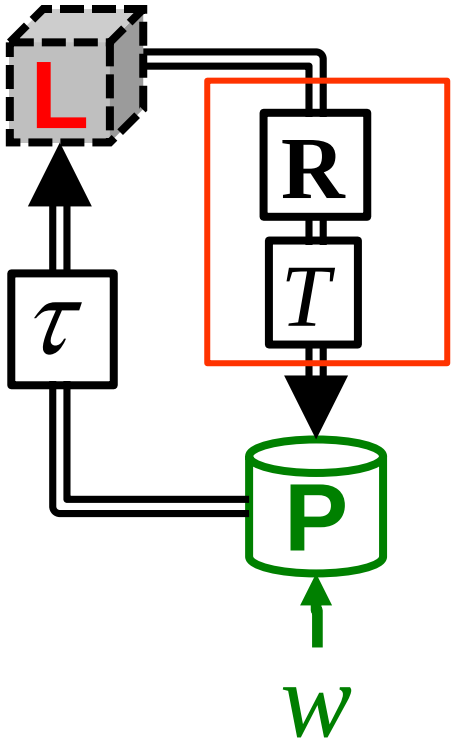
advance



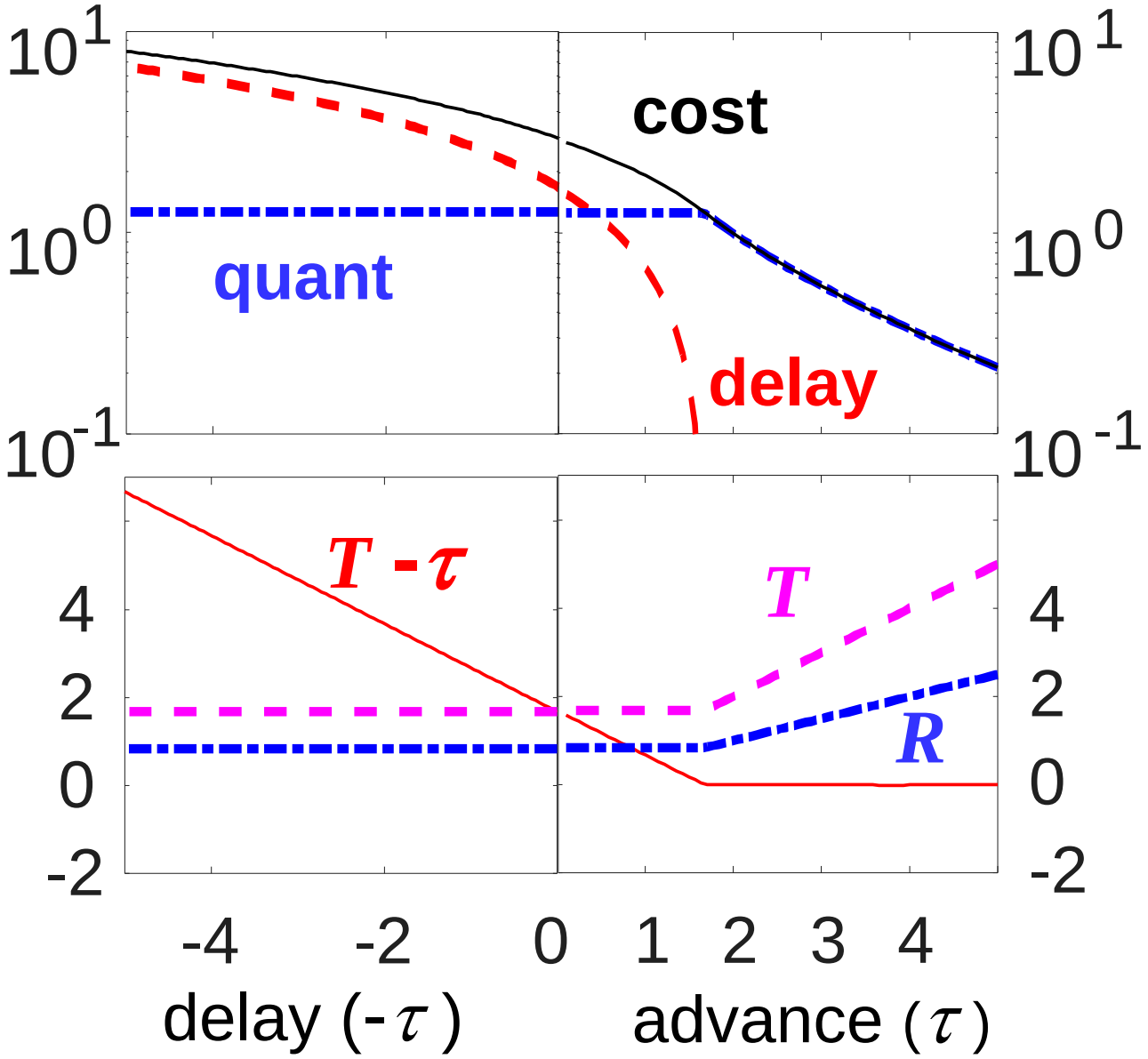
The rest of the system has advance (or delay) = τ

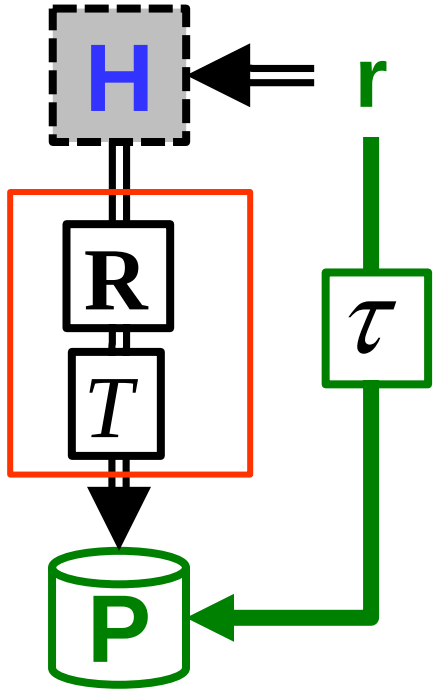
Y.N., N.M. and J.D (2015), Hard Limits on Robust Control over Delayed and Quantized Communication Channels with Applications to Sensorimotor Control.

$$R = \lambda T \rightarrow \underbrace{\max\left(T-\tau,0\right)}_{\text{delay}} + \underbrace{\left(2^{\lambda T}-1\right)^{-1}}_{\text{quant}}$$



delay (-τ)





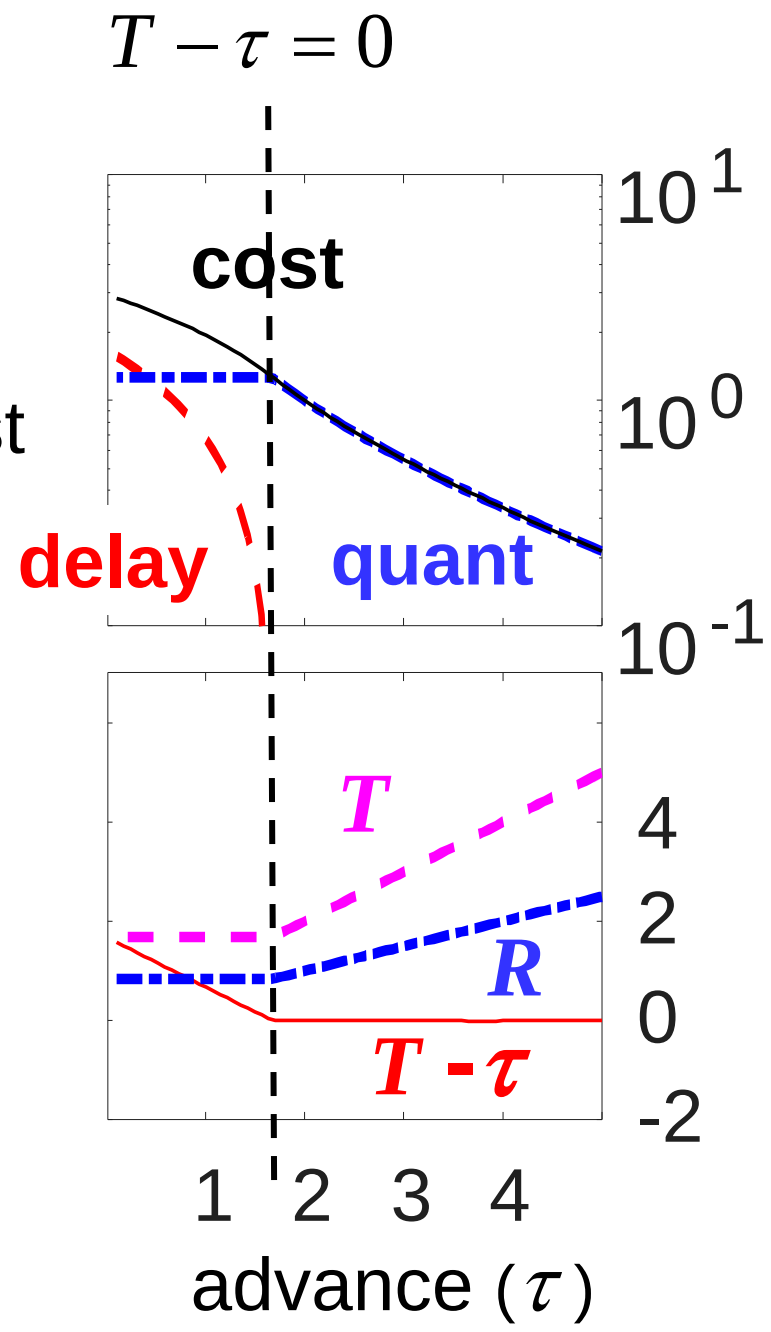
advance (τ)

$$\max(T-\tau,0)+\left(2^{\lambda T}-1\right)^{-1}$$

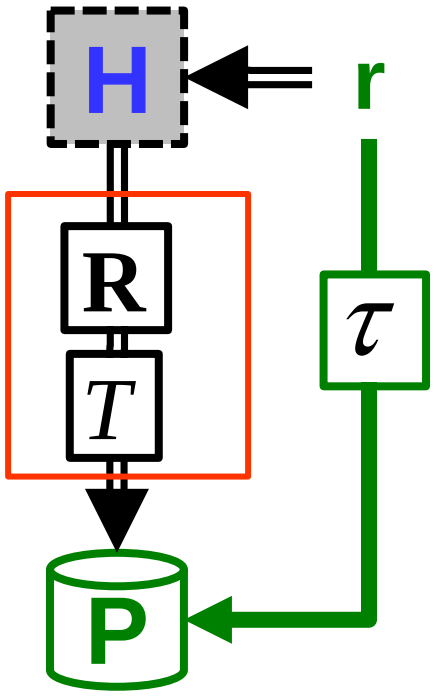
delay **quant**

Large **advance** → **quant** dominates cost and cost → 0

Bandwidth (R) and delay (T) increase

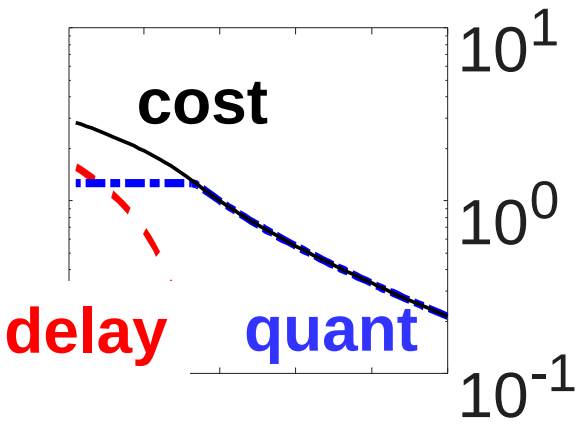


advance



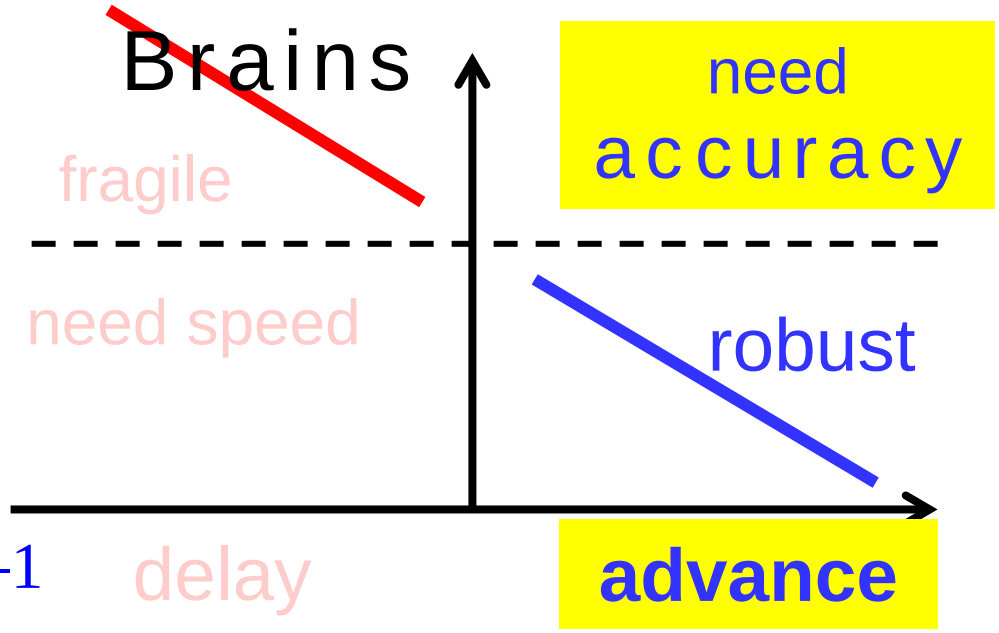
advance (τ)

Large advance → **quant** dominates cost and cost → 0

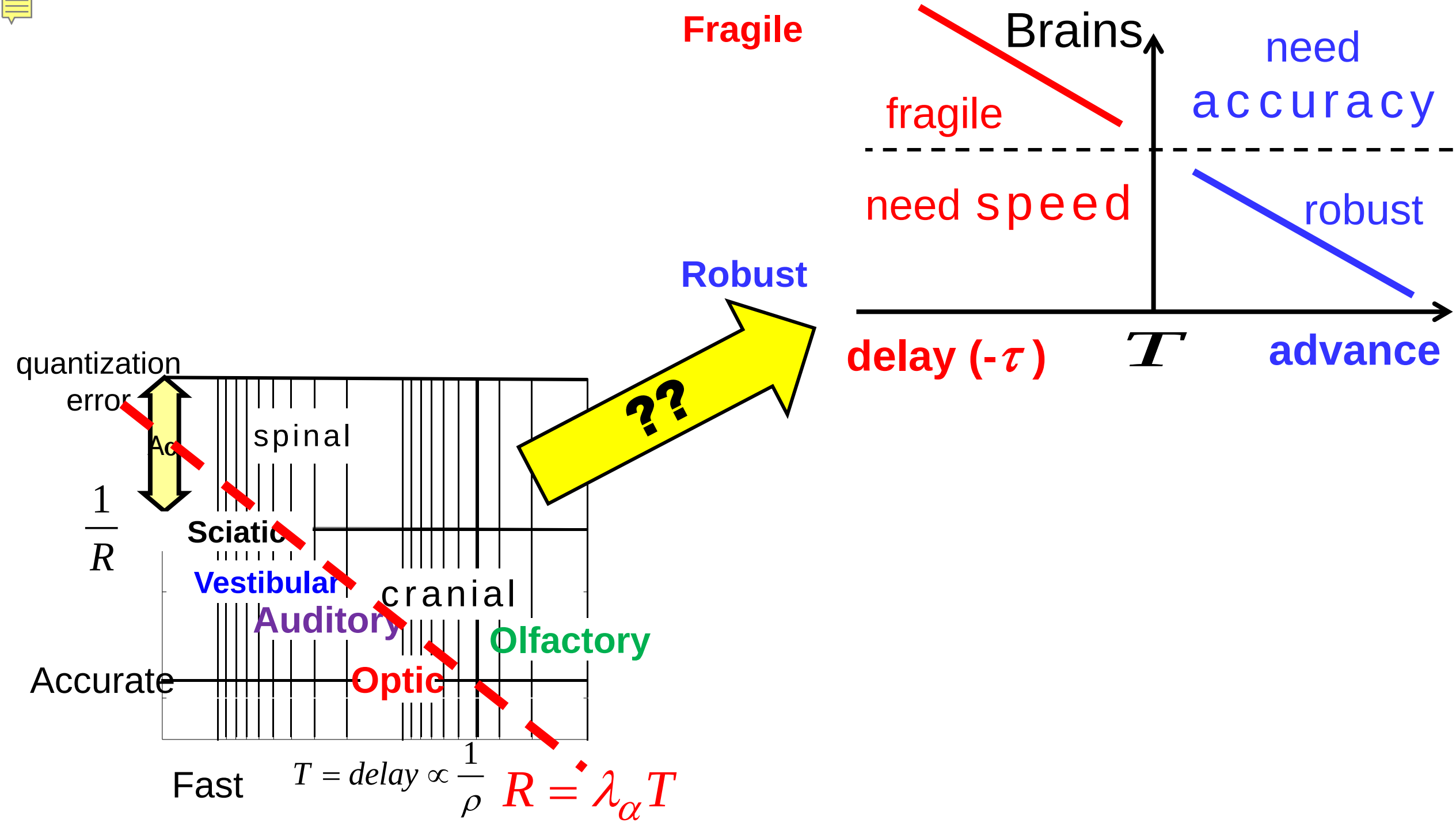


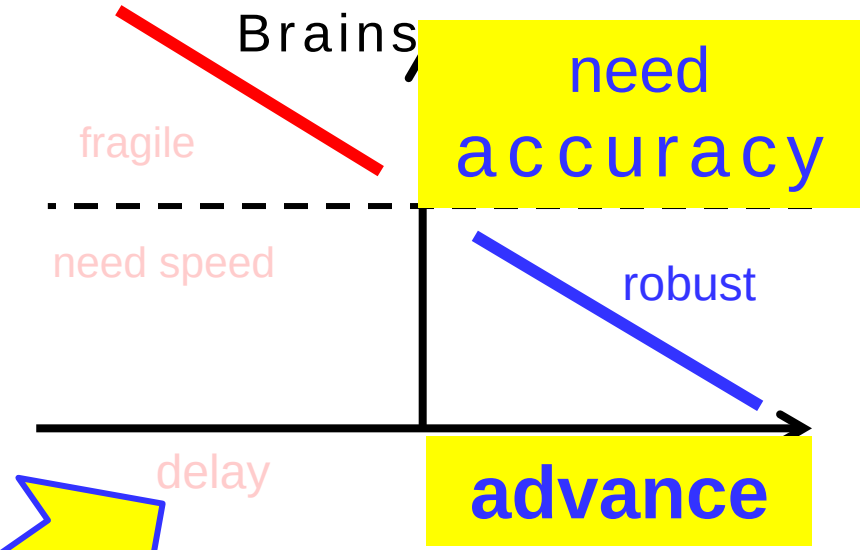
$$\max(T-\tau,0)+\left(2^{\lambda T}-1\right)^{-1}$$

delay **quant**

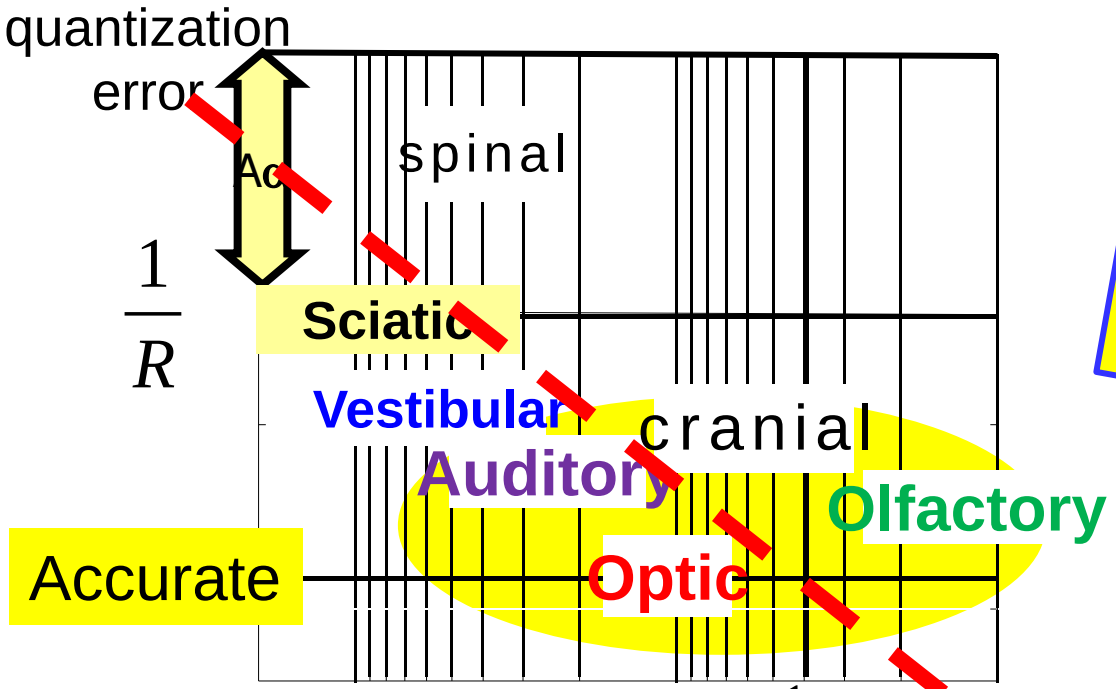


advance

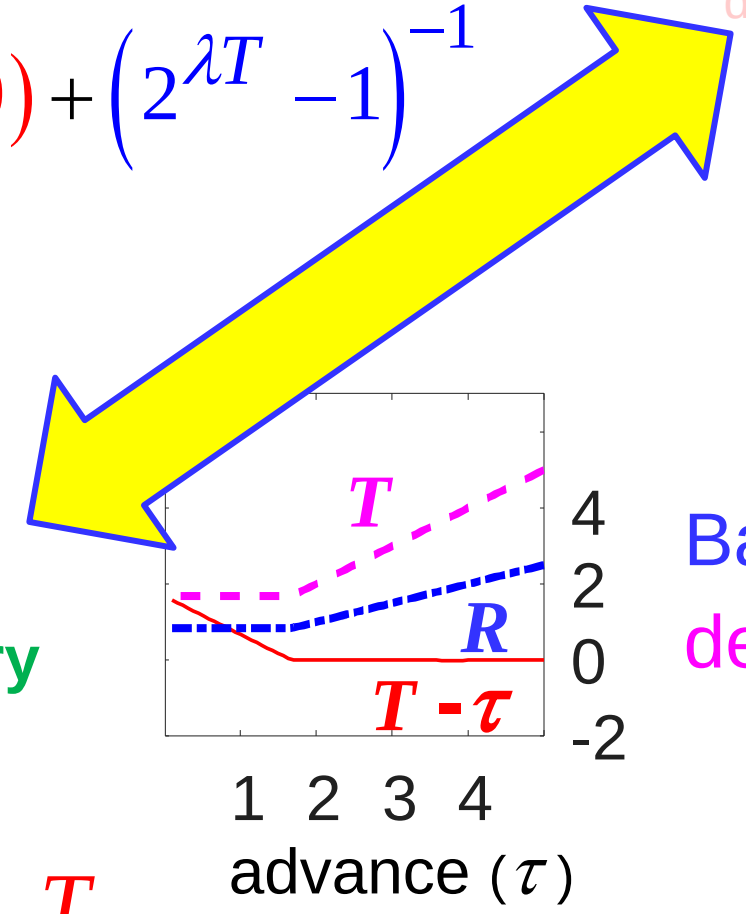

Fragile
Brains
need
accuracy
fragile
need speed
robust
Robust
delay (-τ)
T
advance
??
quantization
error
Ac
spinal
1/R
Sciatic
Vestibular
cranial
Auditory
Olfactory
Accurate
Optic
Fast
T = delay ∝ 1/ρ
R ≐ λ_α T



$$R = \lambda T \rightarrow \max\left(T - \tau, 0\right) + \left(2^{\lambda T} - 1\right)^{-1}$$



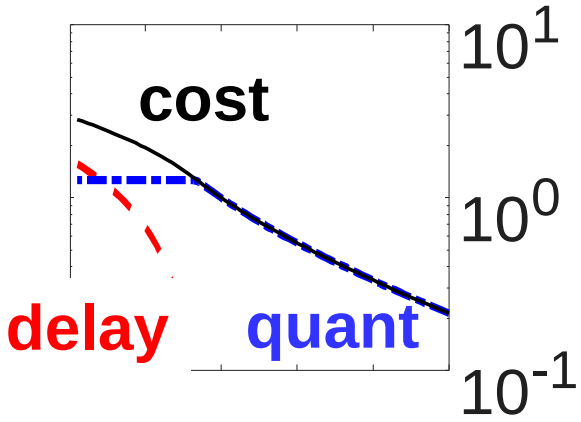
Fast $\quad T = delay \propto \frac{1}{\rho}$ $\quad R \doteq \lambda_\alpha T$

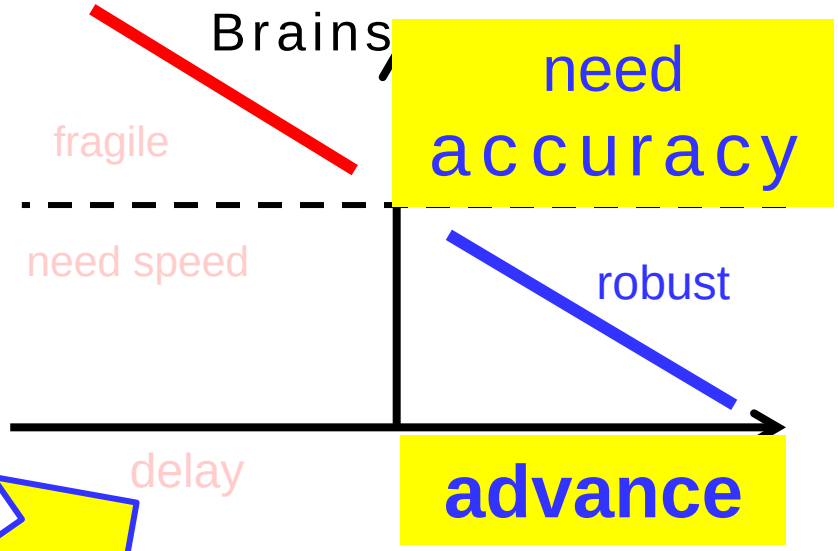


Bandwidth (R) and delay (T) increase

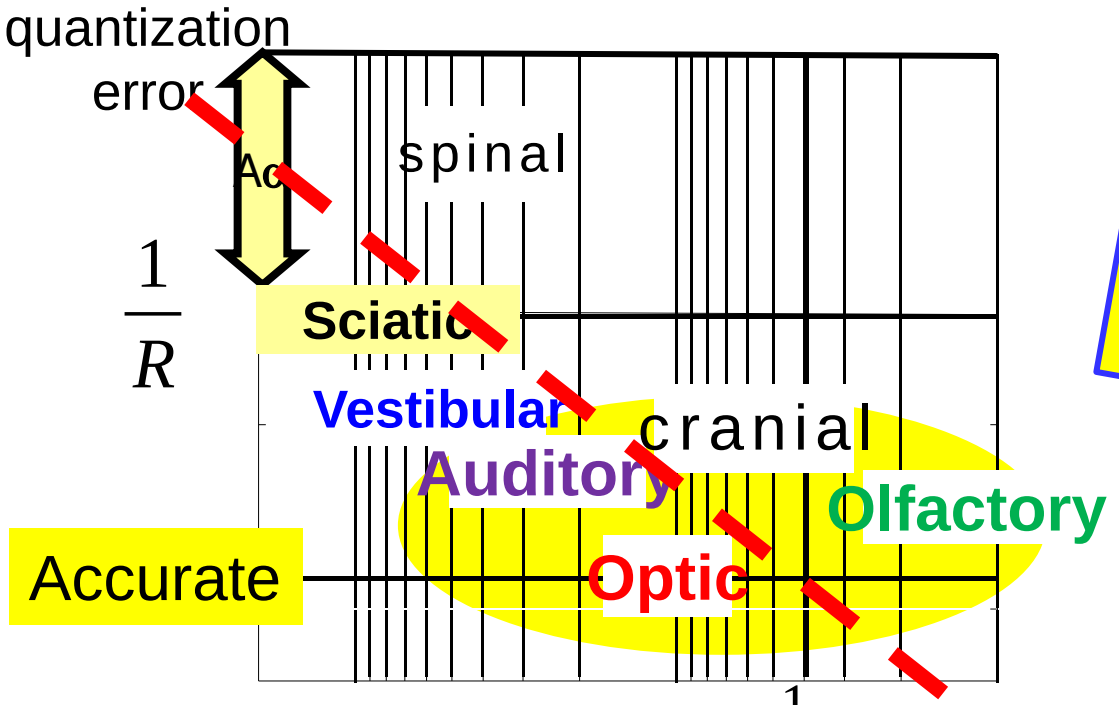
advance

Large advance →
quant dominates cost
and cost → 0



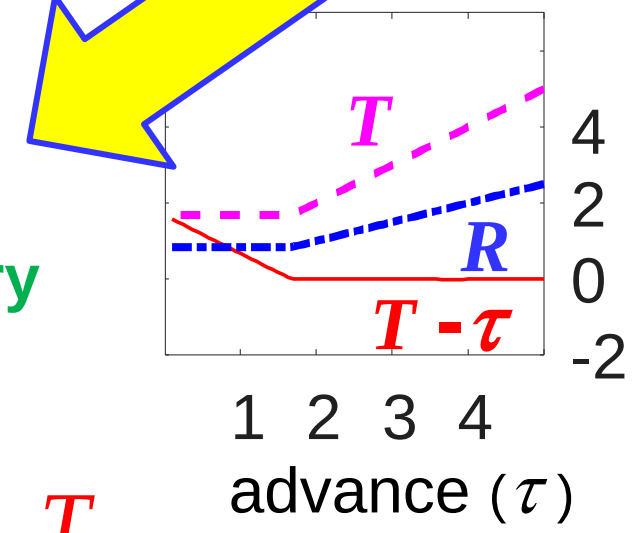


$$R = \lambda T \rightarrow \max(T-\tau, 0) + \left(2^{\lambda T} - 1\right)^{-1}$$



$$T = delay \propto \frac{1}{\rho}$$

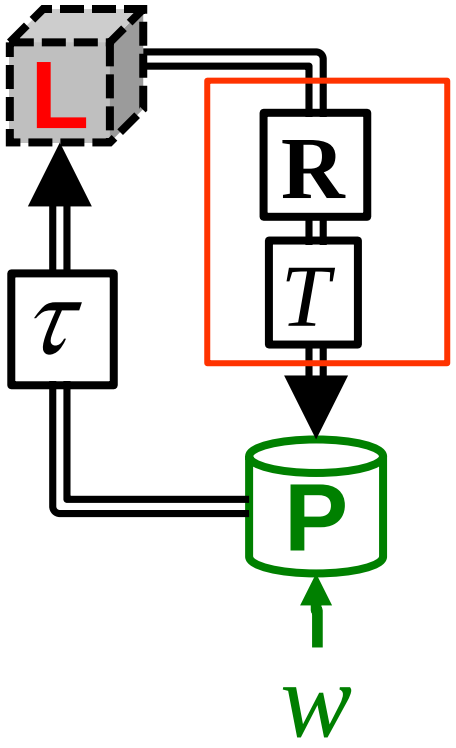
$$R \doteq \lambda_\alpha T$$



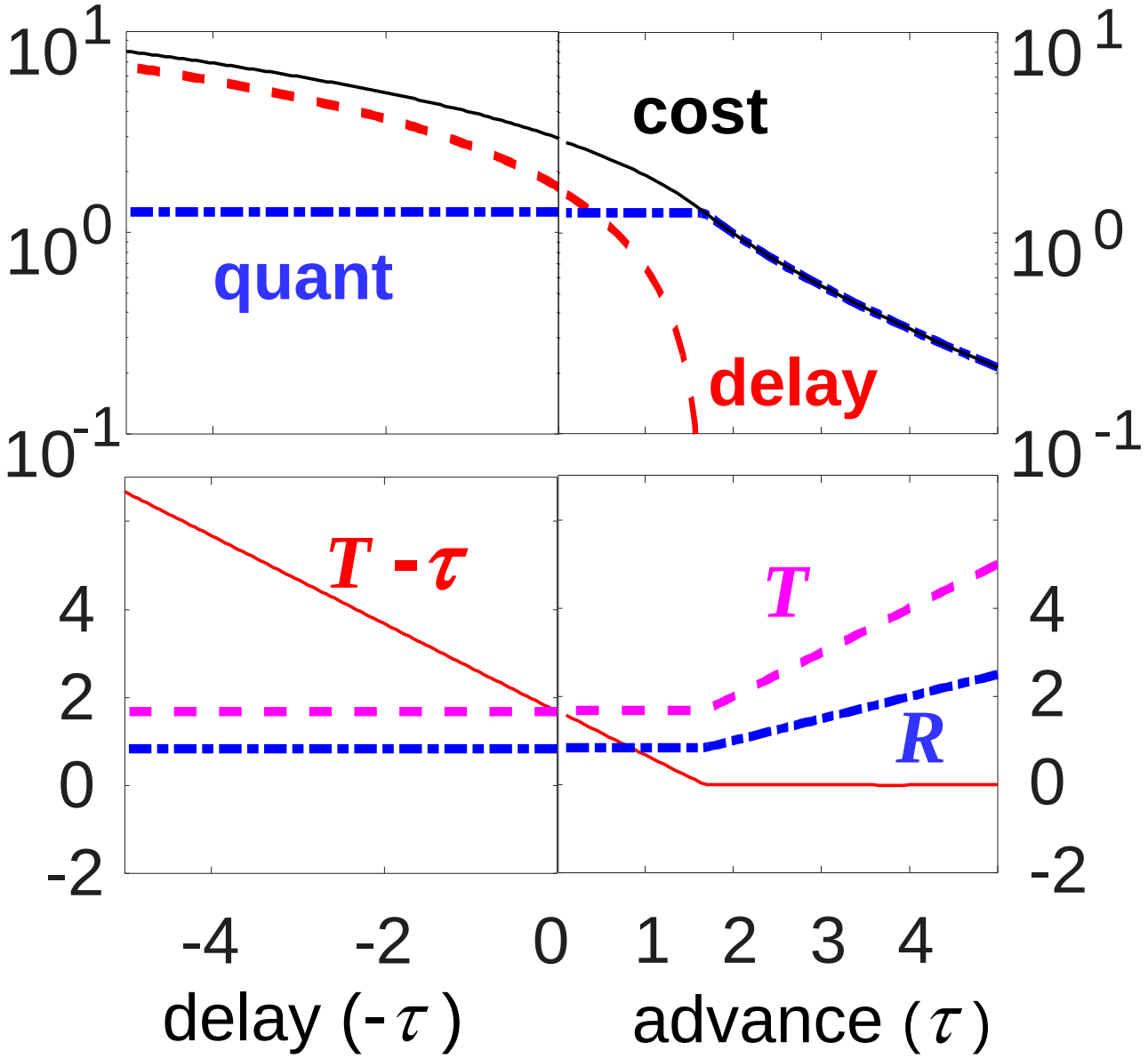
Bandwidth (R) and
delay (T) increase

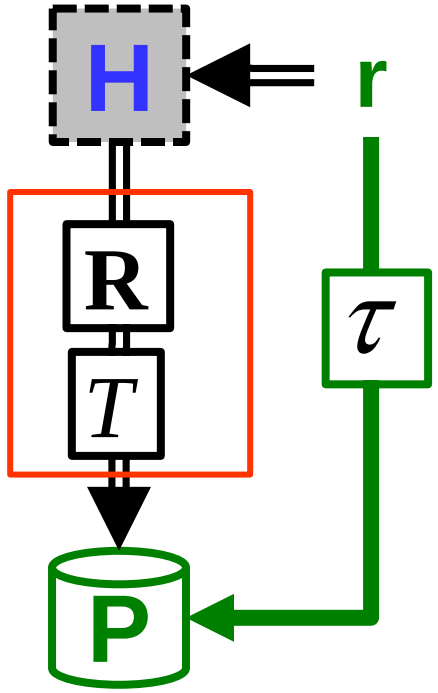
advance

$$R = \lambda T \rightarrow \underbrace{\max\left(T-\tau,0\right)}_{\text{delay}} + \underbrace{\left(2^{\lambda T}-1\right)^{-1}}_{\text{quant}}$$



delay (-τ)

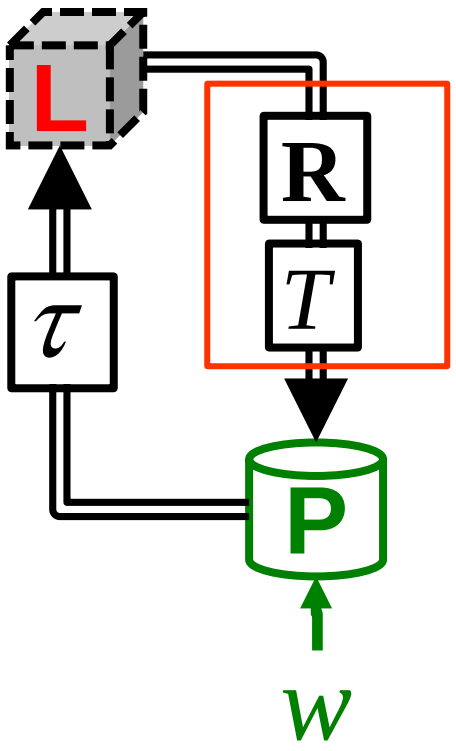




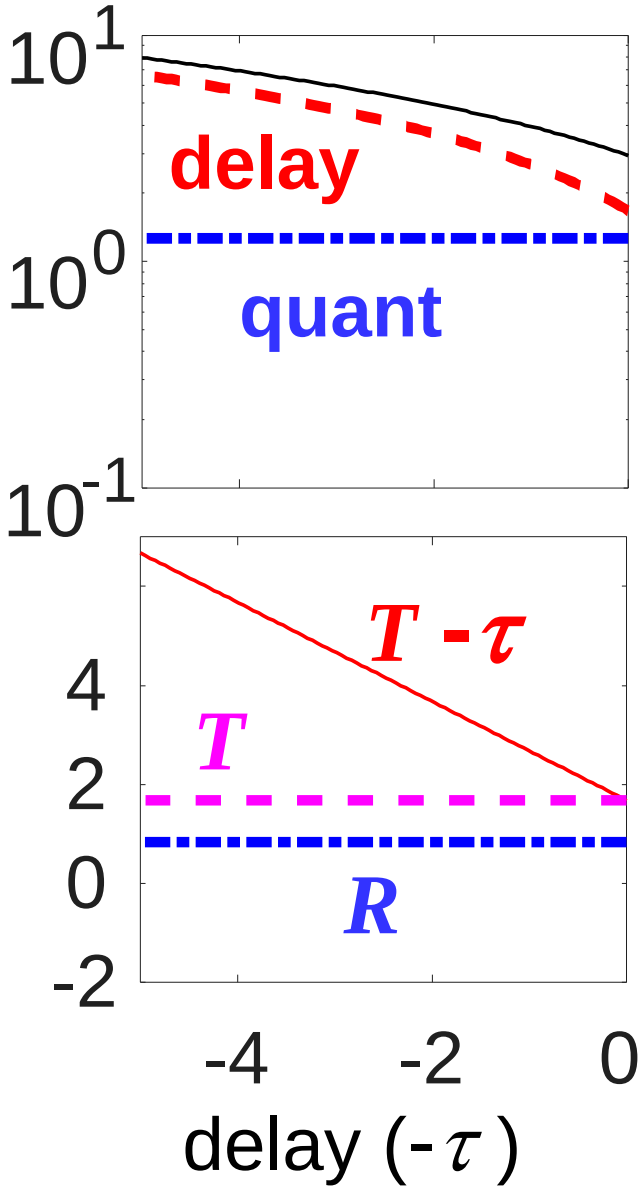
advance (τ)

delay (-τ)

$$R = \lambda T \rightarrow \underbrace{\max(T-\tau, 0)}_{\textbf{delay}} + \underbrace{\left(2^{\lambda T} - 1\right)^{-1}}_{\textbf{quant}}$$

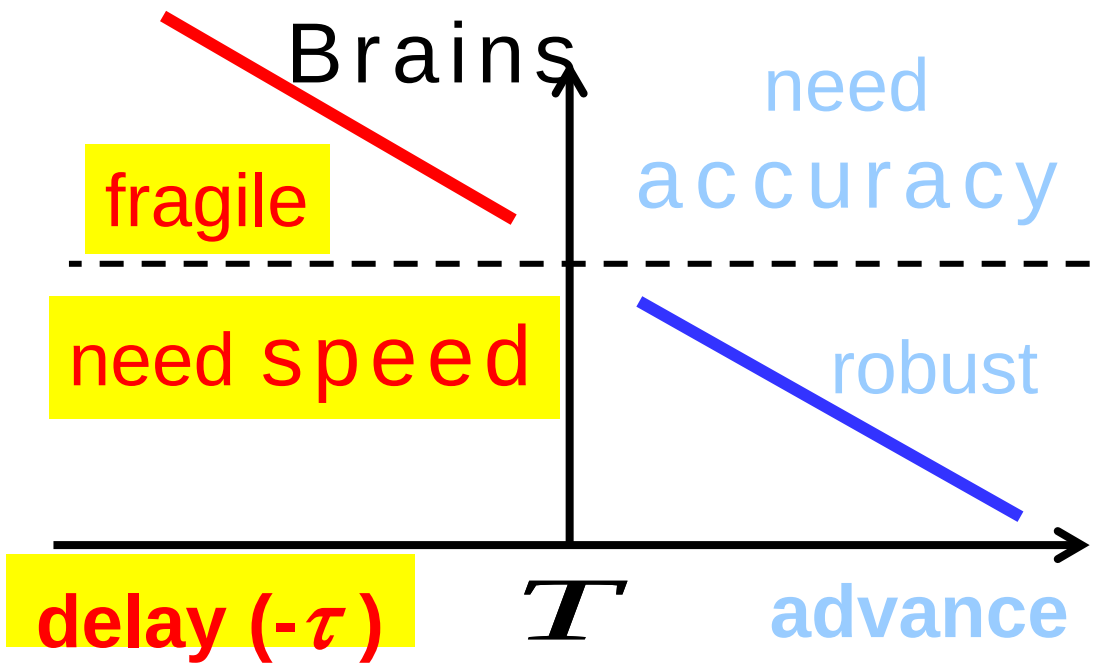


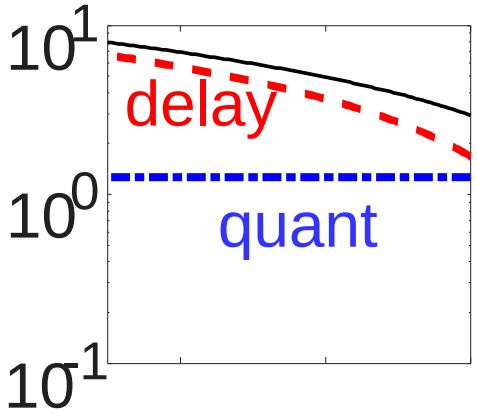
delay (-τ)



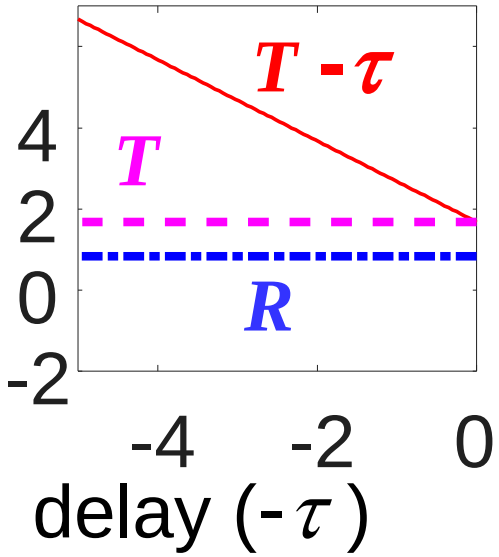
Large **delay** → **delay** dominates cost and cost → ∞

Bandwidth (R) and delay (T) low



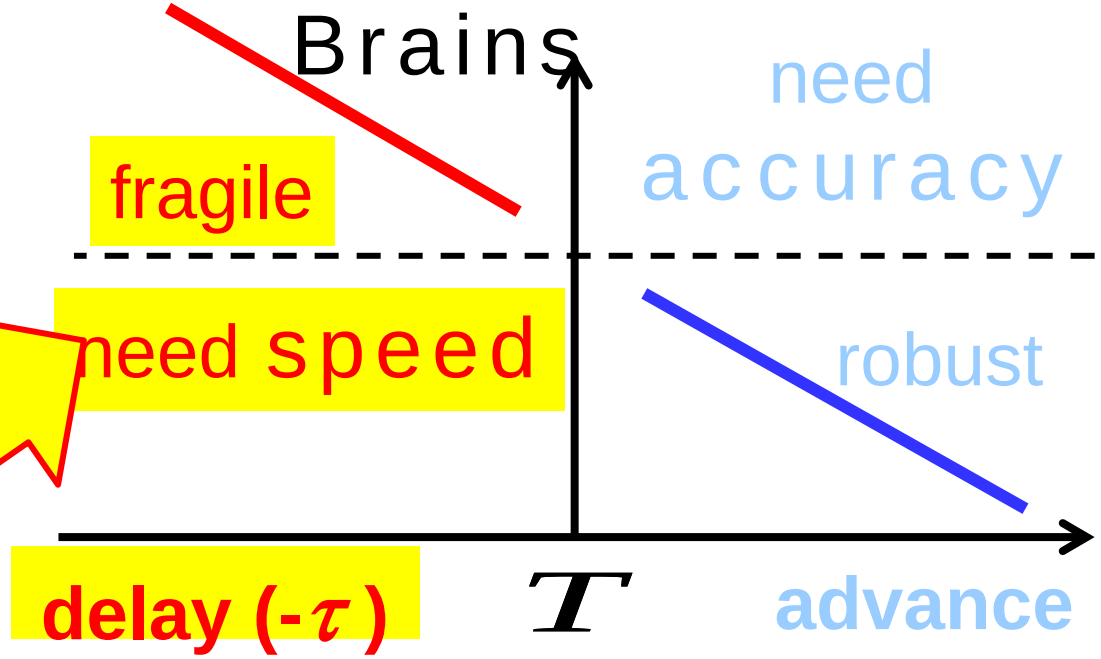


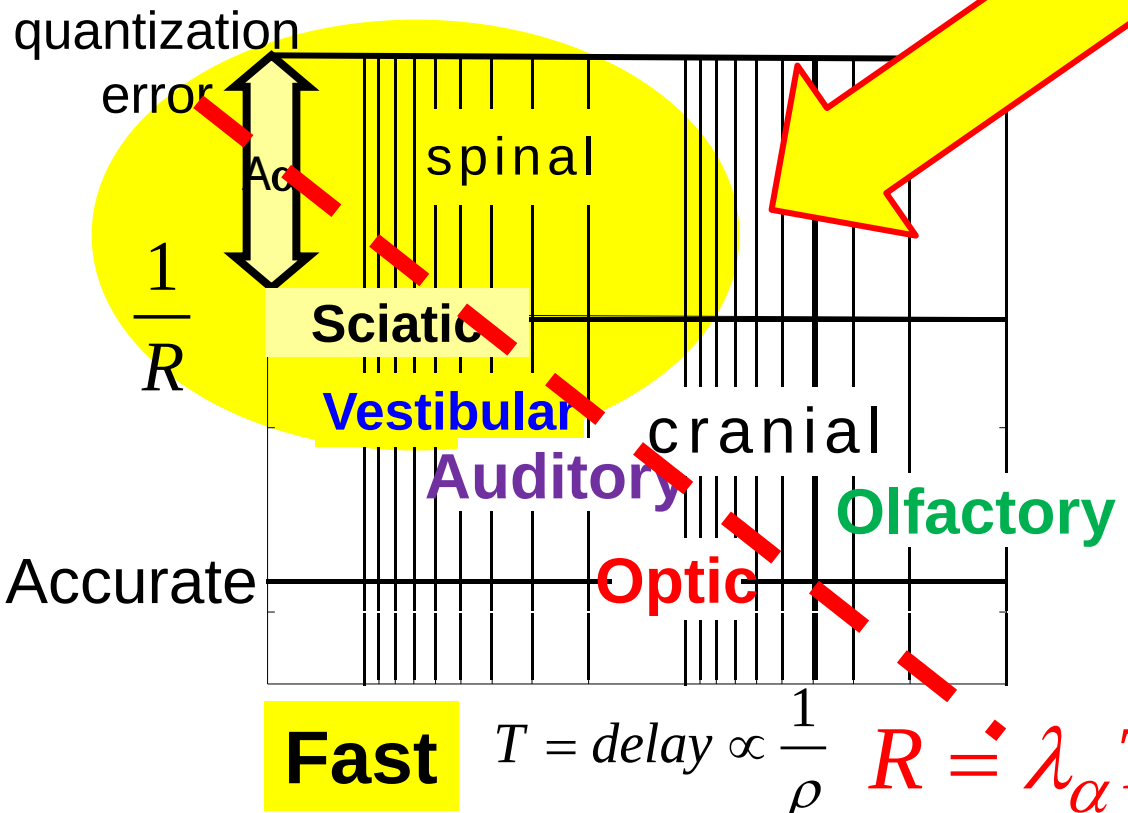
Large delay →
delay dominates cost
and cost → ∞



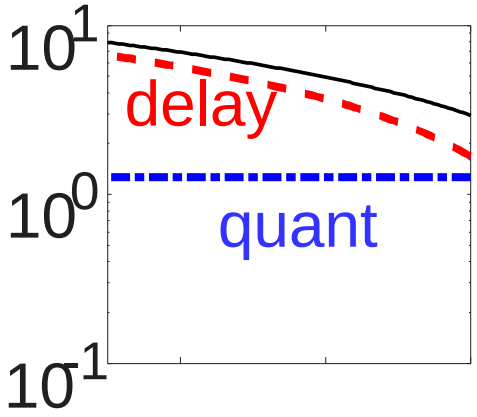
Bandwidth (R) and delay (T) low

delay (-τ)





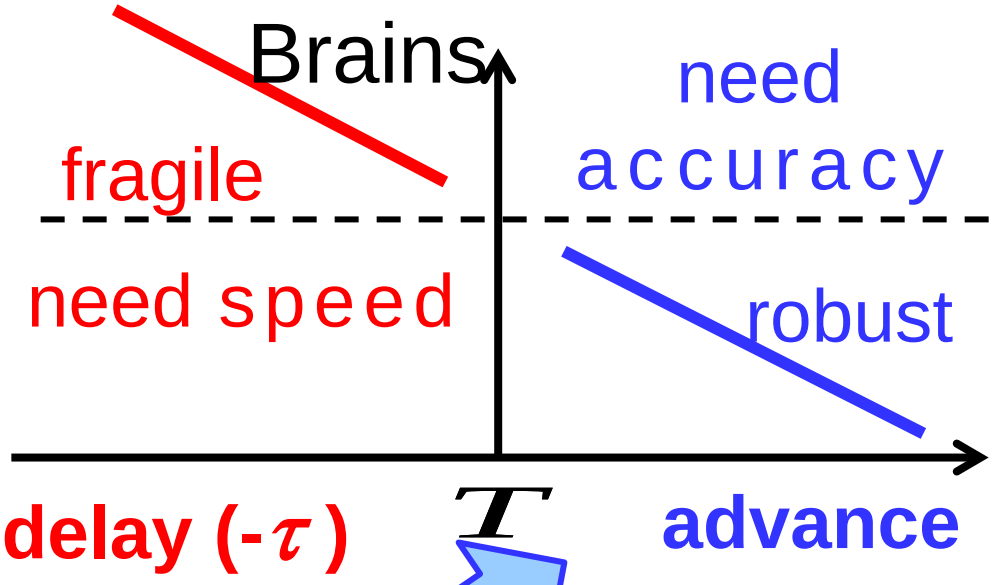
$T = delay \propto \frac{1}{\rho}$ $R \doteq \lambda_\alpha T$



Large delay → **delay** dominates cost and cost → ∞

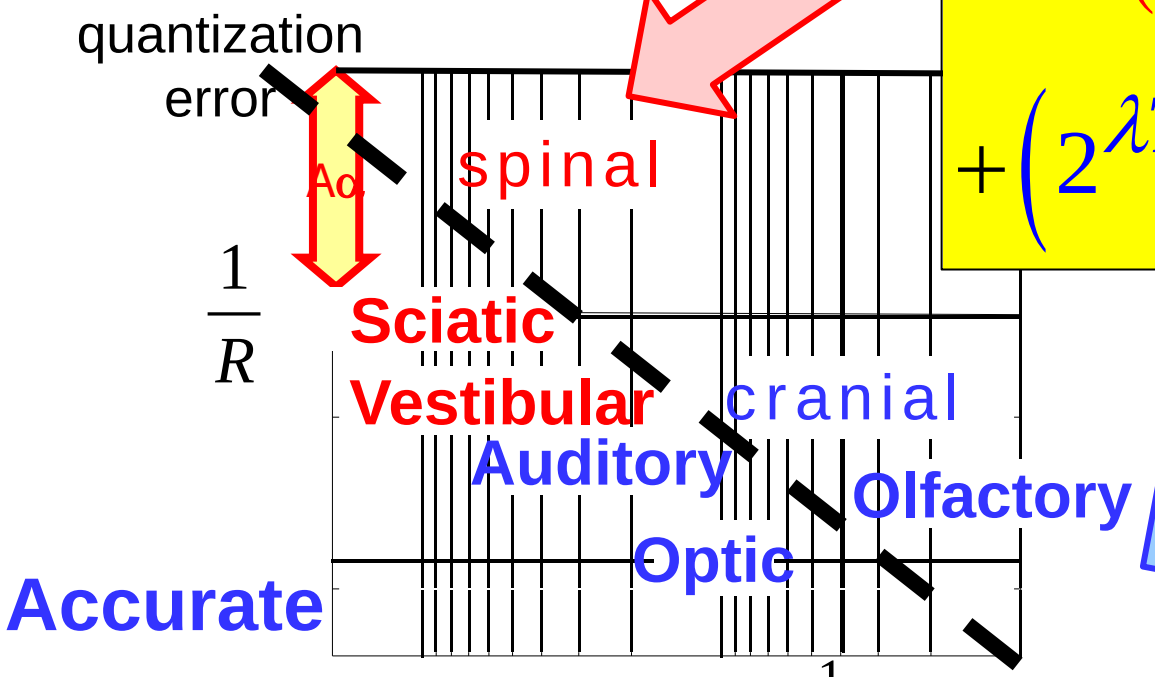
delay (-τ)

delay dominates
and cost $\rightarrow \infty$



Bandwidth (R) and
delay (T) low

$$\max(T-\tau,0) + \left(2^{\lambda T}-1\right)^{-1}$$



$$T = delay \propto \frac{1}{\rho}$$

$$\dot{R} = \lambda_\alpha T$$

quant dominates
and cost $\rightarrow 0$

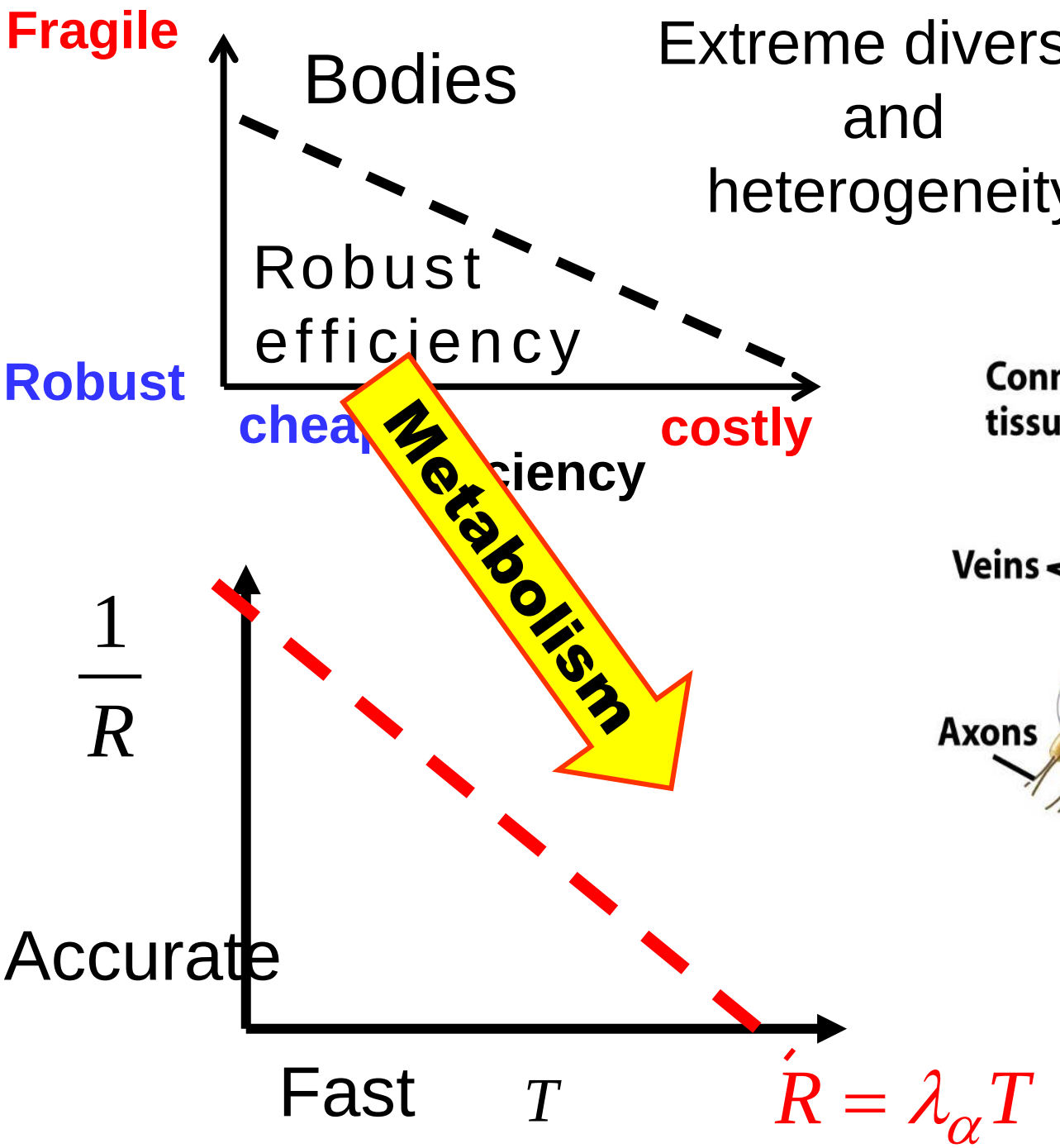
Bandwidth (R) and
delay (T) large

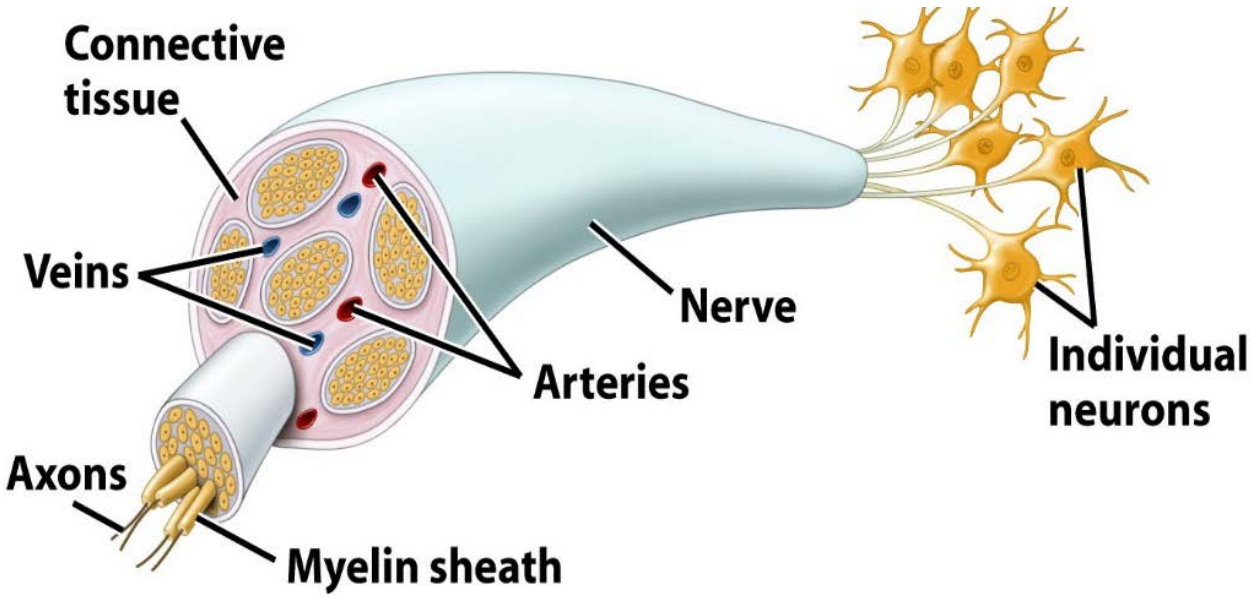
advance

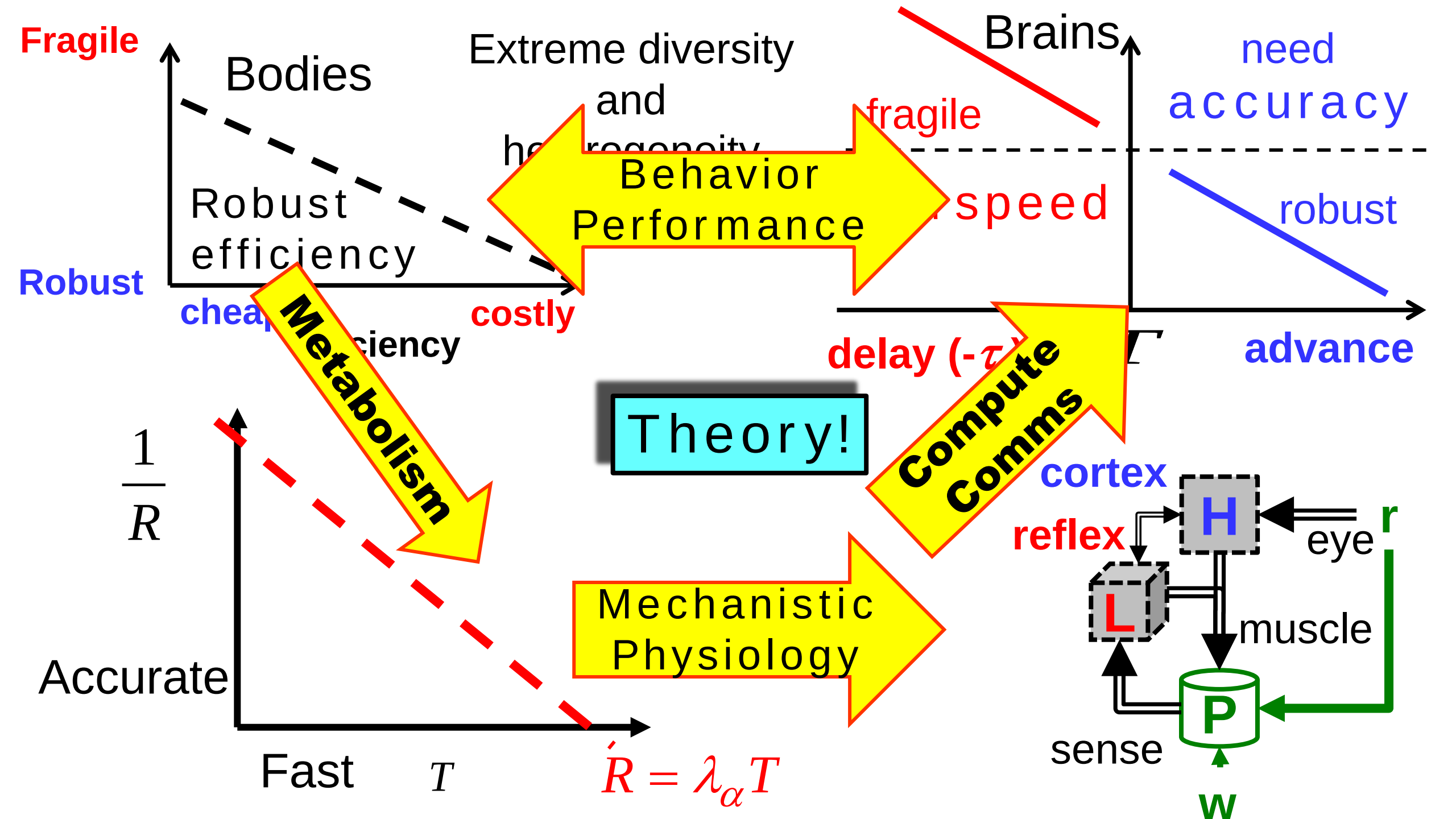
MIMO system extension

Y.N., S. You (2015), A linear programming framework for networked control system design, Necsys.

- MIMO system with
  - Uniform quantizers throughout
  - Potentially distributed control and information sharing
- Method
  - Use any existing (distributed) controller synthesis methods to design the controller
  - Solve a **linear program** to design the uniform quantizers
- Benefits
  - Achieve BIBO stability
  - Provide upper bound for the achievable cost

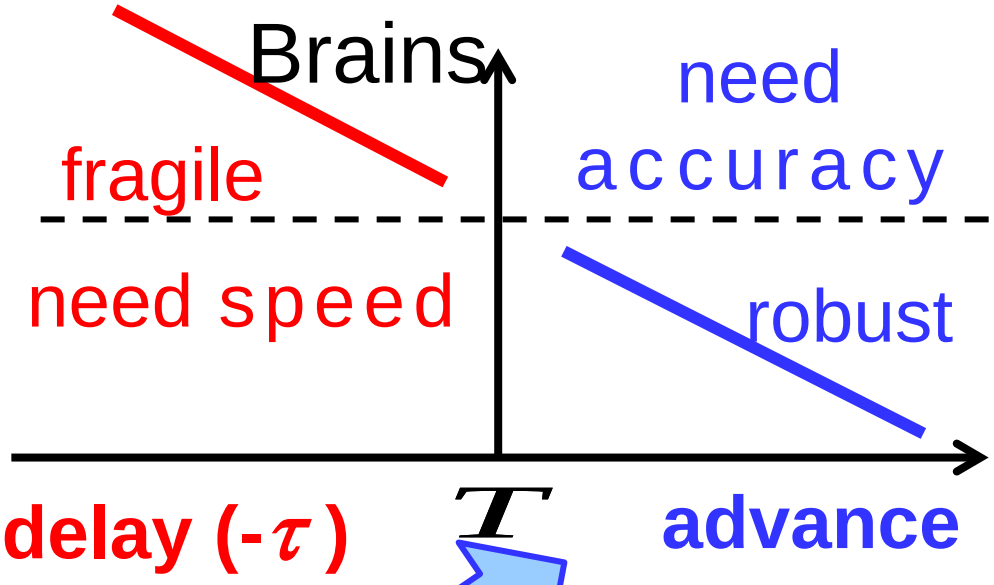

Fragile
Bodies
Extreme diversity
and
heterogeneity
Robust
efficiency
Robust
cheap
costly
ciency
Metabolism
$\frac{1}{R}$
Accurate
Fast
T
$\dot{R} = \lambda_\alpha T$


Connective tissue
Veins
Nerve
Arteries
Individual neurons
Axons
Myelin sheath


Fragile
Bodies
Extreme diversity
and
Robust
efficiency
Robust
costly
Behavior
Performance
Metabolism
Brains
fragile
need
accuracy
speed
robust
delay (-τ)
advance
Theory!
Compute
Comms
1/R
Accurate
Fast
T
Mechanistic
Physiology
cortex
reflex
H
L
P
eye
r
muscle
sense
w

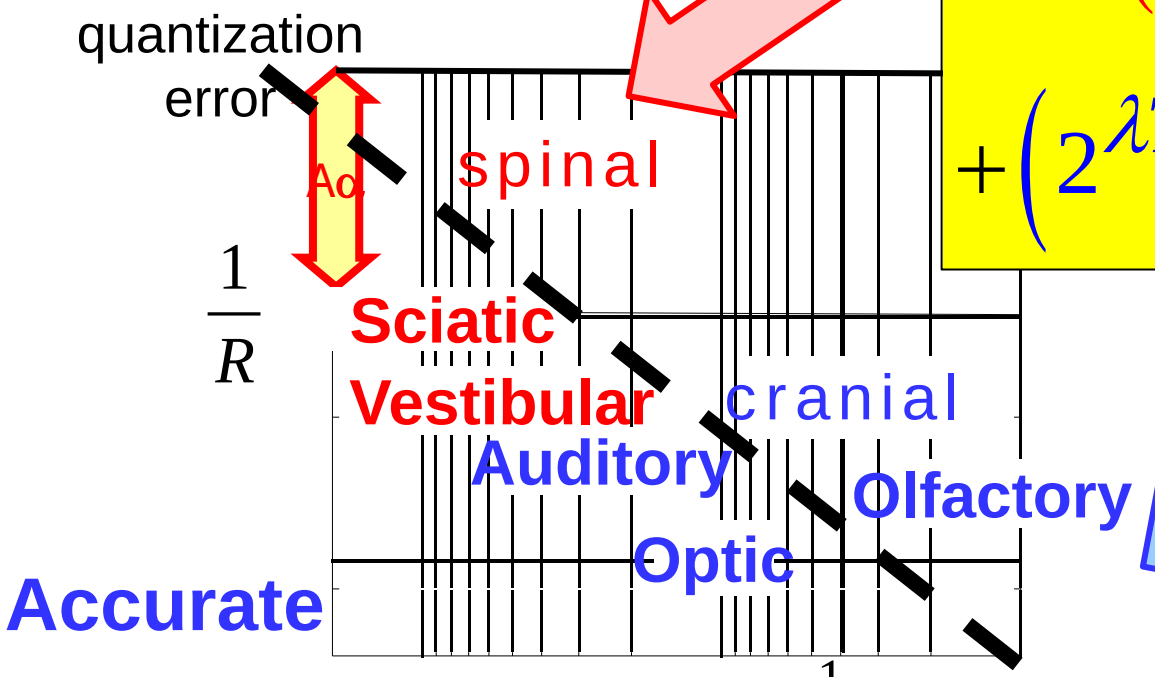
delay (-τ)

delay dominates
and cost $\rightarrow \infty$



Bandwidth (R) and
delay (T) low

$$\max(T-\tau,0) + \left(2^{\lambda T}-1\right)^{-1}$$



$$T = delay \propto \frac{1}{\rho}$$

$$\dot{R} = \lambda_\alpha T$$

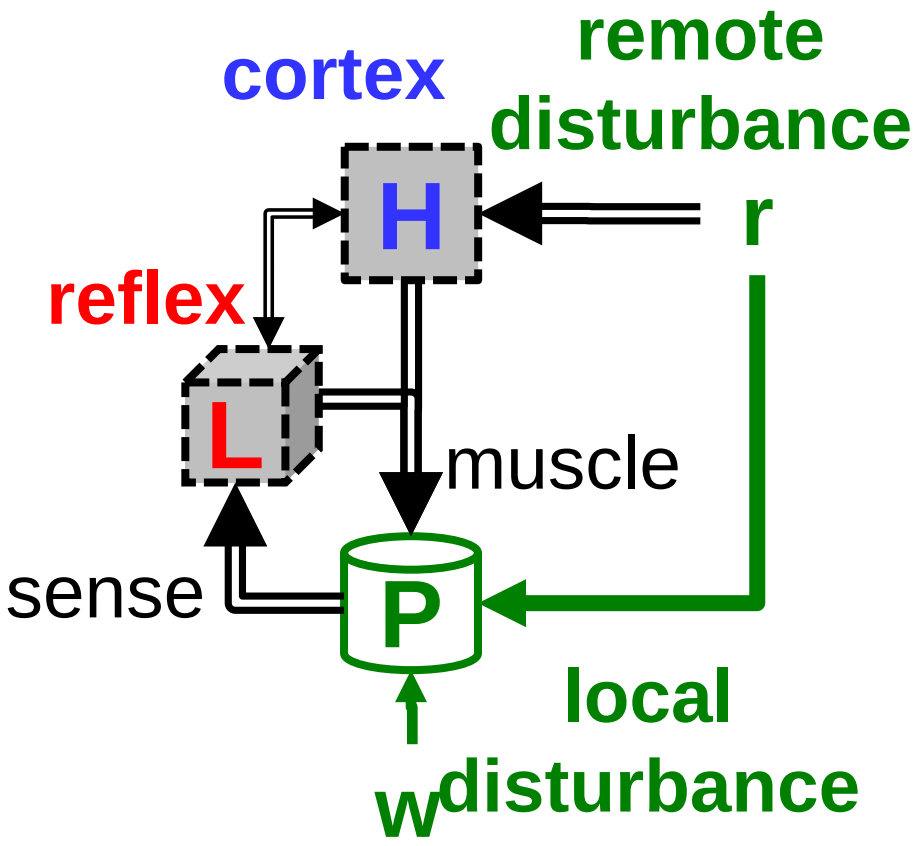
quant dominates
and cost $\rightarrow 0$

Bandwidth (R) and
delay (T) large

advance

Low layers
past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
~~unconscious~~
~~>>10Mb/sec~~
~~distributed~~
~~local~~

~~layered/recursive~~
dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$



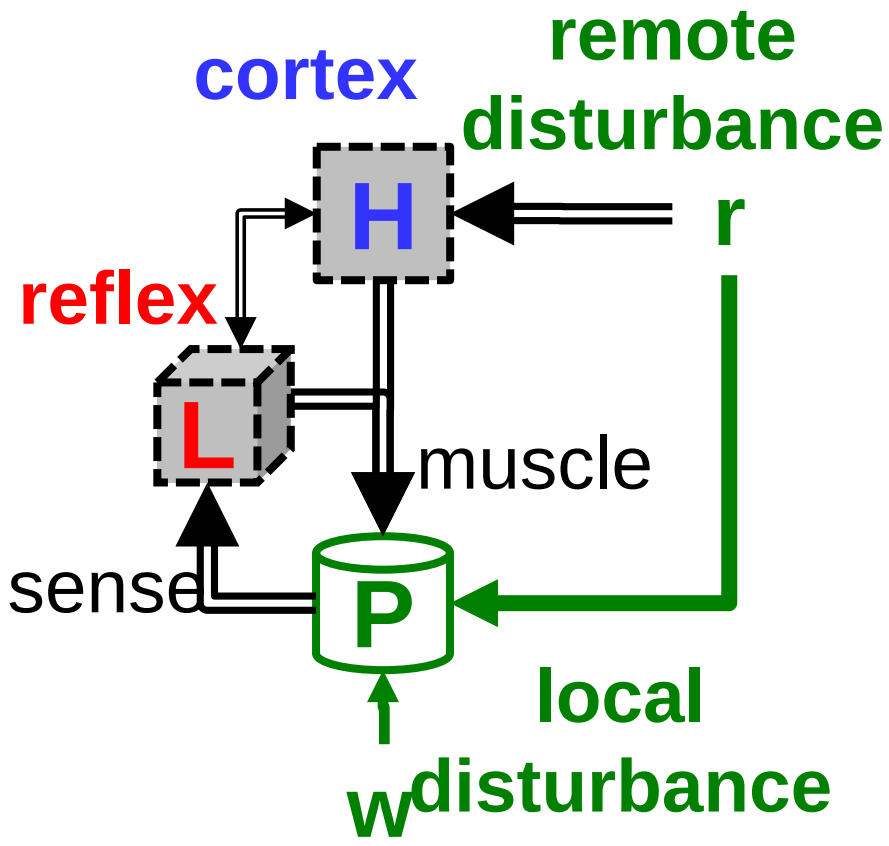
High layers
future
planning
advance warning
stable (virtual)
large ||r| *but*
small ||CL|| <<1
need accuracy
~~conscious~~
~~<100 bits/sec~~
centralized
global

Low layers

past
reflex
delayed
unstable (real)
small ||w|| *but*
large ||CL|| >1
need speed
unconscious
>>10Mb/sec
distributed
local

layered/recursive

dynamic/feedback
saturation
digital/quantized
worst case $||\bullet||_\infty$

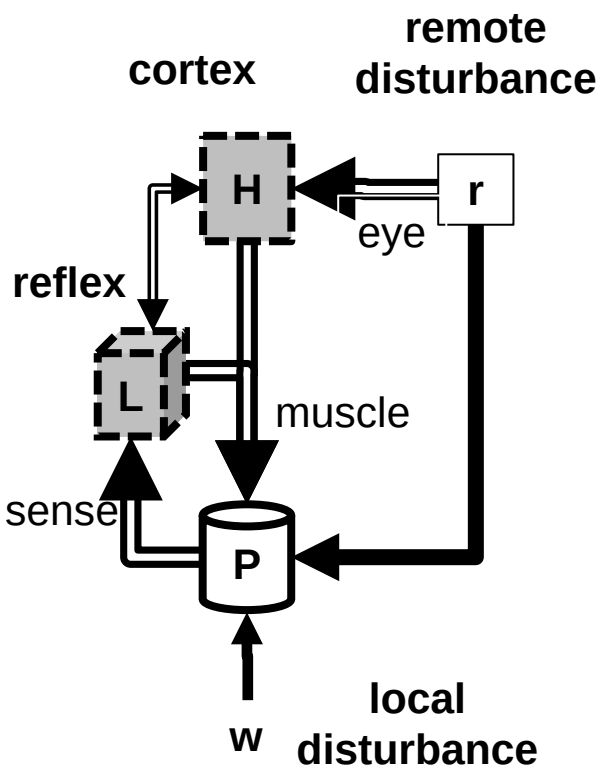


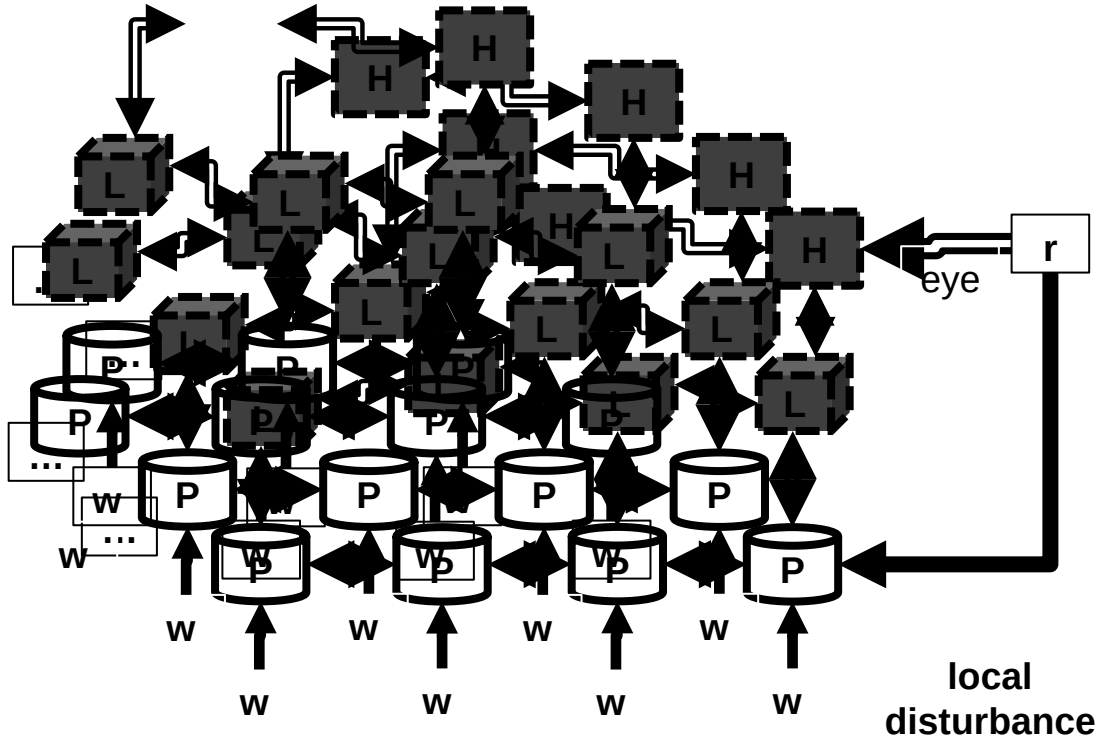
High layers

future
planning
advance warning
stable (virtual)
large ||r| *but*
small ||CL|| <<1
need accuracy
conscious
<100 bits/sec
centralized
global

Low layers	General	High layers
past	layered/recursive	future
reflex	dynamic/feedback	planning
delayed	saturation	advance warning
unstable (real)	digital/quantized	stable (virtual)
	worst case ||●||	
small ||w|| *but*	**scale and sparsity**	large ||r|| *but*
large ||CL|| >1	**(state, sense, act,**	small ||CL|| <<1
need speed	**compute, comms)**	need accuracy
distributed	**co-design (control, plant,**	centralized
local	**comms, sense, act)**	global
	other nonlinearities	
unconscious		conscious
>>10Mb/sec		<100 bits/sec

Nikolai Matni





Simple

Large-scale distributed heterogeneous

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- *A theory of dynamics, control, and optimization in layered architectures,* M. & Doyle submitted to ACC ’16

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- *Localized optimal control*, Wang, M., Doyle, many papers
  - LLQR: CDC ’14
  - LLQG, LLQR + Act. Regularization: submitted to ACC ’16

- Preprints available at **www.cds.caltech.edu/~nmatni** or by request